The Source of the Ownership System*
A reexamination of the anarchy model

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0. Abstract
The main purpose of this paper is to illustrate clearly how the distribution is determined on the assumption that the private ownership is not regarded as absolute truth.

Under this assumption, members in society settle into a natural distribution after struggle. I show it is effective to divide two cases, CASE 1 and CASE 2 on the basis of the utility of members of a society at the natural distribution.

In CASE 1, redistribution does not occur. In this case, private ownership will be established if an effective law can be prepared. If the effective law can not be prepared, the distribution becomes unstable.

In CASE 2, redistribution occurs. In this case, the distribution of goods is determined through a political process. This case leads a society to public ownership. But, even in this case, if members in a society value goods highly relative to the state without struggle, the society approaches private ownership.

1. Introduction
The distribution of income depends on many factors. Not all of them are ones through market mechanism. Actually, there are many redistribution processes in a society. The distribution which is determined through market mechanisms forms only a part of the income. Much economic literature has paid attention only to the income through market mechanism. It seems to me that this attitude should be reformed. On the contrary, market mechanisms should be considered relatively as one of the mechanisms people build up. So we should explain how income distribution is determined by investigating how people build up the institution.

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In this paper, production behavior is neglected. Under this assumption, the neoclassical theory explains by using the Edgeworth box that individuals' rational behavior cause an exchange of goods between each other if an initial endowment is given in the economy. But, it is conceivable that the individuals' rational behavior cause a struggle concerning goods.

Exchange occurs only if the property right is established. If the property right is not established in a society, exchange should not happen. Of course, the property right is not given by God. It is people who build up the property right. Therefore, we must investigate how the property right is established. Otherwise, we could not explain how the distribution is determined from individuals' rational behavior.

Thus we investigate how the distribution is determined on the assumption that the property right is not established in a society. We call this society an anarchy society. In section 2, the anarchy model owing to Bush [4], Bush-Mayer [5], and Buchanan [2] is represented. In section 3, we divide the two cases, in one of which redistribution does not occur under the rule, in the other of which redistribution does occur under the rule. In section 4, we represent a concept of an effective law, which is necessary to keep the rule. In section 5, we examine the relationship between the above two cases and an economic structure. We conclude the above arguments in section 6.

2. The Anarchy Model

We consider a society consisting of two individuals, N and S. And we consider all consumer goods in the society as one good, $X$. We assume that the amount of $X$ in the society is fixed and $X$ is divisible. We also assume that there is no production behavior.

Now, we assume that the society is anarchy. An anarchy society is defined as follows:

**DEFINITION**: A society is said to be anarchy if the society has no law and so the property right is not established in the society.

Therefore, individuals in the society can expend efforts stealing from other individuals and must expend efforts preventing others from stealing from them.

We use the following notations:

- $e^tX$—the initial income of individual i.
- $\bar{X}=\sum e^tX=e^sX+e^nX$.
- $X_i$—natural income of individual i. It is equal to his initial income plus or minus the amount transferred to or from him through interaction with individual j.
- $E_i$—the level of effort expended by individual i taking income from individual j and protecting his own income from j.
- $a_i$—the ability of individual i concerned with $E_i$. That is, effectiveness of $E_i$. We assume $a_i>0$ and $a_i$ is constant.

$(i=S,N; i\neq j)$

Now, we assume that the preferences of the two individuals are represented as follows:

$$U_i=U_i(X_i, E_i) \quad (i=S,N) \quad (1)$$

Concerning these utility functions, I put the following assumptions:

(A-1) $U_i$ is a differentiable function.
(A-2) $UX_i>0$ and $UE_i<0$, 

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where \( U_X \) and \( U_E \) are the partial derivatives of \( U_i \) with respect to \( X_i \) and \( E_i \).

(A-3) \( U_i \) is strictly quasi-concave on its domain.

(A-2) means that \( X \) is a normal good but \( E \) is an unpleasant good. Note that \( U_i \) decreases as \( E_i \) increases.

The budget constraint that \( i \) face is as follows:

\[
X_i = iX + a_i E_i - a_j E_j \\
\text{where } i, j = S, N; S \neq N
\]

and \( X_i \geq 0, 0 \leq E_i \leq T \), where \( T \) is the maximum level of effort \( i \) can expend.

