An Optical Decoding Architecture for the Random Iteration Algorithm of Iterated Function System Codes

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A novel optical architecture based on the random iteration algorithm to decode iterated function system (IFS) codes is presented. The optical part of this system performs the affine transformations of IFS. The code values in the affine transformations of IFS are first translated into the modulation voltages of the electro-optic modulators (EOMs), with a specific probability. Then, the multiplication and addition operations can easily be performed. With the O/E and A/D converters, the light intensity is converted into an electrical signal to address a specific pixel at the display device. A fractal image is then progressively decoded pixel by pixel and superimposed on the display device. The computer simulation and some experimental results for the proposed architecture are provided for verification. The decoding speed is fast, the image quality is high, and the resolution of the generated fractals depends on the resolution of the display device itself.

Key words: random iteration algorithm, IFS codes, affine transformation, fractals

1. Introduction

Fractal techniques for image coding have recently attracted a great deal of attention. Barnsley found that a finite set of specific contractive transform functions of iterated function system (IFS) can be used to generate a fractal image. IFS achieves large amounts of data compression and its decoding algorithms are simple. However, it is computation-intensive for encoding and decoding a fractal image. Clearly, this precludes the use of such a method for real-time applications. The parallelism and high-speed nature of optics are very suitable for image processing. In this paper, we propose an optical decoding architecture to rapidly generate a fractal image to solve this problem.

There are two algorithms (deterministic and random iteration) for decoding (or generating) fractals with IFS. Tanida et al. present an optical fractal synthesizer (OFS) to generate fractal images from IFS with the deterministic algorithm. In this technique, all the points in an image shown on the CRT are duplicated and then optically transformed with one of the affine transformations. The transformed images are combined into a single image and fed back through a CCD camera for the next iteration. After a sufficient number of iterations, a fractal image is obtained whose shape is determined by the characteristics of the optical affine transformations. This technique solves the discontinuity problem caused by rotation, magnification, or reduction of a digital image. However, this OFS can only decode a fractal image which consists of a regular pattern or texture because only two affine transformations in this architecture operate with an equal probability. Also, the decoding speed and the image quality of the generated fractals are limited by the physical resolution of the optical system and the CCD devices used in OFS.

In this paper, we present a novel optical architecture based on the random iteration algorithm to decode the IFS codes. The random iteration algorithm generates an image point by point, and introduces the probability theory to reduce the total amount of computation. A probability that indicates the importance of one transformation relative to others will be assigned to each transformation in the random iteration algorithm. Instead of electronic calculation, a high-speed optical architecture is proposed to implement the random iteration algorithm. This architecture is based on the following: The multiplication result is the output light intensity when a light beam passes through an electro-optic modulator (EOM) with a given modulation voltage. The addition result is obtained when two light-beam intensities are simultaneously detected by a photodetector. Thus, with some additional electronic circuits, we will be able to generate a complicated high-quality fractal image.

2. Iterated Function System and Random Iteration Algorithm

An iterated function system (IFS) is an extension of classical geometry which uses contractive affine transformations to express the relationship between different parts of an image. As IFS achieves a high degree of data compression and its decoding algorithm is simple, it has been proposed as a method for fractal image coding and has attracted much interest for the compression of general images.

The general form of a contractive affine transformation $W(\cdot, \cdot)$ can be defined as

$$W(x_{i-1}, y_{i-1}) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_{i-1} \\ y_{i-1} \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$  \hspace{1cm} (1)$$

where the coefficients $(a-f)$ are within the range $(-1 \text{ to } 1)$ and $|ad-bc|<1$. A probability $p_x$ is assigned to each affine transformation $W_x$, and the sum of all the probabilities $p_x$ is 1. The coefficients $(a-f)$ and a specific probability $p_x$ are thus called the IFS codes. The coefficients $(a-f)$ of the IFS codes represent the rotation, skewing, contraction, and translation of images. For a fractal image with self-
similarity, there are $N$ affine transformations such that

$$F = \bigcup_{k=1}^{N} W_k(F)$$

(2)

where $F$ is a fractal image and $W_k(F)$ is a sub-image obtained by one of the affine transformations. Since the number of these coefficients is small, the image compression ratio is thus high.

The random iteration algorithm for decoding the IFS codes was proposed by Barnsley. Its procedures are summarized as follows:

1. Initialize; choose an arbitrary point $(x, y)$.
2. For $n=1$ to 10,000, do step (3) to (6).
3. Randomly select one of the affine transformations $W_k$ with a probability $p_k$.
4. Apply the transformation $W_k$ to the point $(x, y)$ to get a new point $(\tilde{x}, \tilde{y})$.
5. Set $(x, y) = (\tilde{x}, \tilde{y})$.
6. If $n>10$, plot the new point $(\tilde{x}, \tilde{y})$.

