Comparison of maxillary and mandibular dental arch forms by studying Fourier series developed from mathematically estimated dentitions

By

Hiroko MIKAMI, Michiko NAKATSUKA and Yasutomo IWAI

Department of Oral Anatomy, Osaka Dental University, Osaka, Japan

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Summary: We conducted a Fourier analysis on data obtained using correlation and principal component analyses of parallel-standardized dental study models; both maxillary and mandibular dental arches were predominantly round square in shape. The present study compared and determined the contribution ratio and reproducible coefficients of amplitudes (factors affecting dental arch forms), and demonstrated that the 1st to 4th and the 1st to 6th Fourier harmonics reproduced maxillary and mandibular dental arch forms, respectively. The correlation analyses of the constant term and amplitudes demonstrated that significant differences in the 2nd harmonic amplitude was strongly correlated with the curvature of anterior teeth and the length-to-width ratio in maxillary dentitions. By comparison of the constant term and amplitudes between different arch types, we did not observe significant differences in the constant term and the 1st amplitude of maxillary dentitions and in constant term and all amplitudes of mandibular dentitions. Nevertheless, the study revealed high contribution ratios of the 1st (in mandibular dentitions) and the 2nd (in maxillary dentitions) amplitudes essentially affecting the reproducibility of arch forms. The 1st amplitudes demonstrated a bow-like arrangement of all arch types, while the 2nd amplitudes adjusted the anterior-teeth curvature and in particular demonstrated the length-to-width ratio of maxillary dentitions. The 3rd and the 4th amplitudes were also determinants of the anterior-teeth curvature of maxillary dentitions. The 6th amplitude was necessary for reproduction, but showed no difference between varying mandibular dental arch types. Collectively, we conclude that the establishment of a Fourier series significantly reproduced maxillary but not mandibular dental arch forms.

Introduction

For clinical purposes, dental arch forms are expediently classified as square, round square, round and round V-shaped according to Thompson's descriptive morphological study¹⁾. The widely used conventional classification depends mainly on subjective visual examinations that define the dental arch as upsilon, elliptique, parabolique and hyperbolique^{2, 3)}. Some objective methods using mathematical evaluation and quantitative analyses were developed to classify the dental arch forms by translating the measuring data of various morphological elements recorded on plaster casts into algebraical functions^{4–19)}. A latter study has also verified hyperbolic-, parabolic- and elliptic-shaped dental arches by a conventional quadratic equation analysis²⁰⁾. Another study further superimposed measurements developed from reference points designated on dental arches with catenary,

conic section, elliptic function and fourth polynomial arches, and suggested that the fourth polynomial formula is mostly acceptable for analysing dental arch forms²¹⁾. Various functions observed by the above methods could be used for mathematical explanations, it is difficult to apply the same functions for the purposes of comparing distinguishing characteristics of different dental arch types.

Morphological studies by using Fourier analysis have been proposed as being effective for statistical evaluation of the mandibular fossa, tooth, inferior border of mandible, the dental arch, positional relationships between the palatine and dental arch and the pathway of masticatory movements^{22–35)}. The present study was undertaken to extract the essential factors that affect dental arch forms by using Fourier arithmetical (geometrical) analysis of the data obtained in our previous studies concerning maxillary and mandibular dentitions^{36, 37)}. The Fourier

series, reproducing coefficients for statistical analysis by intersection with certain harmonics, definitely aims at the estimation and comparison of the numerical information representing arch sizes and forms of maxillary and mandibular dental arch forms.

Materials and Methods

1. Materials

The study was performed following the Declaration of Helsinki Ethical Principles for Medical Research (1964). The protocol (No.05507: Morphological Analysis of dental arches) was approved by the Ethic Review Board of Osaka Dental University. In the study, 396 sets of paired maxillary and mandibular dental casts of young adult students (18 to 26 years old; male: 257, female: 139) at Osaka Dental University were prepared to have a total height of 60 mm with the occlusal plane adjusted to be at a height of 30 mm from the mandibular cast base^{36, 37)}. We selected 62 (male: 36, female: 26) maxillary and 53 (male: 29, female: 24) mandibular arches from dental plaster casts showing normal dentition and occlusion, and the dentitions of the standardised specimens were preliminarily classified as square, round square, round and round V-shaped arches^{1-3, 36, 37)}.

2. Methods

Designation of references and lines for examination

We designated the midpoints of the incisor edge ($I1_R$, $I1_L$, $I2_R$ & $I2_L$), the summits of the cuspids (C_R & C_L), the buccal cusps of premolars ($P1_R$, $P1_L$, $P2_R$ & $P2_L$) and the mesial buccal cusps of the first and second molars ($M1_R$, $M1_L$, $M2_R$ & $M2_L$) to be the main reference points of the maxillary and mandibular dental arches. The midpoint of line ($I1_R$ – $I1_L$) was defined as reference point A. From A, let a vertical line to intersect line ($M2_R$ – $M2_L$; original line) at reference point B. Then let line (A–B) to intersect line (C_R – C_L) at reference point E. Point O (origin, pole) was the reference midpoint at the original line ($M2_R$ – $M2_L$), and was designated to study the relationships between the right and the left radius vectors $r\theta n$ ($n = 1, \dots, 7$)_{R&L} (θ = angles between line (O-reference points) and the original line ($M2_R$ – $M2_L$)) by developing a spherical (polar) coordinate system^{27, 36, 37)} of the maxillary and mandibular dentitions (Fig. 1).