\( i \) maximizes \( U_i \) subject to the budget constraint. Using Lagrangian,

\[
L = U_i(X_i, E_i) + \lambda(X_i - iX + a_i E_i + a_j E_j)
\]

\[
L_{X_i} = U_{X_i} + \lambda.
\]

\[
L_{E_i} = U_{E_i} - a_i \lambda.
\]

Putting these two equations to zero gives the marginal rate of substitution between \( E_i \) and \( X_i \) in the optimal point.

\[
- \frac{U_{E_i}}{U_{X_i}} = a_i
\]

Figure 1 shows the above discussion. \( X_i \) and \( E_i \) are assumed to be endogenous variables. And \( iX \) and \( a_i \) are assumed to be exogenous variables. Figure 1 shows that \( S \)'s utility maximization given \( E_N \) results in the solution, \( (E_S, X_S) \). Then, if we move \( E_N \), the solution changes in response to this. Therefore, the following relation holds:

\[
E_S = E_S(E_N).
\]

This is a reaction function of \( E_S \) for \( E_N \). Similarly, we can write a reaction function of \( N_N \) for \( E_S \):

\[
E_N = E_N(E_S).
\]

Writing these reaction functions into one,

\[
E_i = E_i(E_j), \quad i \neq j.
\]

Figure 2 shows an example of one case where the relationship between reaction curves of both individuals is represented.

These curves intersect on the point, \( (E_S^*, E_N^*) \). This point is an equilibrium in the sense that both individuals need not change \( E_i \) so long as they know the other's present choice of \( E_j \). Namely, this equilibrium is a kind of Nash-type one. \( X_i^* \) in this equilibrium can be solved as follows:

\[
X_i^* = iX + a_i E_i^* - a_j E_j^*.
\]

This \( X_i^* \) is called a natural distribution. So, we can get the following definition.

**DEFINITION**: A natural distribution is a distribution under which no individual in
the anarchy society has an incentive to change his behavior independently.

The following theorem is shown by Bush = Mayer [5].

Bush = Mayer's THOREM: For the anarchy society, if assumptions (A-1), (A-2) and (A-3) holds, then a natural distribution exists.

proof. See Bush = Mayer [5]//

We, moreover, put the strong assumptions from now on in order to avoid the trivial problem.

(A-4) There is no corner solution in the natural distribution.

(A-5) The natural distribution is determined uniquely.

Natural distribution is a kind of solution of non-cooperative game. But, effort \((E_i)\) is an unpleasant good for both individuals, they should realize that they could be better off if they did not expend their efforts. Therefore, it is possible that both individuals will agree that they should reduce the level of their effort to zero. If so, we can say that they have made a rule that \(E_i (i=S,N)\) is zero. Then, we can write the definition as follows:

DEFINITION: We say both individuals make a rule that \(E_i=0\) if they make a contract in the anarchy society that they reduce their effort to zero.

Well, what becomes of the distribution under this rule made by both individuals? Since effort is an unpleasant good for both individuals, they accept less distribution than the natural distribution under the rule. Let us put \(^0X_i\) so as to satisfy the following equation:

\[
U_S(X_i^*, E_i^*) = U_S(0, X_i, 0). \tag{6}
\]
the case of $N$

$^0X_N \leq X_N^A$.

Figure 4 unifies the cases of $S$ and $N$ into one diagram. $N$'s diagram is turned upside down. $E_S^*$ and $E_N^*$ in figure 4 correspond to the $E_S^*$ and $E_N^*$ on figure 2. Of course, $E_S^*$ and $E_N^*$ need not have the same quantity.

The region surrounded by $^0X_S$ and $^0X_N$ in figure 4 is the one that both individuals accept. Therefore, the distribution under the rule is determined inside this region. Mathematically, this region can be written as follows:

$$A(^0X_S, ^0X_N) = \{ (X_S^A, X_N^A) | ^0X_i \leq X_i^A \text{ for } i \}.$$ (7)

If the distribution is determined outside the region, either $S$ or $N$ will reject this distribution, even by main force. Therefore, we can get the following proposition.

PROPOSITION 1: The distribution of both individuals under the rule is determined inside the region which is Pareto superior to the natural distribution.

3. The Relationship between the Distribution under the Rule and the Initial Income

We can take the relationship between the distribution under the rule and an initial income into consideration by means of dividing the following two cases.

CASE 1

The initial income is inside the region which is Pareto superior to the natural distribution.

$^0X_i \leq X_i$ for $i$. (8)

For example, a point $C$ in figure 4. In this case, the initial income is accepted. Concerning a point $C$, $S$ has less income than the natural distribution, to be sure, but since $S$ prefers a point $C$ to a struggle condition, he will give priority to the initial income, a point $C$.

CASE 2

The initial income is outside the region which is Pareto superior to the natural distribution.

