Profile Measurement by Projecting Two Gratings with Different Pitches

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A new method for measuring objects with steps is proposed in which two gratings with different pitches are projected on the object surface. The method is demonstrated with sinusoidal gratings made of laser interference fringes and also sinusoidal gratings formed by liquid crystal controlled by computer. An object with step height of 20 mm is measured with standard variation below 3%.

Key words: profile measurement, two gratings, step height, phase-shifting technique, interference fringe, liquid crystal grating, sinusoidal grating

1. Introduction

The profile of a solid object is obtained by projecting a grating on the object surface and observing the deformed pattern in a direction different from the projection. To obtain information finer than the grating pitch, a phase-shifting technique is used. However, if the step of the object surface is so steep that the height change between any adjacent pixels is larger than a certain height P corresponding to the grating pitch, the integer grating order becomes ambiguous and the profile cannot be determined.

In this paper, we report a new method for measuring object with steep steps. Two gratings with different pitches are projected on the object sequentially and two phases of sinusoidal intensity variations of the gratings are calculated by a phase-shifting technique. Integer grating orders can be obtained from the difference of two heights calculated from the phases. We can determine step height uniquely if it is smaller than a certain height corresponding to an equivalent effective pitch \( \lambda \) defined later.

Sinusoidal gratings are obtained by laser interference fringes or by projecting liquid crystal gratings controlled by a computer. An object with step height of 20 mm is measured with standard variation below 3% by these gratings.

2. Principle

The optical system for the present method is shown in Fig. 1. In this figure, point A is the projection center of gratings and point O is the center of the imaging lens of the object. The Z axis is taken in the direction of the optical axis of imaging system. The point A is assumed to be located on the X axis. Equal-intensity positions of the projected gratings are assumed to form planes such as \( m \) in Fig. 1, and all the planes are perpendicular to the X-Z plane. \( P_0 \) is a point on the object surface and \( P_0' \) is the corresponding to \( P_0 \) on the X-Z plane. \( \alpha \) is the angle after subtraction of the angle \( P_0' \cdot AO \) from \( \pi/2 \), \( \xi \) is the angle after subtraction of the angle \( P_0' \cdot OA \) from \( \pi/2 \) and \( a \) is the distance between two points O and A. If the angle \( \alpha, \xi \) for the point \( P_i \) can be measured, the distance \( Z_i \) from the X-Y plane for the point \( P_i \) can be calculated from the equation:

\[
Z_i = a/(\tan \xi + \tan \alpha) \quad .
\]

The angle \( \xi \) can be obtained by the coordinates of the image point P in the image sensor. For example, in the optical system with no lens distortion, the angle \( \xi \) is given by

\[
\xi = \tan^{-1} X / Z \quad .
\]

To obtain the angle \( \alpha \), a known grating pattern expanding from point A is projected onto the object surface. Now suppose that the bright plane of the projected grating is the plane \( m \) in Fig. 1. The angle \( \alpha \) can be obtained from a bright line on the object image by knowing in advance the value of \( \alpha \) for the bright line.

If the angle corresponding to the pitch of the projected grating is \( \alpha_0 \), the angle \( \alpha \) is given by

\[
\alpha = (n + \delta n) \alpha_0 \quad ,
\]

where \( n \) is the integer grating order and \( \delta n \) is the fractional order. The latter is determined from the phase of the sinusoidal intensity variation of the grating projected at the point \( P_0 \). This phase can be calculated from the four frames of intensity data recorded with four phase-shifted gratings. When the phase shift is \( \pi/2 \), \( \pi \), \( 3\pi/2 \) and \( 2\pi \), the fractional order \( \delta n \) at each point is then given by

\[
\delta n = (1/2\pi) \tan^{-1}((I_1 - I_3)/(I_4 - I_2)) \quad .
\]

The value \( \delta n \) is between \( \pm 1/2 \) by examining the signs of the numerator and denominator in Eq. (4).