Photography of the maxillary and mandibular dental casts

A Nikon AD100 digital camera fitted with an AF MICRO Nikkor 70–180 mm, 1:4.5–5.6 D lens (Nikon, Tokyo, Japan) was set up on a King L4 Copy Stand (Asanuma, Tokyo, Japan). The axis of the lens was aligned vertically to the occlusal plane (standard plane) of mandibular casts (focusing distance = 70 mm, $f = 29$), and the distance between the lens and standard plane was

Fig. 1. Reference points, lines, and angles.

A: the midpoint of line ($I1_R$ – $I1_L$), B: the point developed by a vertical line intersecting line ($M2_R$ – $M2_L$) from A, E: the point developed by intersection of line (A–B) with line (C_R – C_L), O (origin, pole): the midpoint at the line ($M2_R$ – $M2_L$) (original line), $r\theta n$ ($n = 1, \dots, 7$)_{R&L} (θ = angles between line (O-reference points) and the original line ($M2_R$ – $M2_L$)).

adjusted to 310 mm so that the photographs could keep in equal proportion with the casts. The digital data of the maxillary and mandibular dentitions were processed with Adobe Photoshop CS (Adobe System; Cal., USA) and Adobe Page Maker (ver. 6.5J, Adobe System) on a Dell Optiplex GX200 computer (OS: Microsoft Windows XP professional; DELL Japan, Kanagawa, Japan), and the equal-proportioned images of plaster casts were printed out using an Epson LP-8300C colour printer (Epson, Nagano, Japan) as has been described in our previous studies^{36, 37)}.

Establishment and development of reference items

By connecting reference points printed on photos, we established 6 items for the preliminarily differentiated (according to Thompson's classification) maxillary and mandibular dental arches: ① $\angle R + \angle L$, ② $(A-B)/(C_R-C_L)$, ③ $180^\circ - \angle(C_R-A-C_L)$, ④ $(A-E)/(C_R-C_L)$, ⑤ $(A-$

B)/(M2_R–M2_L) and ⑥ (rθ5–rθ4)_R+(rθ5–rθ4)_L (Fig. 1). The definition of items has been described in our previous studies^{36, 37}.

Fourier series

The Fourier series is a system that its optional periodic functions of 2π periodicity can be represented by the composition of trigonometric functions with different periodicities and amplitudes³⁵. The present study was evaluated by Origin (R) version 7 (Origin Lab Co, MA, USA) with the following formula^{38–40}:

$$Y(\theta_i) = a_0 + \sum (a_i \cos \theta_i + b_i \sin \theta_i) \quad (i = 1, \dots, 7)$$

$2a_0$: a constant term representing the dental arch size^{26–31, 35}.

a_i, b_i : Fourier coefficients.

$\sqrt{a_i^2 + b_i^2}$: the synthetic wave of cosine (cos) and sine (sin) waves with a same harmonic; power spectrum (amplitude) representing the dental arch form.

The $2a_0$ is also an area obtained by dividing the $Y(\theta_i)$ graph with a periodicity $T^{26–33, 38–40}$. On the other hand, a_i and b_i are Fourier coefficients of cos and sin waves, respectively, and $\sqrt{a_i^2 + b_i^2}$ is developed to indicate the amplitude obtained by synthesizing the cos and sin waves of the same harmonic. The amplitude ($\sqrt{a_i^2 + b_i^2}$) also expresses the size implicating the synthesized wave, so that we can evaluate how the weight (contribution ratio) affects the wave. Standardization of the constant term $2a_0$ has been proposed for the correction of individual differences, although it increases the risk of error during processing the finite term in a Fourier series. In the present study, we did not standardize the constant term $2a_0^{25–27}$.

We defined 14 reference points on each maxillary and mandibular cast and allowed the wave to rise and fall (i.e., go up-and-down) for fewer than 7 times in a period²⁷. The present Fourier analysis defined and investigated $i = 1, \dots, 7$ ($i = 1$ representing the wave of the 1st harmonic and so on). Subsequently, the morphological characters of the constant term $2a_0$ (representing the arch size) and the amplitude (of a same harmonic) $\sqrt{a_i^2 + b_i^2}$ (representing the arch form) were estimated as follows:

- 1) Comparison of the contribution ratio and cumulative contribution ratio expressed in different arch forms by studying the amplitude of different harmonics.
- 2) Comparison of the constant term $2a_0$ and the amplitude $\sqrt{a_i^2 + b_i^2}$ of differentiated maxillary and mandibular arch forms.

