Circularity Ratio

— A Certain Quantitative Expression for the Circularity of a Round Figure —

By

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In the course of our quantitative studies ('70) by electron microscope on the central nervous system in the cat, in a similar but different way which Conradi reported ('69), the cell bodies of the neurons in the ventro-lateral part of the anterior horn (C_{6-7}) were divided into two groups in accordance with over (group I) and under (group II) 700 micron squared in area for quantitative estimations. During observations it has been noticed that the cell-bodies and their nuclei are various in shape, especially those of the nuclei in the group II are irregular (Fig. 1). Accordingly, the attention has been directed to express these irregular forms in shape as areal and lineal quantitative correlations. Although, Higashi ('69) has reported such an attempt concerning the human tumors, these are different points from ours in respect to pathologic ones.

As had been expected though incidentally in a strict sense, a ratio constant of $4\pi A$ to L^2 was found out, of which the A stands for the area of circle and the L for the length of circumference. The constant will be useful for the quantitative expression of irregularities. If a circle is regular the ratio takes maximum value 1.00, and if it is irregular the ratio takes value under 1.00. For the sake of easy comprehension on such ratio, the following two special cases are cited: the value 0.79 or 0.61 is taken when a figure is regular square or triangle (Fig. 2).

As the area (A) and the circumference (L) of a circle can be measured by planimeter and curvimeter, the value of the ratio is easily calculated.

In such a way the values of the ratio of the nuclei in groups I and II, were calculated and were compared with each other. Table 1 shows the values of the ratio of nuclei in groups I and II, and Figure 3 is a

scatter diagram of such values, in which the ordinate indicates the area (A) in micron squared and the abscissa the second power of the circumference (L^2) in micron squared. From the table and the diagram it is clear that the values from the group I are observed to be located collectively near the maximum except for two nuclei, and those from the group II are scattered, and there is significant difference between the mean values ($\bar{X}$) of both groups.

Putting aside the problem of describing different sorts of individual irregular shapes of circles, the value may be utilized for quantitative comparison of round figures, in other words, it is of immense value for the expression as to how each irregularity is nearer to or farther from a regular round figure, especially for electron micrographs if the value of the ratio is statistically taken into account.

References

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Explanation of Figures

- Fig. 1. Representative photomontages of the cell-bodies and their contours in groups, I and II. The nucleus in the group II is very irregular in shape. The values of the ratio of the nuclei are 0.95 (Figure 1A) and 0.48 (Figure 1B) in these cases. A, $\times 3,800$, B, $\times 1,500$.
- Fig. 2. An explanation for the circularity ratio (CR).
- Fig. 3. Scatter diagram and histogram of the values of RC calculated from the nuclei in groups I and II. The ordinate indicates the area (A) of the nucleus in micron squared and the abscissa the second power of the circumference (L^2) of the nucleus in micron squared.
- Table 1. The values of the CR calculated from the nuclei in groups I and II.

B10 μ

Fig. 1B

Fig 2

$$C R (\text{circularity ratio}) = \frac{4\pi A}{L^2}$$

A.. AREA OF A CIRCLE

L.. LENGTH OF THE CIRCUMFERENCE

REGULAR CIRCLE

CR = 1.00

REGULAR SQUARE

CR = 0.79

IRREGULAR CIRCLE

CR = 0.82

REGULAR TRIANGLE

CR = 0.61

Fig 3

number	group I	group II
1	0.94	0.96
2	0.93	0.48
3	0.95	0.99
4	0.90	0.95
5	0.96	0.70
6	0.97	0.62
7	0.94	0.89
8	0.69	0.64
9	0.94	0.22
10	0.94	0.61
11	1.00	0.34
12	0.97	0.89
13	0.90	0.69
14	0.99	0.80
15	0.98	0.32
16	0.96	0.91
		0.87
		0.57
$\bar{x}$	0.92	0.69