Applying the procedures above to the IFS codes, a fractal image can be generated. The results at early iterations have large error if the initial point is not correctly chosen within the range of the fractal image. This error disappears by correctly choosing the initial point or discarding the early iteration points (10 in the algorithm above) for the decoding procedure. Finally, the decoded image converges as the number $n$ of the iteration increases.

3. Electro-Optic Light Beam Modulator

In certain types of crystals such as KH$_2$PO$_4$ and LiNbO$_3$, the application of an electric field results in a change in the refractive index: This is the electro-optic effect. Since the refraction index changes at the modulation rate of the electric field, this effect provides a means of controlling the intensity of the propagating beam of light in real time. A typical arrangement of an EOM consisting of an EO crystal placed between two crossed polarizers of different polarization is shown in Fig. 1(a). The modulator is usually biased to the 50% transmission point by a fixed retardation (or phase difference). This bias can be achieved by applying a voltage $V = V_e/2$ where $V_e$ is the half-wave voltage or, more conveniently, by using a naturally birefringent crystal as shown in Fig. 1(a) to introduce a phase difference of $\pi/2$. Figure 1(b) illustrates the relationship between the transmittance ($T$) and the applied voltage. This relationship is given by

$$T = \frac{I_t}{I_0} = \frac{1}{2} \left[ 1 + \sin \left( \frac{\pi V}{V_e} \right) \right],$$

(3)

where $I_t$ and $I_0$ are the input and output beam intensities, respectively. Therefore, the intensity modulation is a nonlinear replica of the modulating voltage about the bias point. As shown below, the coefficients of the IFS codes can be represented by the modulation voltages. The intensity of the light beam is thus modulated by the EO effect and the multiplication operation is achieved in real time.

4. Optical Decoding Architecture for Fractals

An optical architecture for performance of contractive affine transformations of IFS is proposed. To implement Eq. (1), the light intensity is used to represent the pixel position, $x$ and $y$, within the range (0-1), and the coefficients ($a$-$f$) are encoded by the modulation voltages ($V_a$-$V_f$) of the EOM with

$$V_t = \frac{V_e}{\pi} \sin^{-1} \left( \frac{\xi}{2} \right), \quad \xi = a \cdot f.$$  

(4)

The range of the modulation voltage is within ($-V_e/2$ to $V_e/2$) and their corresponding transmittances are ($T_a$-$T_f$) which are in the range 0-1. To use the EOM, we rewrite Eq. (1) as

$$2 \left[ \begin{bmatrix} T_a & T_b \\ T_c & T_d \end{bmatrix} \right] \begin{bmatrix} x_{i-1} \\ y_{i-1} \end{bmatrix} + \begin{bmatrix} T_e \\ T_f \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

(5)

with $T_t = (\xi+1)/2$ and $\xi = a \cdot f$. The terms inside the large brackets in Eq. (5) are calculated by optics, while the others are performed by electronics.

The proposed optical architecture is shown in Fig. 2. In optical implementation, each of the light signals ($x_{i-1}$, $y_{i-1}$ and the fixed input) is passed through a 50/50 beam splitters. Either the coherent or incoherent light source can be used in our system. Synchronization between the operation of EOMs, O/E and E/O converters, adders, and the delay element is necessary. For each iteration, the modulation voltages corresponding to the IFS codes are first applied with a specific probability. Then, the E/O and O/
Fig. 2. The optical architecture of the random iteration algorithm.

Table 1. IFS code example. Fractal tree.

<table>
<thead>
<tr>
<th>k</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>p_k</th>
</tr>
</thead>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.42</td>
<td>-0.42</td>
<td>0.42</td>
<td>0.42</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>0.42</td>
<td>0.42</td>
<td>-0.42</td>
<td>0.42</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

E converters, adders, and delay elements act simultaneously. The multiplication operation is performed when a light beam passes through an EOM. On the other hand, the addition operation is conducted at the photodetector by detecting the three separate light beams simultaneously. While performing the two operations outlined above, the O/E devices detect and amplify the light intensity by four times to compensate for optical loss due to the beamsplitters and get the factor “2” in Eq. (5). After subtracting the previous value “x_{i-1} + y_{i-1}” and a constant “1,” both x_i and y_i are obtained. The E/O device then converts these two electrical signals into light signals for the next iteration. The addressing circuit performs the analog-to-digital (A/D) conversion and then transfers the (x_i, y_i) signal into the range of image size (for example, 512 by 512). Therefore, the x and y coordinates of a pixel position in an image are obtained simultaneously and used to address the particular point on the display device. For the reasons mentioned in Section 2, we discard the early iteration points such as n = 1–10 to avoid large errors. With repeated iterations, a fractal image can be generated and the image quality improves if the iteration number increases.