One-factor ANOVA ($p < 0.01$) and Kruskal Wallis test ($p < 0.01$) for unequal variances were processed using Excel Statistics 2000 for Windows (Social Survey Research Information Co., Tokyo, Japan). Significant data were assessed by Scheffe's test for *post hoc*.

- 3) Comparison of measured data with the constant term $2a_0$ and the amplitude $\sqrt{a_i^2 + b_i^2}$.

One-factor ANOVA ($p < 0.01$) and correlation coefficient analysis ($p < 0.01$) on the relationships between the present Fourier analysis and the results obtained in previous studies were also performed^{41–47}.

In this study, we in particular estimated the recurrence in percentage (recurrence %) by superimposing the amplitude of different harmonics with the summated amplitudes $\sqrt{a_i^2 + b_i^2}$ (from the 1st to the 7th harmonics), and defined the contribution ratio for each dental arch type of the preliminarily differentiated maxillary and mandibular dentitions. The cumulative contribution ratio was the contribution ratio developed from the accumulation of the amplitudes from the 1st to the 7th harmonics, and thus reproduced arch type forms of the maxillary and mandibular dentitions with a high recurrence of no less than 90%^{27, 35}.

Results

1. Contribution ratio and cumulative contribution ratio of maxillary and mandibular dental arch forms (Table 1)

By studying the amplitudes, we compared the cumulative contribution ratios expressed in different maxillary arch forms, and observed significant values (mostly close to 90%; bold values) in the 1st to 4th (1st+2nd+3rd+4th) harmonics of square and round square arches, in the 1st to 5th (1st+2nd+3rd+4th+5th) harmonics of round arches, and in the 1st to 3rd (1st+2nd+3rd) harmonics of round V-shaped arches. Additionally, we observed a prominent increase in cumulative contribution ratios of the 1st to 3rd (1st+2nd+3rd) harmonics; the 2nd amplitude of the 2nd harmonic had the highest contribution ratios (33.87% to 37.82%; bold italic values) in all maxillary arch forms (Table 1; a) maxillary arch).

In contrast, contribution and cumulative contribution ratios of amplitudes of square, round square and round V-shaped mandibular dentitions were also investigated (Table 1; b) mandibular arch). We noticed that the 1st amplitude (of the 1st harmonic) had the highest contribution ratios (36.77% to 46.23%; bold italic values) in mandibular arch forms. The data of mandibular dentitions indicated that the 1st to 5th (1st+2nd+3rd+4th+5th) harmonics of square arches showed a shift (from 46.23% to 88.19%) of the cumulative contribution ratio towards 90%, while the 1st to 6th (1st+2nd+3rd+4th+5th+6th) harmonics of the round square and round V-shaped arches also showed cumulative contribution ratios which were close to 90% (bold values; Table 1; b) mandibular arch).

2. Average morphology of each dental arch form of maxillary and mandibular dentitions (Fig. 2)

We calculated constant terms ($2a_0$) and mean ampli-

Table 1. Contribution ratios and cumulative contribution ratios of harmonics developed from means amplitudes of different dental arch types (%)

a) maxillary arch

Arch form	Contribution ratio (%)	Amplitude						
		1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
Square (Sq, 13 cases)	Contribution ratio	25.07	35.46	25.40	3.68	3.76	3.71	2.92
	Cumulative contribution ratio	25.07	60.53	85.93	89.61	93.37	97.08	100.00
Round square (R-s, 36 cases)	Contribution ratio	25.39	35.90	25.68	33.38	3.39	3.36	2.95
	Cumulative contribution ratio	25.39	61.29	86.97	90.31	93.69	97.05	100.00
Round (R, 3 cases)	Contribution ratio	23.95	33.87	24.46	4.87	4.89	4.87	3.09
	Cumulative contribution ratio	23.95	57.82	82.28	87.15	92.04	96.91	100.00
Round V-shaped (R-v, 10 cases)	Contribution ratio	26.74	37.82	26.86	2.00	2.06	2.06	2.46
	Cumulative contribution ratio	26.74	64.56	91.42	93.42	95.48	97.54	100.00

b) mandibular arch

Arch form	Contribution ratio (%)	Amplitude						
		1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
Square (Sq, 11 cases)	Contribution ratio	46.23	10.49	4.14	16.28	11.06	8.65	3.16
	Cumulative contribution ratio	46.23	56.72	60.86	77.13	88.19	96.84	100.00
Round square (R-s, 36 cases)	Contribution ratio	42.15	10.43	5.45	15.26	11.54	10.86	4.32
	Cumulative contribution ratio	42.15	52.57	58.02	73.28	84.82	95.68	100.00
Round V-shaped (R-v, 6 cases)	Contribution ratio	36.77	12.32	4.53	17.22	12.82	12.20	4.14
	Cumulative contribution ratio	36.77	49.09	53.62	70.84	83.65	95.86	100.00

All the types of a) maxillary and b) mandibular dentitions showed 90% (approx.) of cumulative contribution ratio (bold values). We observed that the 2nd harmonic had the highest contribution ratio in all maxillary arch forms, and the 1st harmonic had the highest contribution ratio in all mandibular arch forms (bold italic values).