$^0X_i > X_i$ for $i$. (9)

For example, a point $D$ in figure 4. In this case, the initial income is rejected. Concerning a point $D$, $N$ rejects his initial income because he would prefer to struggle with $N$ rather than to accept his initial income. Therefore, in CASE 2 redistribution occurs at least $(^0X_i - X_i)$. So, we get the following proposition:

PROPOSITION 2: Redistribution occurs when the initial income is outside the region which is Pareto superior to the natural distribution.

This redistribution does not necessarily lead to an equality in their income. A point $F$ in figure 5 is an example.
The process of these cases may occur without passing through the route to the natural distribution. If both individuals know the other's preference, ability, and initial income, they need not have passed through the route to the natural distribution. Therefore, the initial income might move directly to the distribution under the rule.

4. Effective Law

For a while, we pick up the CASE 1 in the section 3. The rule is made. Distribution is determined. Then, do we have nothing to worry about? The answer is "NO". If an individual knows the other makes no effort to take an income from him, he will find it to his advantage to take income from the other in spite of making the rule. Namely, both individuals always have an incentive to break the rule. Therefore, an effective law that has a compelling power is required. Well, what is an effective law? We define an effective law as follows:

**DEFINITION:** An effective law makes an individual expend no effort taking from the other's income when he maximizes his expected utility.

The concept of an effective law is indebted to Becker [1]. In his study, social loss consists of damages from crimes, costs of apprehension and conviction and costs of carrying out punishment imposed. And the punishment system is arrayed so as to minimize the social loss. But his study assumes the existence of a social police system and judicial power. Our analysis does not assume them. Our concept: an effective law is a law that makes members of the society abide by the rule in the environment that we do not assume the existence of a police system and judicial power.

That \( E_i \) becomes zero when \( i \) maximizes his expected utility means that the solution in figure 1 is a corner solution. In order to have a corner solution, \( i \)'s budget constraint must have a slope which satisfies the following condition:

\[
-\frac{U_{E_i}}{U_{X_i}} \leq a_i.
\]  

(10)

at the point where \( X_i = X \) and \( E_i = 0 \), that is, the point which indicates \( i \)'s initial income.

Or, supposed

\[
-\frac{U_{E_i}}{U_{X_i}} \bigg|_{X_i=iX} = m_i,
\]  

(11)

we can write

\[
m_i \geq a_i.
\]  

(12)

But, this possibility was excluded by the assumption (A-4) which we put before. Therefore, we remove the assumption (A-4) from now on. Of course, we consider that what makes the equation (10) or (12) possible is a power of the law here.

If (12) holds for both individuals, the law is effective. In order that the law may be effective, we require a punishment for efforts that both individuals break the rule. Let the law impose a punishment tax \( \tau \) on one unit of effort \( (E_i) \). We define this \( \tau \) is a power of the law.

**DEFINITION:** A power of the law is defined as a punishment tax \( \tau \) on one unit of effort.

Without loss of generality, we consider the case where \( S \) breaks the rule. Then, if \( S \) is detected breaking the law, \( S \)'s income is as follows:

\[
X_S = X - \tau E_S \quad (\tau > 0).
\]  

(13)

Of course, \( S \) is not necessarily detected. If \( S \)
is not detected in breaking the law, S's income is as follows:

$$X_s = sX + asE_s. \quad (14)$$

Let $P$ a probability that $S$ is not detected breaking the law. And let $(1-P)$ a probability that $S$ is detected. $(0 \leq P \leq 1)$.

Consider a vector space $(E_s, X)$ space. Let a point $(0, sX)$ is $O$. See figure 6. Any point in the line indicating the equation (14) is represented by the vector normalized with respect to $E_s$, $e_1 = (1, as)$. Similarly, concerning (13), the following vector is used, $e_2 = (1,-\tau)$. Thus, a vector representing the expected budget constraint can be written as follows:

$$e_3 = Pe_1 + (1-P)e_2 = P(1, as) + (1-P)(1,-\tau) = (1, asP-(1-P)\tau). \quad (15)$$

Thus we can replace the condition (12) with as follows:

$$ms \geq asP - \tau(1-P). \quad (16)$$

Solving about $\tau$

$$\tau \geq \frac{asP - ms}{1-P} \quad (17)$$

Namely, in order to have an effective law, $\tau$ must be larger as a probability that stealing is not detected is increasing and as the preference of the good $X$ is increasing. The above discussion brings us to the following proposition.

**PROPOSITION 3:** An effective law works if a power of the law satisfies this equation;

$$\tau \geq \frac{asP - ms}{1-P} \quad \text{for } \forall i. \quad (18)$$

Next, we must take a feasibility of a power of the law. Is the equation (17) possible? In the two individuals case, $N$ must exercise this effective law alone. Can he do so?

