Broad-Band Light-Wave Correlation Topography Using Wavelet Transform

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(Accepted January 18, 1995)

The absolute longitudinal distance between two points can be determined by the corresponding correlation peaks of two light-waves from a broad-band light source. Using this technique, the height of three-dimensional objects can be measured without 2π phase ambiguity. We can also detect the absolute position of scattering seeds in sub-surface or bulk materials such as defects, dislocations or impurities of high purity materials. The wavelet analysis is used to determine the correlation peaks. This technique can be applied to measurement of thickness of a few hundred microns.

Key words: interferometry, correlation, broad-band spectrum, wavelet analysis

1. Introduction

Correlation topography based on phase-shifting interferometry with a broad-band light source is one of the novel techniques to measure the absolute longitudinal distance between two points. This technique is also known as white-light interferometry. In correlation topography, the zero-order interference fringe or the envelope peak of interferometric fringes is detected. It has been successfully applied to thickness measurement, profilometry and material identification.

Although interferometric techniques with a coherent light source have the advantage of high wavelength accuracy, it is difficult to determine the absolute fringe order, and to solve the 2π phase unwrapping problem. The method of exact fraction, on the other hand, which can determine the absolute distance using two or three light sources of different wavelength requires complex analysis. To avoid those problems we used correlation topography with broad-band light source.

The light-wave correlation method allows absolute measurement with high accuracy. We measured a three-dimensional shape with an accuracy of less than one wavelength without 2π phase ambiguity. To acquire an interferogram, phase-shift is introduced to one arm of the interferometer by moving a mirror using a mechanical or a piezoelectrical apparatus.

The wavelet transform is linear like the Fourier transform. The kernels, which are called wavelets, are derived from a mother wavelet by only a scale change in the space domain. This analysis is suitable for localized self-similar signals, and it has been applied to signal compression, optical information processing and image analysis. A remarkable feature of this method is that it represents one-dimensional data in both the space and frequency domains; we can easily obtain a self-similar pattern from two-dimensional wavelet coefficients, and the peak points of the interferometric fringe envelope can be determined.

In this paper we discuss the use of the wavelet transform to analyze white-light interferometric fringes. The envelope peaks of interferograms are easily detectable by the wavelet transform spectrum.

2. Broad-Band Light-Wave Correlation

A two beam interferometric fringe with a coherent light source is written as,

\[ I(z) = 1 + \cos \frac{2\pi z}{\lambda} , \]

where \( \lambda \) is wavelength and \( z \) is path difference between two beams. Using a broad-band light source \( \beta(\lambda) \) if we measure a transparent material which has a thickness \( d \) and a refractive index \( n(\lambda) \), the two beam interferometric fringes are given by:

\[ I(z) = \sum_{\tilde{\lambda}} B(\tilde{\lambda}) \left[ 1 + \cos \frac{2\pi}{\tilde{\lambda}} (z - n(\tilde{\lambda})d) \right] d\tilde{\lambda} . \]

If the material has optical dispersion, the effective position \( z_0 \) where the optical path difference is equal to zero depends on its wavelength. The phase of fringes is written as:

\[ \phi = \frac{z - (n(\lambda) - 1)d}{\lambda} . \]

The fringe of the highest visibility is observed at the point of minimum deviation of the phase \( \phi \) from its wavelength. So, we determine the position \( z_0 \) from \( d\phi/d\lambda = 0 \), as

\[ z_0 = \left[ n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda} \right] d - 1 \]

where \( n_0 = n - \lambda (dn/d\lambda) \) is referred to as the group refractive index determined by the group velocity in dispersive media. In this case the peak of the central fringe does not coincide with the envelope peak of the fringes. The envelope peak is needed to obtain the correlation. A typical interferogram is shown in Fig. 1(a). The correlation peak can be determined by fitting the fringe peaks with a polynomial (Fig. 1(b)). An experimental setup using a
Twyman-Green type interferometer is shown in Fig. 2. A sample under test is set within one arm of the interferometer. Collimated light from a Halogen lamp with a spatial filter is used as a light source. We scan the reference beam by controlling a linear stage with a stepping motor (Micro Control Co., Ltd. model UE 31pp). The positional resolution is 0.1 μm. A cooled CCD camera of Hamamatsu Photonics Co., Ltd. (model C3640) is used. The dynamic range of this camera is 14 bits because we must detect both a large correlation peak of the surface reflection and a small peak of scattering sources. It is 1,024 by 1,024 pixels and pixel size is 19 μm by 19 μm without any separation for data transfer area. The frame rate is 4 seconds. As the amount of the data of one frame is more than 1 Mb, total data exceed half giga-bytes. We used a magneto-optic data storage system with a workstation of HP 9000 series 735. The sample under test shown in Fig. 3 consists of a cover-glass on a mirror. Figure 4 shows an interferogram of a pixel. In this case we can observe four correlation peaks corresponding to front-surface reflection, rear-surface reflection, reflection by mirror and multiple reflection.