tudes ($\sqrt{a_i^2 + b_i^2}$ curves) developed by Fourier coefficients of the 1st to the 4th harmonic waves (dotted line: mean of the original data, yellow line: the 1st harmonic wave, green line: the 1st and/to 2nd (1st+2nd) harmonic waves, blue line: the 1st to 3rd (1st+2nd+3rd) harmonic waves, and red line: the 1st to 4th (1st+2nd+3rd+4th) harmonic waves; Fig. 2A–2D; a) maxillary arch) and of the 1st to the 6th harmonic waves (dotted line: mean of the original data, yellow line: the 1st harmonic wave, green line: the 1st to 2nd (1st+2nd) harmonic waves, blue line: the 1st to 3rd (1st+2nd+3rd) harmonic waves, orange line: the 1st to 4th (1st+2nd+3rd+4th) harmonic waves, sky blue line: the 1st to 5th (1st+2nd+3rd+4th+5th) harmonic waves, and red line: the 1st to 6th (1st+2nd+3rd+4th+5th+6th) harmonic waves; Fig. 2A, 2B, 2D; b) mandibular arch) for drawing average curves of maxillary and mandibular arches, respectively.

Our previous study showed that using a combination of the 1st to 2nd (1st+2nd) harmonic waves reproduced the maxillary arch forms better than using just the 1st harmonic wave, and the 1st to 2nd (1st+2nd) harmonic waves overlapped with the original data in particular at the molar segments of all maxillary arch types³⁵. The present analysis demonstrated that the 1st to 3rd (1st+2nd+3rd) harmonic waves (blue line) improved the adaptability of the anterior teeth segment, and the 1st to 4th (1st+2nd+3rd+4th) harmonic waves (red line) were nearly a complete fit

with the original data (dotted line) of all maxillary arch forms (Fig. 2A–2D; a) maxillary arch).

In contrast, we observed that the 1st harmonic wave (yellow line) demonstrated curves very different from the original data (dotted line) of mandibular dentitions. The 1st to 4th (1st+2nd+3rd+4th) harmonic waves (orange line) barely reproduced arches resembling the original data of the anterior-teeth segment but not molar segments in different mandibular arch types (Fig. 2A, 2B, 2D; b) mandibular arch). Moreover, the present study indicated that the 1st to 5th (1st+2nd+3rd+4th+5th) harmonic waves (sky blue line) showed an increase in adaptability at the molar regions. However, adaptability decreased at the anterior teeth segment when compared to the adaptability of the 1st to 4th (1st+2nd+3rd+4th), 1st to 5th (1st+2nd+3rd+4th+5th) and 1st to 6th (1st+2nd+3rd+4th+5th+6th) harmonic waves (red line; Fig. 2A, 2B, 2D; b) mandibular arch). The findings indicate that the present Fourier series has a lower reproduction of mandibular dental arch forms, and suggest that maxillary dentitions are more specifically differentiated in the morphological characters estimated in the present study.

3. Superimposition of the average dental arch forms of maxillary and mandibular dentitions (Fig. 3)

Subsequently we superimposed the average forms

Fig. 2A. Reproduction of the square dental arch with a Fourier series.

Fig. 2B. Reproduction of the round square dental arch with a Fourier series.

Fig. 2C. Reproduction of the round dental arch with a Fourier series.

Fig. 2D. Reproduction of the round V-shaped dental arch with a Fourier series.

The 1st harmonic waves of all arch (a) maxillary arch, b) mandibular arch) forms rather deviated from original data, while the 1st to 2nd (1st+2nd) harmonic waves overlapped with the original data in particular at the molar segments. Notice that the 1st to 4th (1st+2nd+3rd+4th) harmonic waves overlapped nearly identical to the original data.

of maxillary dental arches (the 1st to 4th (1st+2nd+3rd+4th) harmonic waves of each arch form; black line: square arch, red line: round V-shaped arch, blue line: round square arch and green line: round arch), and found that square arches (black line) had a distinct projection in the cuspid regions and round V-shaped arches (red line) had a pointed anterior-teeth region (Fig. 3; a) maxillary arch). The two ends (molar regions) of the round arches (green line) showed an increase in values of measurements, but the other arch types showed a decrease followed by an increase of the values at the molar regions (Fig. 3; a)

maxillary arch).

On the other hand, by superimposing the average forms of mandibular dentitions (the 1st to 6th (1st+2nd+3rd+4th+5th+6th) harmonic waves of each arch form; black line: square arch, red line: round V-shaped, blue line: round square arch) for comparison of the arch forms, the examination merely demonstrated that the round V-shaped mandibular dental arches (red line) was in particular characterised by a pointed anterior-teeth region (Fig. 3; b) mandibular arch).

Fig. 3. Superimposition of average dental arch forms.

Superimposition of the 1st to 4th (1st+2nd+3rd+4th) harmonic waves of different average maxillary arch forms and the 1st to 6th (1st+2nd+3rd+4th+5th+6th) harmonic waves of different average mandible arch forms for the purpose of elucidating arch form traits.

