How does the Normal Force on the Rigid Rod Behave during Falling?

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Abstract: In our previous studies, we investigated the friction which acts on a rigid body during falling before colliding with a floor. In the next step, we want to know how the normal force behaves during falling. As is well known, the maximum static friction depends on the normal force. Therefore, it is important to know the behavior of the normal force for the purpose of researching how the friction acts on the rigid rod. After this study will be completed, we will investigate what condition will give rise to sliding or floating in falling.

Keywords Fall, Rigid rod, Friction, normal force, Coefficient of Friction.

1. Introduction
According to Ministry of Internal Affairs and Communications, the rate of the over 65 aged people is 24.1 in Japan in 2012. Aged people often fall and sometimes are injured critically. Therefore, it is needless to say that prevention of falling is very important. We have investigated this human fall problem by using the model of the rigid rod in recent years. In our previous studies, the velocity of the upper end of the rod just before colliding with a floor and behavior of the friction have been clarified by solving the equation of motion of the rigid rod.

We will show how the normal force behaves during falling by similar method as was described in our previous papers. In detail, let us explain the procedure about the calculation in the next paragraph. We wish this result will help us to understand the mechanism of the damages of the aged people by fall.

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2. The model of the rigid rod
As we mentioned in our previous studies, we assumed that human body can be regarded as the rigid body with homogeneous mass density for simplicity. The real situation of falling can be regarded as a rotation around the foot approximately. This model is shown in Fig.1.

Fig.1. The model of the rigid body with homogeneous mass density
In this case, the equation of the motion can be expressed as follows:

\[ \frac{m}{3} \ddot{\theta} = - \frac{1}{2} mgl \cos \theta \]  

(1)

\[ \frac{m}{3} l^2 \]  

is the moment of inertia around the z axis which is perpendicular to x-y plane as is shown in Fig.1. Each overdot denotes a time derivative.

The solution of the above equation (1) is written as

\[ \dot{\theta}^2 = \frac{2g}{l} (1 - \sin \theta) \]  

(2)

under the initial condition of

\[ \dot{\theta}(0) = 0 \quad \text{and} \quad \theta(0) = \frac{\pi}{2} \]  

(3)

3. Calculation of the behavior of the normal force

In the next step, we will consider the behavior of the normal force. It is one of the constraint forces including the friction. Let us indicate the resultant force \( R \) in Fig.2. The angle \( \alpha \), which is also shown in Fig.2., depends on \( \theta \) and time \( t \).

First, we will find out the magnitude of this force \( R \) as below. For this purpose, it is needed that the equation of motion (1) is described both in the radial direction and in the tangential direction again.

Then, in the radial direction, the equation of motion (1) can be written as,

\[ \frac{m}{2} \ddot{\theta} = mg \sin \theta - R \cos \alpha \]  

(4)

The equation of motion (1) in the tangential direction is

\[ \frac{m}{2} \ddot{\theta} = -mg \cos \theta + R \sin \alpha \]  

(5)

The product of equation (5) multiplied by \( \cos \theta \) is

\[ \frac{m}{2} \dot{\theta} \cos \theta = -mg \cos^2 \theta + R \cos \theta \sin \alpha \]  

(6)

On the other hand, The product of equation (5) multiplied by \( \sin \theta \) makes new equation as follows:

\[ m \frac{l}{2} \dot{\theta} \sin \theta = mg \sin^2 \theta - R \sin \theta \cos \alpha \]  

(7)

The subtraction the equation (7) from the equation (6) is

\[ m \frac{l}{2} (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) = -mg + R \sin (\theta + \alpha) \]  

(8)

At last, by the substitution equation (1), (2) in equation (8), it changes to the next expected equation.

\[ N = R \sin (\theta + \alpha) \]

\[ = mg + \frac{mg}{2} (\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) \]

\[ = mg + \frac{mg}{2} \left(-\frac{3}{2} \cos^2 \theta - 3 \sin \theta + 3 \sin^2 \theta \right) \]

\[ = mg \left(3 \sin^2 \theta - 1\right)^2 \]  

(9)

In the equation (9), \( N \) is the normal force.

![Fig.2. The normal force N and friction F](image)

The solution (9) is expressed by the graph in Fig.3.. We can find out that the normal force \( N \) is zero in one angle which is about 19.5 degree. It is also easily known that \( N \) is positive during falling from the form of the square. This result is consistent with the fact that normal force usually acts vertically in the upper direction. When \( \theta \) is 90 degree, \( N \) have to be mg which the rod weigh. This matter can be taught by the solution (9).
Furthermore the solution teaches us that the normal force \( N \) is less than \( mg \).

![Graph of N in the unit of mg](image)

**Fig.3. The behavior of the normal force \( N \)**

### 4. Discussion

In our previous study, we calculated the velocity just before colliding with a floor. It is about 25.5 km/h. Many people have been involved in traffic accidents and seriously hurt for many years.[1] Although it seems difficult to estimate exactly, it would be supposed that some traffic accidents may happen with the velocity similar to this result. In fact, fall also sometimes leads to the serious damages such as bone fracture or traumatic subdural hematoma[2]. Falling with slipping is also dangerous as well as falling without slipping. The accurate damage of fall the in our previous another study[3], we showed that the velocity in both cases is same not as expected. It is very important and inevitable to investigate a change in the acceleration to elucidate its damage to human body such as brain, chest, femur or neck precisely[4],[5],[6].

However, we are still interested in when the rigid rod begin to slide on the ground.

For this purpose, we have to know the behavior of the friction \( F \). In fact, we have investigated about it before.

The result is shown in Fig.4, and the formulation to express this graph is written as below. We have to cut its derivation for lack of space.

\[
F = -\frac{3mg \cos \theta}{4} (2 - 3 \sin \theta) \quad (10)
\]

![Graph of friction](image)

**Fig.4. The behavior of the friction \( F \)**

In dynamics, it is generally admitted that the relation between the normal force and the friction force as is shown below.

\[
\mu N \geq F \quad (11)
\]

In this expression, \( \mu \) the is coefficient of friction and generally depends on materials. In most cases, coefficients of friction range from near 0 to greater than 1. The motion thought in this paper has to satisfy the condition (11). Otherwise the rotation around the origin without slipping cannot be realized.

Now, we will take the assumption that \( \mu \) is 0.5 In this case, let us consider about the discussion above in more detail. In Fig.5, the relation between the magnitude of friction and maximum of friction is shown.

![Graph of \( \mu N \) or \( F \) in the unit of mg](image)

**Fig.5. The behavior of the friction \( F \)**

Some findings are easily found out as written below. There is one angle which is named \( \theta_o \) and is nearly equal to 37 degree.

1. \( 0 \leq \theta \leq \theta_o \quad F \geq F_{\text{max}} \)
2. \( \theta_o \leq \theta \leq 90 \quad F \leq F_{\text{max}} \)
Since the situation (1) is impossible in actual falling, the rigid rod cannot fall with fixed lower end. Instead, it will slide or float. It depends on the dynamical situation, which phenomenon occurs. We have to clarify what the dynamical situation is. It needs further consideration.

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**References**