3. Wavelet Data Analysis

The simplest way to obtain the peak of the envelope is to fit the fringe peaks with a polynomial function (Fig. 1(b)), because the correlation peak does not coincide with the peak of the zero-order fringe peak in the presence of dispersion. In this method only the fringe peak positions are used, and therefore high S/N ratio cannot be expected. Taking the auto-correlation of the data, we can also obtain the distance of the peaks. This technique has a low spatial resolution because the shape of the interferometric fringes are not always the same in dispersive media.

The Fourier transform is a powerful technique to analyze frequency components in the signal. Fourier analysis is appropriate for expressing smooth signals, but it is not good to inspect localized, non-stationary or noisy signals.

We used the wavelet transform technique since it has a significant feature allowing inspection of both periodicity and locality of one dimensional signal. Compared with the windowed-Fourier transform, the wavelet transform has a high frequency accuracy at low frequency area and a high spatial resolution at high frequency area. As it is impossible to make an orthogonal set of the kernels in the continuous wavelet transform, the result of transformation depends on both original signals and wavelets. This is not a serious problem in the case of our analysis to find a peak position of the fringe envelope.
A kernel of the wavelet transform is defined as:

\[ h_{a,b}(x) = \frac{1}{\sqrt{a}} h \left( \frac{x-b}{a} \right), \tag{5} \]

where \( a \) is a dilation factor and \( b \) is a translation factor. We used the Morlet wavelet because of its similarity to the interferometric fringes. The mother wavelet is written as

\[ h(x) = \exp(i2\pi f_0 x) \exp \left( -\frac{x^2}{2} \right), \tag{6} \]

where \( f_0 \) is the spatial frequency. The spatial and the frequency resolution of wavelets are determined by spread of bases in space and in Fourier transform domains. The Morlet wavelet we use is spread in both the space and the frequency domains. The wavelet transform of one-dimensional data \( s(x) \) can be written by the inner-product as:

**Fig. 5.** The kernels of the wavelet transform represented by Eqs.(5) and (6) when \( f_0 = 1 \) and \( b = 0 \). (a) \( a = 0.5 \), (b) \( a = 1.0 \), (c) \( a = 2.0 \).

**Fig. 6.** Wavelet transform of the interferogram at one pixel. The vertical axis is the position of the sample, and the horizontal axis corresponds to the spatial frequency.

**Fig. 7.** Cross-sectional distribution of the wavelet transform shown in Fig. 6, where \( a = 0.1 \).
\[ W_d(a,b) = \langle h_{a,b} s(x) \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} h^*(\frac{x-b}{a}) s(x) dx. \]  

Figure 5 shows the wavelet bases depending on different factors of \( a \) and \( b \). A wavelet transform spectrum is shown in Fig. 6 in which the horizontal axis is the frequency axis and the vertical axis is equivalent to the position of the original interferogram (Fig. 4). From a cross-sectional distribution at \( a=0.1 \) (Fig. 7) we are able to clearly identify the peaks of the correlation. Considering the fringe frequency, we selected the value \( a=0.1 \). Background noise and the dispersion effect are removed. From the spacing of the peaks and the group refractive index, the thickness of the cover glass is estimated as 145.1 \( \mu \)m.

In the wavelet transform, frequency selectivity is worse when we select the higher frequency, where S/N ratio is reduced though spatial resolution is becoming high. In our case the spatial resolution is estimated as about \( \pm 1 \mu \)m (10% of the kernel envelope) at the selected carrier frequency. The mechanical stability of the moving apparatus is within \( \pm 5 \mu \)m, so the result of the measurement is expected to be 145 \( \mu \)m\( \pm 6 \mu \)m. The spatial resolution can be increased when we select the optimized wavelet kernel and use the precision mechanics of a piezoelectric transducer.

4. Conclusions

Correlation topography is a valuable technique to use in measuring the absolute distance between two points in the range of several hundred micrometers. Using it we are able to get a three-dimensional shape, the absolute depth of a gap, as well as the optical thickness of a transparent or a translucent thin film. This method can be applied to identify the position of scattering seeds or dislocations near a surface.

Wavelet analysis is a powerful technique to obtain the peak position of the envelope of interferometric fringes. The combination of correlation topography and wavelet transform enables us to make highly accurate measurement of an absolute shape without \( 2\pi \) ambiguity.

References