Table 2. Comparison of the constant term and amplitude between different arch types

	constant term $2a_0$	1 st	2 nd	3 rd	4 th	5 th	6 th								
Max			**	**	**										
Mand															
Max	2 nd	Sq	R-s	R	3 rd	Sq	R-s	R	4 th	Sq	R-s	R			
	R-s				R-s	**				R-s	**				
	R				R	**				R	**				
	R-v	**	**		R-v	**				R-v					
Mand	2 nd	Sq	R-s	3 rd	Sq	R-s	4 th	Sq	R-s	5 th	Sq	R-s	6 th	Sq	R-s
	R-s			R-s			R-s			R-s			R-s		
	R-v			R-v			R-v			R-v			R-v		

Max: maxillary arch, Mand: mandibular arch, ** $p < 0.01$

Sq: square, R-s: Round square, R: Round, R-v: Round V-shaped

In maxillary (Max) arches, significant differences of the constant term ($2a_0$; representing the arch size) and amplitudes of the 1st harmonic (representing the arch forms) were not demonstrated (one-factor ANOVA and Kruskal Wallis test). Subsequently, Scheffe's test was performed for estimation of the differences between amplitudes representing arch forms, and significant differences were only observed in the maxillary (Max) arches (** $p < 0.01$).

4. Comparison of the constant terms and amplitudes between different arch types of maxillary and mandibular dentitions (Table 2)

Significant differences of the constant term ($2a_0$; representing the arch size) and amplitudes of the 1st harmonic (representing the arch forms) were not demonstrated in the evaluation of maxillary and mandibular arch types using the one-factor ANOVA and Kruskal Wallis test. However, the analysis indicated that amplitudes of the 2nd, 3rd and 4th harmonics were only significant in the differentiating arch forms of maxillary dentition (Table 2; Sq: square, R-s: Round square, R: round and R-v: round V-shaped dental arch types; $p < 0.01$). Subsequently, we performed Scheffe's test for estimation of the differences between the amplitudes of the 2nd, 3rd, 4th, 5th and 6th harmonics, which were supposed to represent different dental arch types. Significant differences for varying maxillary and mandibular arch forms of each harmonic were summarized as the following (Table 2; $p < 0.01$):

1) Maxillary dental arches

- The 2nd amplitude: significantly differed between a round V-shaped (R-v) and either square (Sq) or round square (R-s) arch (Table 2; $p < 0.01$).
- The 3rd amplitude: significantly differed between a square arch (Sq) and other types (R-s, R and R-v) of dental arch (Table 2; $p < 0.01$).
- The 4th amplitude: significantly differed between a square (Sq) and either round square (R-s) or round (R) arch (Table 2; $p < 0.01$).

2) Mandibular dental arches

No significant differences of the 2nd, 3rd, 4th, 5th and the 6th amplitudes (developed from 2nd to 6th harmonics) were

observed (Table 2; $p < 0.01$).

We summarize the results of morphological superimposition and statistical analysis, thereby confirming that the differences in dental arch types are distinct in maxillary dentitions but indistinctive in mandibular dentitions (also see Results 3 and 4; Figs. 2A–2D and 3; Tables 2 and 3).

5. Comparison of estimated data with the constant term and the amplitude (Table 3)

Relationships between the estimated data obtained in our previous principal component analyses study of maxillary and mandibular dentitions^{36, 37} with the constant terms ($2a_0$; related with the arch size) and amplitudes ($\sqrt{ai^2 + bi^2}$; related with the arch form) of different harmonics were represented by correlation coefficients. The coefficients with values close to/above 0.7, either positive or negative, were indicative for strong correlations in the present analysis (Table 3, $p < 0.01$).

1) The analysis of the maxillary dental arches were summarized as the following (Table 3; a) maxillary arch; $p < 0.01$):

- The constant term ($2a_0$): strongly and positively correlated with either the anterior-teeth width ($C_R - C_L$) or dental arch width ($M2_R - M2_L$), whereas being weakly and positively correlated with the dental length (A–B) of the maxillary dentitions.
- The 1st amplitude: weakly and positively correlated with the dental arch length (A–B) and anterior-teeth width ($C_R - C_L$).
- The 2nd amplitude: strongly and positively cor-