The operation procedures of the proposed architecture are summarized below:

1. The code values (a–f) are converted into the modulation voltages (V_a–V_f) of the EOMs with a specific probability (p_k), respectively.
2. The light intensity signals, x_{i-1} and y_{i-1}, which are emitted from the E/O device pass through BS_i and BS_{i+1}, respectively. Each of them is split into two light beams with equal intensity.
3. These four split signals pass through the EOMs (EOM_1–EOM_4) and are multiplied by the transmittances (T_{x_i}–T_{y_i}) of the EOMs, respectively.
4. A fixed input light passes through BS, and is separated into two light beams. These two light beams pass through...
the EOM₂ and EOM₄, respectively, to obtain the \( T_\alpha \) and \( T_\beta \) terms in Eq. (5).
5. Each O/E device simultaneously detects the three light intensity signals to perform an addition operation and converts the summed signal into the electrical form. This signal is amplified by four times to obtain the first two terms in Eq. (5). Therefore, by subtracting the delayed sum of the previous signals \( x_{i-1} + y_{i-1} \) and a constant value “1,” both \( x_i \) and \( y_i \) are obtained.
6. While the iteration number is greater than ten, the addressing circuit receives \( x_i \) and \( y_i \) signals to address a pixel on the display device. At the same time, \( x_i \) and \( y_i \) signals are fed back to the E/O device and converted into light signals for the next optical iteration.
7. The above procedures proceed iteratively. A fractal image is decoded pixel by pixel and then superimposed on the display device.

5. Simulation and Experimental Results

Computer simulation and some experimental results for the proposed architecture were described here. During the simulation, the initial point is \( (0, 0) \), the reconstructed binary image size is 512 \( \times \) 512, and the sum of the probability for each affine transformation is unity. We first generate a random number between 0 and 1 to select one of the affine transformations. In the optical paths of \( x_{i-1} \), \( y_{i-1} \) and fixed input, each light beam passes through a 50/50 beamsplitter and is attenuated by a factor of “two.” To compensate for this effect and implement Eq. (5), the O/E device detects the light intensity and amplifies the converted signal by four times. Subtracting the sum of the previous signals, \( x_{i-1} + y_{i-1} + 1 \), we can obtain the position signals \( (x_i, y_i) \) to address the pixel and use these for the next iteration. The iteration number depends on the size of the reconstructed image. As the image size grows, this number should be increased for a binary image with a higher quality.

Table 1 shows an example of IFS codes for a fractal tree. Figure 3 shows the image decoded by simulation. As the generated fractals are superimposed pixel by pixel, we can obtain a complicated and high-quality image if the iteration number is sufficiently large. In addition, the decoding speed of the proposed architecture is high because the EO effect is very fast (the other of 10⁻¹² s), and thus the decoding speed only depends on the response time of the E/O and O/E converters and processing time of the electronic circuits.

Experimental results were provided to verify the proposed architecture. Here, only the multiplication and addition operations were tested. In addition, the constant “2” in Eq. (5) was not considered. The experimental set-up is shown in Fig. 4, and consisted of three light sources, three EOMs, two beamsplitters, and a light-power meter (which serves as the E/O converter). The three light sources were \( x_{i-1} = 187 \mu W \), \( y_{i-1} = 632.8 \text{ nm} \), and fixed input = 85 \( \mu W \) obtained with a He-Ne laser together with two beamsplitters. In addition, EOM₁, EOM₂, and EOM₃ were replaced by photographic films with transmittance \( T_\alpha = 0.10 \), \( T_\beta = 0.43 \), and \( T_\gamma = 0.51 \), respectively. In our experiment, the mirror had 19% loss. If this optical loss is considered, then the output power should be 96.51 \( \mu W \). The power detected by the light-power meter was 96.8 \( \mu W \) (0.3% error compared with the expected output power).

6. Conclusion

In this paper, we proposed a novel optical decoding architecture for the random iteration algorithm of the IFS codes. The computer simulation and experimental results verified our proposed architecture. In our architecture, a fractal image is decoded pixel by pixel and then superimposed on the display device. The resolution of the generated fractals only depends on the resolution of the display device itself, and thus a high-quality fractal image can be obtained.

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References