In order to exercise this effective law, $N$ must have an ability which satisfies as follows:

$$asT \geq \frac{asP - ms}{1-P}E_s. \quad (19)$$

Or

$$as \geq \frac{asP - ms}{1-P} \frac{E_s}{T}. \quad (20)$$

where $T$ is the maximum level of effort $N$ can expend. (19) and (20) means that $N$ has at least the ability by which he can keep the effective law if he expends his effort as best as he can. Of course, $N$ does not necessarily have this ability. If $N$ does not have this ability, it is possible that $S$ breaks the law. Thus it may be probable that the society will turn back to anarchy. Therefore, we can say as follows:

**PROPOSITION 4:** The stability of the distribution of a society worsens if the one of the members of the society does not have an ability which enables him to exercise an effective law.
5. The Anarchy Model and an Economic Structure

In the previous section, we took the case when the distribution is determined uniquely. In this section, we consider CASE 2 in section 3. By PROPOSITION 1 we know redistribution occurs in this case. But, we have not yet known where the distribution is determined.

In the previous section, we show a power of the law is required in order to determine the distribution uniquely. PROPOSITION 4 shows it is difficult to determine the distribution uniquely even in CASE 1. Therefore, in CASE 2, it can be said that does rise the possibility that the distribution is determined politically within the region which is Pareto superior to the natural distribution.

If one of the properties of socialism is that the distribution is determined by a usual political process: public ownership, it is quite probable that CASE 2 leads a society to socialism.

PROPOSITION 5: In CASE 2 when redistribution occurs, a society approaches socialism.

Let us compare the following two figures. Figure 7 is the case when both individuals value a lack of effort \((-E)\) high relative to the good \(X\). Figure 8 is the case when both individuals value the good \(X\) high relative to a lack of effort \((-E)\).

In figure 7, the region which is Pareto superior to the natural distribution is broad. The amount of \(X\), the distribution of which is determined politically is large. That is, the case of figure 7 contains a strong property of socialism.

On the other hand, figure 8 shows that the region which is Pareto superior to the natural distribution is narrow. The amount of \(X\), the distribution of which is determined politically is small. That is, the case of figure 8 does not contain a strong property of socialism.

When the value of a lack of effort \((-E)\) approaches infinitely near \(0\), the distribution is determined uniquely. This leads to private ownership which is the characteristic property of capitalism. Then, we can get the following proposition.

PROPOSITION 6: Concerning the case when redistribution occurs (1) if both individuals value a lack of effort \((-E)\) high relative to the good \(X\), the society approaches socialism, (2) if both individuals value the good \(X\) high relative to a lack of effort \((-E)\), the society approaches capitalism.
6. Conclusion

First, we explained a redistribution process in compliance with Bush = Mayer. The case when a redistribution occurs was as follows: When an initial income brings to one of the members of a society less satisfaction than an income received by struggle. Then, he claims they should have more income when a rule is made. The other accepts his claim. Therefore, the other redistributes to him.

I think this redistribution process can be applied to other situations. Concerning the policy issue the rule of which is not fixed, the above explanation seems to be available. For example, the argument of a regulation beneficial to special groups, a compensation for damages, a reform of tax structure and so on.

Next, we presented the concept of an effective law. Everyone has an incentive to break the rule. In order to exercise control over this incentive, it is necessary to have an effective law. An effective law changes the member's budget constraints so that his expected utility maximization behavior results in zero effort. We showed, however, that it is be probable that one of the members of the society does not have the ability to exercise the effective law. If he does not have the ability, the distribution cannot help being unstable.

Next, we showed whether or not the above redistribution process occurs closely related with the distinction between capitalism and socialism. In the case when redistribution does not occur, the distribution is determined uniquely. After the distribution is determined, private ownership is established. On the other hand, in the case when redistribution occurs, it is difficult to determine the distribution uniquely so that it is probable that the distribution is determined through a political process within the region which is Pareto superior to the natural distribution. This direction leads a society to public ownership of this region, which is a characteristic of socialism. Moreover if members of a society become to value good (X) higher and higher relative to the state without struggle (lack of effort, −E), the uniqueness of the determination of the distribution is getting stronger and stronger. This direction approaches private ownership, which is a characteristic of capitalism.

Recent peacefulness all over the world decreases people's relative evaluation of the state without struggle and increases people's relative evaluation of the goods. According to my analysis, the duration of this peacefulness will lead the world to private ownership.

"We must seek a different road to the future." This is an quotation of Gorbachev's U. N. speech in Dec. 7, 1988. This means that even if the system of capitalist countries and socialist countries come to resemble each other, it is difficult for both systems to harmonize ultimately. But, according to our analysis, the road is not different due to lack of a difference of an incentive to production but because of the above analysis.

REFERENCE