Table 3. Correlation analyses of the constant term ($2a_0$) and amplitudes of the Fourier harmonics ($p < 0.01$)

a) maxillary arch

	Constant term $2a_0$	1 st amplitude	2 nd amplitude	3 rd amplitude	4 th amplitude
①	-0.104	-0.008	-0.201	-0.372	-0.233
②	-0.248	0.134	-0.760	-0.514	0.019
③	-0.221	0.088	-0.621	-0.481	-0.195
④	-0.243	0.055	-0.613	-0.460	-0.159
⑤	-0.170	0.364	-0.800	-0.343	0.094
⑥	0.032	0.315	-0.357	0.103	0.274
A-B	0.373	0.487	-0.512	-0.117	0.063
A-E	-0.006	0.196	-0.539	-0.323	-0.006
C_R-C_L	0.735	0.410	0.319	0.477	0.090
$M2_R-M2_L$	0.714	-0.023	0.696	0.416	-0.072

b) mandibular arch

	Constant term $2a_0$	1 st amplitude	2 nd amplitude	3 rd amplitude	4 th amplitude	5 th amplitude	6 th amplitude
①	0.039	-0.021	0.099	0.050	0.023	-0.009	0.042
②	-0.021	0.153	0.132	-0.015	0.203	0.118	0.155
③	-0.030	0.148	0.070	-0.039	0.237	0.034	0.078
④	-0.074	0.148	0.042	-0.010	0.197	0.048	0.147
⑤	-0.273	0.194	-0.091	-0.119	0.051	-0.108	-0.027
⑥	0.046	0.062	0.049	-0.096	0.159	0.074	0.054
A-B	-0.172	0.115	-0.088	-0.146	0.103	-0.151	-0.102
A-E	-0.136	0.135	-0.047	-0.061	0.159	-0.061	0.046
C_R-C_L	-0.175	-0.030	-0.258	-0.135	-0.119	-0.310	-0.281
$M2_R-M2_L$	0.198	-0.161	0.033	0.021	0.037	-0.007	-0.066

The constant term ($2a_0$) showed positive relationship of the $M2_R-M2_L$ of both the maxillary and mandibular dental arches. The amplitudes of the 2nd and 3rd, and the 4th Fourier harmonics were identified correlating with length/width ratio (⑤; strong in the 2nd amplitude) and the curvature of anterior teeth (including ②, ③ and ④; strong in the 2nd amplitude yet weak in the 3rd amplitude), and the curvilinear contour of the dental arch (including ① and ⑥; weakly related), respectively (a) maxillary arch; $p < 0.01$). In contrast, the amplitude of the 4th Fourier harmonic was identified to be comparably correlated with the curvature of anterior teeth of mandibular dentitions (including ②, ③ and ④; b) mandibular arch; $p < 0.01$).

①: $\angle R + \angle L$, ②: $A-B/C_R-C_L$, ③: $180^\circ - \angle C_R-A-C_L$, ④: $A-E/C_R-C_L$, ⑤: $A-B/M2_R-M2_L$, ⑥: $(r\theta_5-r\theta_4)_R+(r\theta_5-r\theta_4)_L$

related with the arch width ($M2_R-M2_L$), but being strongly and negatively correlated with the anterior-teeth curvature (including ②, ③ and ④ = the 1st principal component) and the arch length-to-width ratio (⑤ = the 3rd principal component).

- d) The 3rd amplitude: weakly and positively correlated with the anterior-teeth width (C_R-C_L) and the dental arch width ($M2_R-M2_L$), but showing a negative correlation with the anterior-teeth curvature (including ②, ③ and ④).
- e) The 4th amplitude: weakly and positively correlated with the difference of radius vectors at the transitional region between the anterior and posterior teeth (including ① and ⑥), thereby indicating a positive relationship with the curvilinear contour of the maxillary dental arch.

2) In contrast, the analysis of mandibular dentitions showed that (Table 3; b) mandibular arch; $p < 0.01$):

- a) The constant term ($2a_0$): weakly but positively correlated with the dental arch width ($M2_R-M2_L$), but being weakly and negatively correlated with the arch length-to-width ratio (⑤).

- b) The 1st amplitude: was not significantly correlated with estimated items.
- c) The 2nd amplitude: weakly and negatively correlated with the anterior-teeth width (C_R-C_L).
- d) The 3rd amplitude: was not significantly correlated with the items.
- e) The 4th amplitude: showing a weakly and positively correlation with the anterior-teeth curvature (including ②, ③ and ④).
- f) The 5th and 6th amplitudes: were significantly but relatively weak-and-negatively correlated with the anterior-teeth width (C_R-C_L).

Discussion

1. The contribution ratio and cumulative contribution ratio of dental arches (Table 1)

The present analyses suggested that the amplitudes developed from the 1st to 4th ($1^{st}+2^{nd}+3^{rd}+4^{th}$) and the 1st to 6th ($1^{st}+2^{nd}+3^{rd}+4^{th}+5^{th}+6^{th}$) harmonics could be significantly reproduced arch forms of maxillary and