When the phase difference between adjacent pixels is larger than \( 2\pi \), the integer grating orders become ambiguous. We shall define the height difference corresponding to the grating pitch, or phase of \( 2\pi \) as the effective pitch \( P \). The profile cannot be determined when the height difference between adjacent pixels exceeds the effective pitch. In the proposed method, two gratings with different pitches are projected on the object to determine the integer grating orders. Now suppose that two effective pitches for the two gratings are \( P_i \) and \( P_2 \) \( (P_2 < P_1) \) and the fractional orders are \( \delta n_1 \) and \( \delta n_2 \), respectively. Figure 2(a) shows the relation between calculated fractional height \( \delta n_i P_i \) \( (i=1,2) \)
and true height $H$. The difference of the calculated height $SA = \delta n_1 P_1 - \delta n_2 P_2$ is expressed in a step function as shown in Fig. 2(b). In this figure, $n_1$ and $n_2$ are integer grating orders for effective pitches $P_1$ and $P_2$, respectively. From this figure, we can estimate the integer grating orders $n_1$ and $n_2$ by calculating the difference $SA$. The distribution of difference $SA$ is repeated with a period of about length AB. This length will be called the equivalent effective pitch $\lambda$, which is given by

$$\lambda = \frac{P_1 P_2}{(P_2 - P_1)}.$$

3. Experiment

3.1 Measurements by Laser Interference Fringes

The experimental optical system is shown in Fig. 1. Gratings are generated with a laser Michelson interferometer consisting of a beam splitter and two mirrors, one of which is mounted on a piezoelectric transducer (P.Z.T.) for phase-shifting. The light source is an argon ion laser. The interference gratings (fringes) are expanded by the objective lens and projected on the object surface. The grating pitch can be changed by tilting another mirror. The grating pattern projected on the object surface is observed with an image sensor in a direction different from the projection. The grating pattern is digitized at $512 \times 512$ pixels, and can be shifted by applying a voltage...
to the P.Z.T. A microcomputer provides the required control signals to the camera, the P.Z.T. and the frame memory.

To verify the proposed method, the profile of an object with steps was measured. Figure 3 shows the experimental result of an object which produces a phase shift larger than the effective pitch. Measurements were made with the experimental system $a=672$ mm, $a_0=\pi/4$ in Fig. 1, $P_1=7.2$ mm, $P_2=8.6$ mm and $\Lambda=44.2$ mm. The experimental result shown in Fig. 3(c) agrees well with the step height of 20.0 mm. The average step height of measured values was 19.5 mm.

A tooth model of actual size was also measured with this method. Measurements were made with the experimental system where $a=276$ mm, $a_0=\pi/4$ in Fig. 1, $P_1=3.2$ mm, $P_2=4.0$ mm and $\Lambda=16$ mm. Figure 4(a) shows the result obtained for front teeth. This profile could be measured by a single grating because this model has a small step height. The profile of both tooth and gums was accurately measured.

On the other hand, a back tooth has a complicated profile. Figure 4(b)-(1),(2) show the experimental results obtained by single-pitch and two-pitch methods, respectively. As the step height between tooth and gums is more than half of the effective pitch, the step height cannot be determined by single-pitch analysis. However, by the two-pitch method, the profile could be accurately measured.

### 3.2 Measurements by Liquid Crystal Gratings

A sinusoidal grating was also obtained using a controlled liquid crystal projector instead of a Michelson interferometer. The light intensity projected through the liquid crystal can be changed at 256 levels by the digital output from a computer. A liquid crystal grating may be practical since this gives an accurate phase shift, arbitrary projecting pitch and constant surface brightness controlled by computer. Figure 5 shows sinusoidal intensity changes at a point on the object. Measurements were made with the experimental system at $a=1,073$ mm, $a_0=\pi/4$ in Fig. 1, $P_1=11$ mm, $P_2=13.3$ mm and $\Lambda=63.6$ mm. Figure 6 shows the result obtained for an object with step height of 20.0 mm. The average step height was 20.16 mm and the measurement error distribution was less than $\pm 0.2$ mm.

### 4. Conclusion

A new system for profiling an object with steep steps was constructed based on two projected gratings with different pitches and a phase-shifting technique. An object with step height of 20 mm was measured with standard variation below 3% by interference fringes and liquid crystal gratings.

### References