Harmonic elimination of three phase PWM DC-AC inverter using particle swarm optimization

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Abstract: The purpose of this paper is to improve the output quality of three phase PWM DC-AC inverter. The improvement in output quality is required to reduce the harmonic components. The PWM control can reduce harmonic components by adjusting the width of each pulse. We design the switching phase to improve output quality, also propose an evaluation function for evaluating the frequency components. In order to optimize the switching phase, particle swarm optimization is applied. We confirm the effectiveness of the proposed method comparing with other methods. Moreover, we confirm the effectiveness by using the implementation circuit of the three phase inverter.

Key Words: three phase, harmonic components, DC-AC inverter, PWM control, optimization, particle swarm optimization

1. Introduction
In electrical power systems of recent years, a lot of switching conversion technologies are used \cite{1, 2}. The PWM DC-AC inverter is one of such conversion technologies \cite{3}. PWM technique controls the width of output pulse. In general, the switching angles of the PWM DC-AC inverter are determined by the sinusoidal comparing method \cite{3}. The adequate determinations of pulse width to improve of the output quality of inverter are studied \cite{4}.

In Ref. \cite{4}, Rajaram et al. studied to improve the output quality of three phase PWM DC-AC inverter by using Firefly algorithm or Fireworks algorithm. The reduction of the harmonic components is required to improve the output quality. However, the procedure of Ref. \cite{4} is insufficient to reduce of harmonic components. The reason why the procedure cannot reduce harmonic components is the optimization method is possible to generate infeasible solutions. The initial generated solutions of Ref. \cite{4} satisfy the given constraint condition. However, according to the state update, the system
may generate the infeasible solution which does not satisfy the constraint condition. Therefore, the output of the procedure of Ref. [4] does not reduce the harmonic components sufficiently.

We have proposed the switching phase design method to improve the output quality of single phase PWM DC-AC inverter [5, 6]. In the proposed method, we applied particle swarm optimization (abbr. PSO) to optimize the switching phase. The proposed PSO can be satisfied the constraint condition to search best solution. Therefore, the proposed PSO is different from other methods and the PSO generates only feasible solutions. We defined the evaluation function for optimization. As a result, the proposed method achieved to design more effective switching phase than the conventional methods. Based on our design procedure for single phase PWM DC-AC inverter in Refs. [5, 6], we propose a switching phase design procedure for three phase PWM DC-AC inverter in this paper. For the design procedure of three phase inverter, we will extend the single phase design procedure. We confirm the effectiveness of the propose method by the numerical simulations and the laboratory measurements.

2. Three phase DC-AC inverter

A lot of three phase inverter circuits are used for the electric motor control to realize the energy saving. The objective three phase PWM inverter is composed by six switches as shown in Fig. 1. The output of the three phase PWM inverter is the line voltage $V_{u,v}$ as shown in Fig. 1. These six switches are operated periodically under a constrain condition. The control signal of each phase has $2\pi/3\omega$ [s] phase difference. Each phase of the three phase inverter corresponds to the single phase half-bridge inverter. Figure 2 shows the example of the single phase half-bridge inverter. The phase voltage is generated with the switching operation as shown in Fig. 2. Note that the output of the single phase half-bridge inverter corresponds to the phase voltage of the three phase PWM inverter.

The output waveform becomes a square waveform which contains only two level voltages as shown in Fig. 3. The output voltage $V_o(t)$ is described by Fourier series as follows:

$$V_o(t) = \sum_{n=-\infty}^{\infty} C_n(x)e^{jn\omega t},$$

where, $C_n(x)$ means Fourier coefficients, $x$ denotes a switching phase vector, and $\omega$ represents the

![Fig. 1. Example of the three phase PWM DC-AC inverter. The output is the line voltage $V_{u,v}$.](image)

![Fig. 2. Example of the basic operation of the single phase PWM DC-AC inverter. The output voltage $V_o$ corresponds to the phase voltage of the three phase inverter.](image)
fundamental angular frequency. For simplicity, we normalize the period as $2\pi$ without loss of generality. Because of the symmetric property of the output waveform, we pay attention to only a quarter of the period.

