Particle-Tracking Calculation of Classical Transport in High-Beta Field-Reversed Configuration Plasma

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In this paper, classical particle transport processes in field-reversed configuration plasma is investigated by particle-tracking calculations. The end-loss rate is found to increase with ion temperature, and the temperature dependence is much stronger than that of the Bohm scaling and the empirical scaling.

Field-reversed configuration (FRC) plasma is sustained by the poloidal field that is generated by the diamagnetic toroidal plasma current. As a result, it has a high-volume-averaged beta value of about 0.8, which is advantageous for designing nuclear fusion reactors [1].

The transport properties of FRCs, however, are still unclear [2, 3]. Several studies have reported the particle lifetime due to classical transport in FRCs. Clemente et al. calculated the radial particle flux employing the simplified Ohm’s law [4, 5]. On the basis of their result, the particle confinement time was found to depend on magnetic field, density, and resistivity on the separatrix surface. The use of the generalized Ohm’s law has no significant effect on the confinement time [6]. Consideration of anisotropic pressure [7] modifies the analytical form of the equilibrium flux function, thereby changing the confinement time.

The aforementioned studies, however, neglect the kinetic nature of high-beta FRCs. The ion orbit patterns of FRCs can be of three types: betatron, figure-8, and small-gyroradius drift orbits [8]. In particular, the gyroradius of betatron particles are comparable to the field-null radius; they are quite kinetic in nature and cannot be described by a fluid model. Since electromagnetic fluctuations with wavelength shorter than the ion gyroradius has less effect on transport, Coulomb scattering is considered to dominate the transport of large-gyroradius ions in FRCs [9].

In the present report, we calculate the particle loss rate through the end of confinement region by using a particle-tracking method. Since ions are initially loaded inside the separatrix and over a wide range in a velocity space, we consider the particle effects of kinetic ions. In addition, as the transport process, we consider pitch angle scattering.

We use an equilibrium FRC state that satisfies the Grad–Shafranov equation for the particle-tracking calculation. The external magnetic field is maintained constant for the entire calculation; therefore, the plasma pressure is also constant. The ion temperature is two times as high as the electron temperature and is assumed to be uniform inside the separatrix, which on the basis of the experimental evidence is a valid assumption [10, 11]. Consequently, the plasma density increases with decreasing temperature. The radius of calculation region $r_w$ is 0.17 m and the external magnetic field is 0.2 T. When the ion temperature is 100 eV (50 eV for electrons), the field-null density is $6.6 \times 10^{20} \text{ m}^{-3}$. Note that $r_w$ is the same as the inner radius of the theta pinch coil of the NUCTE-III machine [12]. The calculated equilibrium profiles for the flux and density are shown in Fig. 1, where $n_{\text{max}}$ and $\psi_w$ are the field-null density and calculation boundary flux, respectively.

Slowing down collisions are neglected to keep the average kinetic energy (i.e., the ion temperature) constant. The pitch angle scattering reported in Ref [13] is reproduced. The number of ion-loading meshes is 500 in the $r$ direction and 128 in the $z$ direction. The present calculation is only for ions inside the separatrix. At each loading position, $7 \times 7 \times 7$ superparticles with $\pm 3, \pm 2, \pm 1, 0$ times the thermal velocity in the three-dimensional direction start to move in the magnetic field. A Maxwellian ion velocity distribution is assumed, and the superparticle weight (i.e., their particle number) is determined from the distribution.

Fig. 1 Radial profile of poloidal flux and density at midplane.

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Because of pitch angle scattering, plasma ions leave the separatrix and move to the open-field region. Eventually, they move through the mirror end, which is defined as the axial position $z = z_M$. The end-loss ions are counted, and the ratio increases with time, as shown in Fig. 2. Here the end-loss ratio is the number of end-loss ions per initial ion number confined in the separatrix, and the normalization time $\tau = 0.21 \mu s$. The calculation is carried out for 25, 50, 75, and 100 eV ions. As the ion temperature increases, the end loss increases despite a higher collision frequency.

Because the plasma pressure is constant for all the calculations, the classical diffusion coefficient is proportional to $T^{-1.5}$. The particle confinement time derived from classical physics, therefore, increases with temperature. However, the result of the calculation is considerably different from the classical prediction based on a fluid model; we attribute this difference to the high-beta nature of FRCs. When kinetic ions in betatron and figure-8 orbits are subject to pitch angle scattering, the step size of the diffusion process is comparable to the separatrix radius. To clearly show the effect of the finite Larmor radius (FLR) on particle transport, the conventional kinetic parameter $s$ is introduced [4]. The FLR parameter is

$$s = \int_{r_i}^{r_o} \frac{(r/r_i) dr}{r_s},$$  

where $r_i$ is the ion Larmor radius. For our approximation, $r_i = \sqrt{2m_i T_i}/(q_i B)$, where $m_i$ and $q_i$ are the mass and charge of plasma ions, respectively. In the neighborhood of the field-null, $r_i$ increases with $1/B$. Since the gyroradius of betatron particles is limited to the field-null radius, our estimate of $s$ is less than a statistically averaged value of $s$ by probably a factor of 2 to 3. Suppose the number of plasma ions exponentially decreases with time, then we can estimate the particle decay time from the end-loss ratio shown in Fig. 2. Its relation to the FLR parameter $s$ is shown in Fig. 3. Although the temperature range in our present calculation is similar to typical experimental conditions, the entire range of the FLR parameter $s$ is in the kinetic regime. The data shown in Fig. 3 are fit to

$$\tau_N \propto s^{1.74}.  \tag{2}$$

The above scaling has a much stronger dependence on the plasma temperature ($\sim T^{-2.37}$) than the Bohm scaling ($\sim T^{-0.5}$). The classical transport rate, however, is much smaller than the Bohm rate; thus, the particle decay time is on the order of milliseconds. The discrepancy between our classical scaling and the empirical scaling [14] suggests the presence of more active fluctuations induced by FRC plasma instabilities. Exploratory research on inherent instabilities that cause proper radial transport scaling will be focused in the future study.