mandibular dentitions, respectively. Among them, the highest contribution ratio of the 2nd and 1st harmonics of maxillary and mandibular dentitions elucidated that they were the most essential factors in affecting the reproducibility of the maxillary and mandibular dental arch forms, respectively. It was consistent with the previous studies, which reported that the 1st to 3rd (1st+2nd+3rd) harmonics was essential representing characters of dental arch forms^{26, 28, 30)}. In the present study, we also identified the 1st to 4th (1st+2nd+3rd+4th) harmonics to reproduce the maxillary dental arch forms (cumulative contribution ratio $\div$ 90%). Alternately, the cumulative contribution ratio of the 1st to 5th (1st+2nd+3rd+4th+5th) harmonics of mandibular dentitions was nearly 85% inclusively reproducing the original dental arch forms; the 1st amplitude was almost 50% of the contribution ratio^{26, 28, 30)}. The results also indicated that the cumulative contribution ratios of the 1st to 3rd (1st+2nd+3rd) harmonics of mandibular dental arches were nearly 60%; the contribution ratios of the 3rd and 4th amplitudes were about 5% and 15%, respectively. In summary, the results elucidated that the 1st to 3rd (1st+2nd+3rd) harmonics did not represent traits of arch types in mandibular dentitions as they did in maxillary dentitions (Table 1). Therefore, the present study concludes that the mandibular arch does not show any unique element of the maxillary dental arch forms.

2. The average form for each dental arch form (Fig. 2; Tables 2 and 3)

We superimposed and compared the average form with the original data of each arch form, and found that the 1st and/or 2nd (1st+2nd) and the 1st to 4th (1st+2nd+3rd+4th) harmonic waves showed higher reproducibility for maxillary and mandibular dental arches, respectively. However, the 1st to 2nd (1st+2nd) harmonic waves showed a higher reproducibility of dental arch forms compared to only using the 1st harmonic wave in maxillary dentitions³⁵⁾. In addition, by comparison of the estimated data with the constant term ($2a_0$) and the amplitude ($\sqrt{a_1^2 + b_1^2}$) of different maxillary dental arch forms, the analytical results indicated that the 2nd amplitude was strongly and positively correlated with the arch width ($M2_R - M2_L$), and strongly yet negatively correlated with the anterior-teeth curvature ((2)+(3)+(4)) and the arch length-to-width ratio ((5)). The relationships suggested that the 1st to 2nd (1st+2nd) harmonic waves reflected all maxillary dental arch forms, and the 1st to 3rd (1st+2nd+3rd) harmonic waves nearly fitted the original data (described in Results 4 and 5). The present results demonstrated that the 1st, 1st to 2nd (1st+2nd) and the 1st to 3rd (1st+2nd+3rd) harmonic waves essentially represented the dental arch form characters, and the 1st to 4th (1st+2nd+3rd+4th) harmonic waves showed an increase in the anterior-teeth conformity (including (2), (3) and (4)) with the original data, as well as the reproduced maxillary dental arch types almost completely (described in Result 5).

The analysis suggested that the advanced harmonic waves fitted more with the original data of maxillary dentitions. However, low harmonics of mandibular dentitions resulted in an increase of the reproducibility of the anterior-teeth segment, and high harmonics resulted in the conformity of an increase in molar segments but with a decrease in the anterior-teeth segment when compared with the original data (described in Results 4 and 5). The present study suggested that the Fourier series reproduced lower conformity of the arch forms, which did not reveal distinctive characteristic of each harmonic in mandibular than in maxillary dentitions.

3. Superimposition of different average arch forms (Fig. 3)

The 1st to 4th (1st+2nd+3rd+4th) harmonic waves for average maxillary arch forms and the 1st to 6th (1st+2nd+3rd+4th+5th+6th) harmonic waves for average mandibular arch forms were further superimposed for elucidating arch form characters. We observed that the square maxillary arch (Fig. 3; a) maxillary arch, black line) had a large arch width but a small anterior-teeth curvature than other types; the round-V maxillary arch (Fig. 3; a) maxillary arch, red line) was contrary to that. The round maxillary dental arch (Fig. 3; a) maxillary arch, green line) was characterised by a gentle transition between anterior- and posterior-teeth curves having small ($r\theta_5 - r\theta_4$)_{R&L} values in the estimated item ⑥ (Table 3; a) maxillary arch). Moreover, the study indicated that round square arch was the average and the most common maxillary dental arch³⁵⁾.

On the other hand, there is no distinct morphological differences between the square (Fig. 3; b) mandibular arch, black line), round V-shaped (Fig. 3; b) mandibular arch, red line) and round square (Fig. 3; b) mandibular arch, blue line) dental arch were found, but a prominent anterior protrusion ((2)+(3)+(4)) of the round V-shaped dental arch in mandibular dentitions was observed; therefore, the study indicated that mandibular dentitions had little differences between dental arch forms (Table 3; b) mandibular arch).

4. Comparison of the constant terms and the amplitudes of different arch forms (Table 2)

Some previous studies have pointed out that the constant term in the formula of a Fourier series represented the dental arch size, while the amplitude of a synthetic wave represented characters for the arch forms²⁶⁻³¹⁾. As what has been described in the present study about the average form of each dental arch type, we found that the 1st harmonic waves were parabolic curves and very similar in all maxillary and mandibular dental arch types. Our study on maxillary dental arch types reported the significant differences of the 2nd amplitude between the round V-shaped arch and the square and round square arches, of the 3rd amplitude between the square arch and

other arches, and of the 4th harmonic between the square and the round square and round arches³⁵⁾ (Table 2). On the contrary, we did not identify significant differences between the 2nd, 3rd, 4th, 5th and 6th amplitudes in mandibular dental arches (Table 2). Hence the present study concludes that the amplitudes are factors affecting characters of different maxillary dental arch types, but do not take part in characters of the specific mandibular dental arch forms.