The three phase inverter outputs the line voltages. The line voltages are represented by difference of each phase voltage as follows

$$V_{u,v}(t) = V_u(t) - V_v(t),$$

(2)

$$V_{v,w}(t) = V_v(t) - V_w(t),$$

(3)

$$V_{w,u}(t) = V_w(t) - V_u(t),$$

(4)

where, $V_u(t)$, $V_v(t)$, and $V_w(t)$ is phase voltages of $u$-phase, $v$-phase, and $w$-phase, respectively. The phase voltages have $2\pi/3[s]$ phase difference in each other. Since the three phase must have a symmetrical property in each phase, each phase voltage has the same switching phase vector $\mathbf{x}$. Each phase voltage is expressed by Fourier series as follows

$$V_u(t) = \sum_{n=-\infty}^{\infty} C_n(\mathbf{x}) e^{jnt},$$

(5)

$$V_v(t) = V_u \left( t - \frac{2}{3} \pi \right) = \sum_{n=-\infty}^{\infty} C_n(\mathbf{x}) e^{jn(t - \frac{2}{3} \pi)},$$

(6)

$$V_w(t) = V_u \left( t - \frac{4}{3} \pi \right) = \sum_{n=-\infty}^{\infty} C_n(\mathbf{x}) e^{jn(t - \frac{4}{3} \pi)},$$

(7)

where, $C_n(\mathbf{x})$ means the $n$-th complex Fourier coefficient of Fourier series of $V_u(t)$ whose switching phase is expressed by the vector $\mathbf{x}$. Based on the time shifting property of Fourier series, the Fourier expressions of $V_v(t)$ and $V_w(t)$ can be expressed by the Fourier coefficients of $V_u(t)$ as above. The line voltage is derived by two any phase voltages as follows.

$$V_{u,v}(t) = V_u(t) - V_v(t)$$

$$= V_u(t) - V_u \left( t - \frac{2}{3} \pi \right)$$

$$= \sum_{n=-\infty}^{\infty} C_n(\mathbf{x}) e^{jnt} - \sum_{n=-\infty}^{\infty} C_n(\mathbf{x}) e^{jn(t - \frac{2}{3} \pi)}$$

$$= \sum_{n=-\infty}^{\infty} C_n(\mathbf{x})(1 - e^{-jn\frac{2}{3} \pi}) e^{jnt}$$

(8)

In Eq. (8), $C_n(\mathbf{x})(1 - e^{-jn\frac{2}{3} \pi})$ denotes the $n$-the Fourier coefficients. Equation (8) indicates that the $3m$-th coefficient becomes 0, therefore, the $3m$-th harmonic order is not necessary to consider. In other words, the line voltage output of the three phase inverter does not contain the $3m$-th order harmonic components. The output of PWM inverter is a kind of rectangle waveforms, therefore, the
output does not contain even order harmonic components. Please note that the actual output of the DC-AC inverter is passed through a LPF.

3. Evaluation function

Various methods have been proposed to optimize the switching phase of the inverter [4, 7, 8]. In Ref. [4], they consider the two evaluation values; one index becomes 0 when the effective value of the fundamental component is close to the desired objective value. The other index denotes the sum of components in the elimination band. The evaluation function of Ref. [4] is the linear combination of these two indexes. In Ref. [7], Kato and Iwamoto evaluate the filter characteristics under specified conditions. In Ref. [8], they proposed the method to evaluate weighted harmonic components. These methods are interesting, however, the quality of the output is insufficient. Therefore, we consider a novel evaluation function to improve the quality of the output waveform.

The decrease of the harmonic components is effective to improve the output quality. In order to optimize the switching phase of the DC-AC inverter, total harmonic distortion (abbr. THD) is applied to the evaluation function [9, 10]. The THD is derived from the ratio of the harmonic components to the fundamental component. The small THD value indicates that the output wave contains a few harmonic components. The THD is calculated as follows.

\[
THD(x) = \sqrt{\sum_{n=2}^{\infty} \frac{|C_n(x)|^2}{|C_1(x)|}},
\]

(9)

where, \( n \) is the order of the harmonics.

The \( THD(x) \) in Eq. (9) is referred the infinity degree. However, the actual output of the DC-AC inverter is passed through a LPF. Namely, the high frequency harmonic components are eliminated by the LPF. The scope of the harmonics of the output waveform is finite. Therefore, we propose the following evaluation function.

\[
F_t(x) = \sqrt{\sum_{n=2}^{K} \frac{|C_n(x)|^2}{|C_1(x)|}},
\]

(10)

where, \( K \) denotes harmonic order to evaluate. If this value becomes zero, it means that the output is composed of only the fundamental component within the desired frequency range.

Since \( F_t(x) \) evaluates only the ratio of harmonic components, the energy of the output waveform may be reduced. Therefore, we have to evaluate the energy of the output waveform. The evaluation function of the energy is the following.

\[
F_p(x) = \left| 1 - \frac{2 |C_1(x)|}{P_d V_{dc}} \right|,
\]

(11)

where, \( P_d \) denotes a desired amplitude value which is a positive arbitrary constant less than 1. Also, \( P_d V_{dc} \) means a desired amplitude value which corresponds to an effective value. If this function becomes zero, the effective value of the output is close to the desired effective value \( P_d V_{dc} \). The quality of the output waveform can be evaluated by the \( F_t(x) \) and \( F_p(x) \).

