54. Decay in the Seismic Vibrations of a Structure by Dissipation of their Energy into the Ground.

By Katsutada SEZAWA and Kiyoshi KANAI.
Earthquake Research Institute, Tokyo Imperial University.
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In the absence of some form of damping resistance, almost any structure would suffer serious damage in an earthquake, owing to resonance. The question of air resistance and damping in the material used in the structure as well as in the foundation have been studied by some investigators, but the dissipation of vibrational energy in the form of seismic waves transmitted into the ground, which seems to be the most important part of the problem, has not yet received the attention of people. From the results of our recent calculations with respect to the decaying of seismic vibrations of various simple and framed structures, the case of shearing as well as of flexural vibrations, analogous to the respective horizontal oscillations of a tall building with rigid floors and one with flexible floors, seem to be the one that we should investigate.

We shall first discuss the case of an n-storied structure with rigid floors. Let the axis of $x$ be drawn vertically downwards with its origin at the free surface of the earth, and let $\rho, \lambda, \mu, \rho' (= m/\alpha_1)$, $G(=12.4E\rho^2/l_1^2)$ be the density, the elastic constants of the earth, and the effective density, and effective rigidity of the structure, where $E, j, l_1, \alpha$ are Young’s modulus, radius of gyration of section, length and total sum of the sectional areas of columns between two adjacent floors. Provided the incident transverse waves in the earth are propagated vertically upwards, the boundary conditions are such that there is no effective shearing stress at the upper end of the structure, while the displacement and the shearing stress at its lower end are continuous respectively with those in the earth resulting from incident transverse waves, and scattered waves, both longitudinal and transverse. By taking principal terms of the scattered waves and neglecting the effect from the formation of surface waves, we obtain the approximate value of the horizontal deflection of a structure of the form

$$u' = \frac{(7\sqrt{\lambda/\mu} + 2 - 4) \cos k'(l + x)}{\sqrt{\left\{3\sqrt{\frac{\rho' G (\frac{\lambda}{\mu} + 2)}{\rho \mu}} \sin k'l\right\}^2 + \left\{4\left(\sqrt{\frac{\lambda}{\mu}} + 2 - 1\right) \cos k'l\right\}^2}} \cdot \cos \left\{\rho t - \tan^{-1}\frac{3\sqrt{\frac{\rho' G (\frac{\lambda}{\mu} + 2)}{\rho \mu}} \sin k'l}{4\left(\sqrt{\frac{\lambda}{\mu}} + 2 - 1\right) \cos k'l}\right\},$$

in which $k' = \rho v / \rho' G$, $l = nl_1$, corresponding to incident transverse waves.

\[ u_1 = \cos (pt + kx) \]. The left hand term under the root sign of the denominator of the above equation represents the effect of dissipation of energy scattered as seismic waves. The larger the ratio of \( G/\mu \) or that of \( \rho'/\rho \), the larger the decrease in the amplitudes of vibrations at the periods corresponding to the resonance condition, \( \cos k'l = 0 \), of the case with no dissipation of energy. It is also evident that the displacement at the lower end is always zero under the resonance conditions. It is possible to prove that the logarithmic decrement of the free vibrations of the same structure is expressed by

\[
\frac{\sqrt{G/\rho'}}{2l} \log \frac{3\sqrt{(\rho'/G/\mu)(\lambda/\mu + 2)} + 4(\sqrt{\lambda/\mu + 2} - 1)}{3\sqrt{(\rho'/G/\mu)(\lambda/\mu + 2)} - 4(\sqrt{\lambda/\mu + 2} - 1)}.
\]

The next problem to be solved is the case of a structure with flexible floors, the incident waves being also transverse waves with the amplitudes orientated horizontally. The upper end is free from bending moment and shearing force, while the displacement and the shearing force at the lower end of the structure and those of the earth are respectively continuous. There is moreover another condition with respect to the inclination of the structure at that end which we may consider as remaining always vertical. The solution of the problems satisfying all these conditions corresponding to incident transverse waves, \( u_1 = \cos (pt + kx) \), particularly at the effective upper end, \( x = -l = (n + \frac{1}{2})l_1 \), and at the lower end, \( x = 0 \), take the form

\[
y_{x=-l} = (\cos \sqrt{p cl} \cosh \sqrt{p cl})\phi, \quad y_{x=0} = (\cos \sqrt{p cl} \cosh \sqrt{p cl} + 1)\phi,
\]

where

\[
\phi = \frac{(7\sqrt{\lambda/\mu + 2} - 4) \cos (pt - \tan^{-1}\gamma / \chi)}{\sqrt{\gamma^2 + \chi^2}}, \quad c = \left( \frac{m}{l_1 E \alpha f^2} \right)^{\frac{1}{4}},
\]

\[
\gamma = 3\sqrt{\frac{\lambda}{\mu} + 2} \frac{E_f^2}{\mu k_f^2} (\sqrt{p cl})^3 (\cos \sqrt{p cl} \sinh \sqrt{p cl} + \sin \sqrt{p cl} \cosh \sqrt{p cl}),
\]

\[
\chi = 4(\sqrt{\lambda/\mu + 2} - 1) (\cos \sqrt{p cl} \cosh \sqrt{p cl} + 1).
\]

The nature of the problem for the case under resonance conditions is similar to that of the preceding case, except with the difference that in the former, however small the ratio of \( E_f^2/\mu k_f^2 \) may be, the amplitudes of vibrations under higher resonance conditions become rather less

\[ Fig. 1. \]
than those at frequencies out of resonance. This may also be seen from Fig. 1, which shows the amplitudes of the case, $E_j^2/\mu k l^3 = 0.55$ at different frequencies corresponding to the incident waves of unit amplitudes transmitted through the earth in which $\lambda = 14 \mu$. The short vertical strips represent the frequencies with which the resonance should take place in the usual sense.