79. Analysis of the Field of Diurnal Variation of Terrestrial Magnetism of the Different Types.

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The observed materials on days of extremely different types of diurnal magnetic variation at our observatory, i.e. Sept. 23 and 24, 1933 and June 21 and 23, 1934,¹ were made available to study its progressive change by courtesy of the directors of all the accessible magnetic observatories in the world (Fig. 1). It may be of interest and importance to give the aspect of potentials of these diurnal magnetic variations on the earth on each of these days. This paper gives a summary of the results of a mathematical analysis of these observed data, mentioned in the previous communication, for such purpose.

Since the appearance of the celebrated memoir (1897) of Arthur Schuster, the method of representing the average state of diurnal magnetic field in series of spherical harmonics has been employed by several investigators. That method is based upon two fundamental assumptions; (I) the existence of a diurnal magnetic potential and (II) the independence of the diurnal variations with longitudes. As it is necessary, for investigating the progressive change of the diurnal magnetic field, to deal with the instantaneous states, the assumption (II) is no more admissible. This makes stress on the urgent requirement for the number and distribution of the magnetic observatories over the earth. Another difficulty arises in the determination of the base value; that is, the separation of the observed magnetic force into the permanent elements and the diurnal variable components. In view of the fact that the most remarkable characteristic variations were observed on these days at the stations situated between the meridians 90° E and 180° E, the mean of the hourly values from 0h th 24h (15h to 15h in G. M. T.) in Japanese Mean Time was obtained for each individual station, and the hourly deviations from this mean were taken as the components of diurnal variations in this analysis.

The instantaneous diurnal magnetic fields of the earth were determined as follows; by means of the determined values of the diurnal variations along the meridians I, II, III and IV in Fig. 1, we have on the parallel circles V, VI and VII, several points at which the hourly values are known. These values are taken on a coordinate paper, and curves are drawn connecting the points of the same hours, whereby are inferred the values before and after that hour.²) We have then, on each meridian, known values at three points in the Northern Hemisphere. With these values and those at stations in the Southern

¹) Proc. 12 (1936), 88.
²) Such curves are shown in Fig. 1 on page 227.
Hemisphere, the diurnal variations at a given hour along the meridian are determined. The ambiguity in this determination is to some degree mitigated by the fundamental assumption (I).

The values at 11\textsuperscript{h} (2\textsuperscript{h} in G. M. T.) on the four days above mentioned (we call them briefly VI21, VI23, IX23 and IX24) are expanded into a series of spherical harmonics (tesseral harmonics). The formulae and notations are, for the most part after Chapman,\textsuperscript{1)} of the following forms;

\[
V = \sum_{n} \left[ \cos n\lambda \sum_{m} P_{n}^{m}(\mu) \left\{ E_{mA}^{n} r_{0}^{m} r_{m-1}^{m} + I_{mA}^{n} r_{m}^{m+2} \right\} \right. \\
+ \sin n\lambda \sum_{m} P_{m}^{n}(\mu) \left\{ E_{mB}^{n} r_{0}^{m} r_{m-1}^{m} + I_{mB}^{n} r_{m}^{m+2} \right\} 
\]

where \( \mu = \cos \theta \) and

\[
P_{n}^{m}(\mu) = \frac{(2m)!}{2^{m} m! (m-n)!} \left\{ \mu^{m-n} (m-n) (m-n-1) \frac{\mu^{m-n-2} + \ldots}{2(2m-1)} \right\}.
\]

The coefficients were determined by the method of least squares within \( \pm 45^\circ \) latitudes. Considering a slow convergency of the harmonic analysis the coefficients were calculated for \( n=1, m=1, 2, 3, 4 \); for \( n=2, m=2, 3, 4 \) and for \( n=3, m=3, 4 \)\textsuperscript{2)} But considering the low accuracy of the materials, perhaps only the main terms merit discussion.

As for a finite discrepancy between the sets of coefficients calculated from \( X \) components and those from \( Y \) components, which, in our case, was not so great as the results of Van Vleuten\textsuperscript{3)}; we are now being occupied in improving our method of calculation.

The external and internal system of equivalent electric currents were calculated and drawn in charts as illustrated in Fig. 2, 3, 4 and 5.\textsuperscript{4)} Although the results may not be quite satisfactory, some of the common characters of types in these cases of two days each in the different seasons, are made appreciable.

1. On VI21 and IX24, the amplitude of the term \( P_{1}^{1}(\mu) \) is about three times as great as those of \( P_{1}^{2}(\mu) \), while on VI23 and IX23 they are of the same order of magnitude.\textsuperscript{5)}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
 & VI21 (P) & & & VI23 (E) & & & IX23 (E) & & \hline
\hline
n & m & \( E_{m}^{n} \) & \( I_{m}^{n} \) & \( E_{m}^{n} \) & \( I_{m}^{n} \) & \( E_{m}^{n} \) & \( I_{m}^{n} \) & \( E_{m}^{n} \) & \( I_{m}^{n} \) \hline
1 & 1 & 7.2 & 3.4 & 5.7 & 1.9 & 4.1 & 1.9 & 3.6 & 2.7 \hline
1 & 2 & 2.2 & 1.3 & 3.4 & 1.7 & 3.8 & 2.0 & 1.7 & 0.8 \hline
1 & 3 & 0.7 & 0.6 & 0.8 & 0.4 & 0.4 & 0.4 & 1.1 & 0.2 \hline
1 & 4 & 0.6 & 0.0 & 0.5 & 0.1 & 1.0 & 0.3 & 0.9 & 0.1 \hline
2 & 2 & 1.1 & 0.33 & 0.31 & 0.25 & 0.34 & 0.30 & 0.87 & 0.39 \hline
2 & 3 & 0.38 & 0.20 & 0.41 & 0.22 & 0.58 & 0.31 & 0.15 & 0.14 \hline
2 & 4 & 0.11 & 0.09 & 0.10 & 0.05 & 0.07 & 0.08 & 0.04 & 0.14 \hline
\hline
\end{tabular}

2) Zonal terms will be discussed in future communications.
4) These figures may be compared with those of the average state of summer: Chapman-Bartels' figure or Fritsche-Nippoldt's figure.
5) The former term is symmetrical and the latter asymmetrical about the equator.
2. Table 1 shows both parts of the coefficients (say $E_m^n$ and $I_m^n$) which appertain, respectively to the external and internal causes. It can be seen that the amplitude ratios $E_m^n/I_m^n$ are not the same. This implies that both current systems do not form similar figures.

3. Amplitude ratios $E_1^1/I_1^1$ are definitely greater than $E_2^2/I_2^2$ on VI23 and IX23. ($E$ type in our observatory.) This relation finds expression in Fig. 2 and 3 as the different positions of the centers of external and internal electric circulations.

4. The figures and tables apparently give some more indications concerning the relations between the type of variations (in our observatory) and the systems of electric circulations. These are being reserved for the full report.

Fig. 1. Distribution of observatories.

Fig. 2. External electric current system at 2h G. M. T. June 21, 1934. Unit, 1000 Amp.

\[ E_m^n = \sqrt{(E_{mA}^n)^2 + (E_{mB}^n)^2}, \quad I_m^n = \sqrt{(I_{mA}^n)^2 + (I_{mB}^n)^2}. \]
Fig. 3. Internal electric current system at 2h G.M.T. June 21, 1934. Unit, 1000 Amp.

Fig. 4. External electric current system at 2h G.M.T. June 23, 1934. Unit, 1000 Amp.

Fig. 5. Internal electric current system at 2h G.M.T. June 23, 1934. Unit, 1000 Amp.