This optimization problem has above two evaluation functions, \( F_t(x) \) and \( F_p(x) \). In this paper, we apply the combination of \( F_t(x) \) and \( F_p(x) \) as represented in Eq. (12).

\[
F(x) = F_t(x) + F_p(x),
\]

(12)

The optimum switching phase corresponds to the smallest value of the objective function.

4. Optimization based on PSO

In order to search the optimum switching phase, we use PSO [11, 12].

Each particle of PSO has the location information which gives the best evaluation value before visited locations. The best location information is called \( p_{best} \). Each particle shares \( p_{best} \) information
in the swarm, and selects the best \( \text{pbest} \) in the swarm. The best information in \( \text{pbest} \) is called \( \text{gbest} \).

The dynamics of the PSO is described as

\[
v_{ij}^{t+1} = w v_{ij}^{t} + c_1 r_1 (p_{best}^{t} - x_{ij}^{t}) + c_2 r_2 (g_{best}^{t} - x_{ij}^{t}),
\]

(13)

\[
x_{ij}^{t+1} = x_{ij}^{t} + v_{ij}^{t+1},
\]

(14)

where, \( x_{ij}^{t} \) denotes the location of the \( j \)-th particle on the \( t \)-th iteration. \( v_{ij}^{t} \) denotes the velocity of the \( j \)-th particle on the \( t \)-th iteration. \( w \) denotes an inertia weight coefficient. \( c_1 \) and \( c_2 \) denote acceleration coefficients. \( r_1 \) and \( r_2 \) denote the uniform random numbers from 0 to 1.

The PSO searches the optimum solution of Eq. (12). The location \( x_{ij}^{t} \) represents the \( i \)-th dimensional value on the \( j \)-th particle. Since each dimension of the location corresponds to each switching phase of the PWM, each dimension must satisfy the following constraint condition.

\[
0 \leq x_{1j}^{t} \leq x_{2j}^{t} \leq \cdots \leq x_{kj}^{t} \leq \pi/2,
\]

(15)

where \( k \) denotes the maximum number of the switching times in the quarter period.

The above constraint condition can be implemented on PSO easier than other meta-heuristics [5, 6]. Therefore, the PSO generates only feasible solutions. The asynchronous update manner is applied for the PSO [13]. In the case of the real switching element, a response time is required. Therefore the following conditions are applied.

\[
x_{ij}^{t} = \begin{cases} 
  x_{i-1,j}^{t} + T_{\text{limit}}, & \text{for } x_{ij}^{t} \leq x_{i-1,j}^{t} \\
  x_{i+1,j}^{t} - T_{\text{limit}}, & \text{for } x_{ij}^{t} \geq x_{i+1,j}^{t} \\
  T_{\text{limit}}, & \text{for } x_{ij}^{t} \leq 0 \\
  \pi/2 - T_{\text{limit}}, & \text{for } x_{kj}^{t} \geq \pi/2 \\
  x_{ij}^{t}, & \text{otherwise}
\end{cases}
\]

(16)

where, \( T_{\text{limit}} \) is a constant which is depended on the response time of the switching element. The time is related to the rise and fall time of the switch.

5. Experiments

We confirm the effectiveness of the proposed method by the numerical experiments. First, we compare the performance of our proposed method to the other proposed methods. In Ref. [4], Rajaram and Palanisamy proposed two methods based on Firefly algorithm and Firework algorithm. These methods determine the switching phase of the three-phase PWM DC-AC inverter to eliminate the desired harmonic components. In order to compare the performance, we simulate the same conditions in Ref. [4]. Table I shows the experimental parameters. The desire elimination harmonic components are 5-th, 7-th, 11-th and 13-th. Our numerical simulation results and the results of Ref. [4] are shown in Table II. The frequency elements in each technique are shown in Table II. Table III shows the switching phase obtained by our proposed method. In this experiment, the desired amplitude value of the output is set as \( P_d = 0.6 \). Because this value is applied in Ref. [4]. The results as shown in Table II indicate that Firefly algorithm and Firework algorithm does not eliminate the desired harmonic components sufficiently. On the other hand, our proposed method by the PSO can eliminate the desired harmonic components completely. The results of Table II also indicate that our proposed method keeps the amount of the fundamental component better than these methods. The reason is Firefly and Firework algorithms generate infeasible solutions. On the other hand, our proposed PSO can generate only deasible solutions. Therefore, the constraint condition (15) is important.