5. Comparison of the estimated data with the constant term and the amplitude (Fig. 2; Tables 1 and 3)

The present study compared estimated data with the constant terms ($2a_0$) and amplitudes ($\sqrt{ai^2 + bi^2}$) for evaluation of how the individual harmonic affected the dental arch forms. By studying the details of morphological characters, we observed that the constant terms ($2a_0$) had a positive correlation with the dental arch width ($M2_R - M2_L$), which was hence an essential factor affecting the arch size of both maxillary and mandibular dentitions³³⁾ (Table 3).

The present correlation analysis revealed that the 1st amplitude of the 1st harmonic was in particular positively correlated with the arch length (A–B) and width of the anterior-teeth segment ($C_R - C_L$) in maxillary but not in mandibular dentitions. We noticed that the 1st harmonics were in nearly bowlike form in all maxillary and mandibular dental arch types (Fig. 2). However, we found no significant analytical differences but weak correlation coefficients between the estimated data and the 1st amplitudes in both maxillary and mandibular dentitions (Table 3: a) maxillary arch and b) mandibular arch). The results were consistent with what has been discussed in sections-1, -2 and -4 of Discussion, and therefore we concluded that both the maxillary and mandibular dentitions could be roughly described to be bowlike-arranged regardless of arch types. Furthermore, the analysis indicated that the 2nd amplitude was strongly and negatively correlated with the arch length-to-width ratio $-(5)(A-B)/(M2_R - M2_L)$, had the highest contribution ratio to determine the gross appearances of dental arches (described in Result 1) and enhanced the 1st to 2nd ($1^{st} + 2^{nd}$) harmonic waves to overlap with the original data (described in Result 2), thereby elucidating the most prominent factor affecting the morphological variations of maxillary dental arch types. Conversely, we found no correlated items in the analysis of mandibular dentitions, so that the 1st to 2nd ($1^{st} + 2^{nd}$) harmonic waves did not reproduce curves resembling the original mandibular dental arches as in maxillary dentitions (described in Result 2). However, the 2nd amplitude showed a relatively weak-and-negative correlation with the mandibular anterior-teeth width ($C_R - C_L$) (described in Result 5) having a contribution ratio of 10% approx. (described in Result 1). The analysis therefore indicated that the 2nd harmonic importantly adjusted the curve shape of the anterior segment of

mandibular dentitions. Furthermore, we did not identify any characteristic items, while demonstrated that the 3rd amplitude expressed less than 5% contribution ratios in all mandibular dental arch types (described in Result 1). We also noticed that the 1st to 3rd ($1^{st} + 2^{nd} + 3^{rd}$) harmonic waves did not show prominent differences with the 1st to 2nd ($1^{st} + 2^{nd}$) harmonic waves. It is thereby elucidated that the 3rd harmonic was not a determinant of mandibular dental arch forms.

The present analysis provided evidence that the 4th harmonic tuned exactly at the transitional region of anterior and posterior teeth (①+⑥) in maxillary dentitions, while being closely related with the anterior-teeth curvature (②+③+④) of mandibular dentitions³⁵⁾. The 4th amplitude showed contribution ratios (above 15%) next to the 1st amplitude, hence providing little effects on the morphology of mandibular anterior teeth segment (Tables 1 and 3; b) mandibular arch). We observed that the 1st to 4th ($1^{st} + 2^{nd} + 3^{rd} + 4^{th}$) harmonics more accurately duplicated the inter-cuspal distance (anterior-teeth width; $C_R - C_L$) in comparison to that of the 1st to 3rd ($1^{st} + 2^{nd} + 3^{rd}$) harmonics in both maxillary and mandibular dentitions. In the present study, we elucidated that the 1st to 4th ($1^{st} + 2^{nd} + 3^{rd} + 4^{th}$) harmonic waves reproduced maxillary dental arch types almost completely, whereas reproduced the anterior-teeth segment but not molar segments in different mandibular arch types (described in Result 2).

By examination of the 5th and 6th amplitudes of the mandibular dentitions, we observed that they were significantly but weakly and negatively correlated with the anterior-teeth width ($C_R - C_L$), showing high conformity at the canine regions. Furthermore, by graded cumulating amplitudes of higher frequency (i.e., the 5th, 6th and then 7th amplitudes), the present study developed a Fourier series showing an increase at the molar regions but a decrease at the anterior teeth segment in adaptability of mandibular dental arch forms (described in Result 2). Thus, we identified that the 6th amplitude therefore seemed to be necessary for reproduction of arch forms, whereas showed no difference between varying arch types of mandibular dentitions (described in Results 2, 3 and 5).

In conclusion, we identify the Fourier series has a lower reproduction of mandibular dental arch forms, and suggest that maxillary dentitions are more specifically differentiated in the morphological characters estimated in the present study.

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