Next, we confirm the performance of our proposed method by using an implementation circuit. In our implementation circuit, the fundamental frequency sets as 50[Hz]. Figure 4 represents our implementation circuit. In this circuit, FET is used as the switching element. The switching is controlled with PIC-16F88 with the built-in oscillator of 8[MHz]. The switching phase is calculated in advance by our proposed method. The obtained switching phase is shown in Table III. The control signal uses these switching phases. The response time of the switching element is measured. Based
Table I. Experimental parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The trial number</td>
<td>50</td>
</tr>
<tr>
<td>The maximum iteration number</td>
<td>5000</td>
</tr>
<tr>
<td>The number of particles</td>
<td>10</td>
</tr>
<tr>
<td>The inertia weight coefficient $w$</td>
<td>0.729</td>
</tr>
<tr>
<td>The acceleration coefficient $c_1, c_2$</td>
<td>1.49</td>
</tr>
<tr>
<td>Criterion time $T_{\text{limit}}$</td>
<td>$\pi/1000$</td>
</tr>
<tr>
<td>The desired amplitude voltage rate $P_d$</td>
<td>0.6</td>
</tr>
<tr>
<td>The number of switching (a quarter period) $k$</td>
<td>5</td>
</tr>
<tr>
<td>The evaluation range $K$</td>
<td>14</td>
</tr>
</tbody>
</table>

Table II. Experimental results.

<table>
<thead>
<tr>
<th>Harmonic Component</th>
<th>Proposed method</th>
<th>Firefly</th>
<th>Fireworks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental component</td>
<td>0.600</td>
<td>0.594</td>
<td>0.601</td>
</tr>
<tr>
<td>Fifth harmonic</td>
<td>0.000</td>
<td>1.160</td>
<td>1.100</td>
</tr>
<tr>
<td>Seventh harmonic</td>
<td>0.000</td>
<td>1.030</td>
<td>0.900</td>
</tr>
<tr>
<td>Eleventh harmonic</td>
<td>0.000</td>
<td>0.920</td>
<td>0.850</td>
</tr>
<tr>
<td>Thirteenth harmonic</td>
<td>0.000</td>
<td>1.030</td>
<td>1.010</td>
</tr>
</tbody>
</table>

Table III. Obtained switching phase vector by our proposed method.

<table>
<thead>
<tr>
<th>Switching phase $x$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.081678</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.295301</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.785961</td>
</tr>
<tr>
<td>$x_4$</td>
<td>0.956779</td>
</tr>
<tr>
<td>$x_5$</td>
<td>1.476782</td>
</tr>
</tbody>
</table>

Fig. 4. Experiment circuit configuration.

The trial number is 50, the maximum iteration number is 5000, the number of particles is 10, the inertia weight coefficient $w$ is 0.729, the acceleration coefficient $c_1, c_2$ is 1.49, the criterion time $T_{\text{limit}}$ is $\pi/1000$, the desired amplitude voltage rate $P_d$ is 0.6, the number of switching (a quarter period) $k$ is 5, and the evaluation range $K$ is 14.

We can say that these results have the same characteristic. Therefore, we confirm the effectiveness of the proposed method by the implementation circuit.

on the measurement value, we decide the parameter $T_{\text{limit}}$ as $\pi/1000$. The input DC voltage is set as $V_{dc} = 5$[V]. The target amplitude of the output voltage is set as 3. The desire elimination harmonic components are 5-th, 7-th, 11-th and 13-th.

Figure 5 shows the phase voltage of the experimental result. Figure 6 represents the corresponding numerical simulation result of Fig. 5. In these figures, the vertical axis denotes the phase voltage and the horizontal axis denotes time. Note that each output of three-phase inverter has a symmetric characteristic, and the output of the three phase inverter is the line voltage.

We can say that these results have the same characteristic. Therefore, we confirm the effectiveness of the proposed method by the implementation circuit.
Fig. 5. The phase voltage of the implementation circuit in the case of the input voltage $V_{dc}$ is 5.

Fig. 6. The phase voltage of the numerical simulation.

6. Conclusion
We proposed the determination method of the switching phase of the three phase PWM DC-AC inverter to improve the efficiency. We compared the performance of our proposed method to the other methods. As the results, we confirmed our proposed method has excellent performance. The effectiveness of the obtained switching phase was confirmed by the implemented circuit experiment.

In future problem, we will evaluate the conversion efficiency of the three phase PWM DC-AC inverter.

References


