54. *On the Motion of the Asteroid Hilda (153).*

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The motion of the asteroid of the Hilda group, like that of the Trojan group, belongs to the case of commensurable periods with the planet Jupiter, which is in fact one of the most interesting subjects in the theory of planetary perturbations. Moreover, it is a case of the proximity of the orbits, attending with some peculiarities of the motion.

The theory of general perturbations, as was treated by one of the authors several years ago,¹⁾ may be applied to this case; but, it cannot be complete by any means, as the convergency of the adopted series and the effect of the neglected quantities become questionable. The method of special perturbations is free from those defects, although it requires a considerable amount of numerical computations proportional to the duration, and applies only to a special object. The method, at least, may serve for verifying, or for correcting the results obtained by the general theory.

In accordance with this idea, we have chosen the asteroid Hilda as the typical object in the group, and followed its motion numerically, from the time of discovery in 1875 to the end of 1956. The action of two principal planets, Jupiter and Saturn, was considered, taking the interval of the integration from 40 days to 160 days, according to the amount of perturbations.

Comparisons with the observations were made in order to verify the calculations, and at the same time to determine the correct values of the orbital elements. The method of procedure is exactly same as for improving the elements, so often used by the authors for many other asteroids.²⁾

The elements determined by Kühnert in 1891³⁾ were adopted for the calculation of the perturbations. They are as follows:

Epoch and Osculation: 1891 Nov. 7.46 U. T.

Ecliptic and mean equinox: 1890.0

M	117.510
ω	55.250
Ω	228.237
i	7.865
φ	9.443
μ	451.317.

The observations selected for the comparison are as in the following table:

1) K. Hirayama: Researches on the distribution of the mean motions of the asteroids; Journ. Col. Sc., Imp. Univ. Tokyo, **41** (1918), Art. 3, 51.

2) Proc. **10** (1934), 204.

3) Berliner Jahrbuch, 1893, 396.

No.	Date U.T.	α	δ	Observation by	Publication
1	1875 Nov. 3.83	45.22+17.52		Palisa, Pola	A.N. 86 381
1'	" " 23.00	42.18+16.25		" "	" 87 355
2	1880 May 27.90	207.63-13.00		B. Peter, Leipzig	" 100 353
2'	" " 27.96	207.63-13.00		Knorre, Berlin	" 101 195
3	1881 July 5.94	289.76-12.30		Palisa, Vienna	" 102 281
3'	" " 14.91	288.22-12.34		" "	" "
4	1883 Nov. 28.98	48.37+16.90		" "	" 109 167
4'	" Dec. 22.83	45.59+15.78		" "	" "
5	1884 Dec. 12.99	93.63+16.64		" "	" 111 275
5'	" " 14.95	93.31+16.62		" "	" "
6	1886 Feb. 7.86	126.96+ 8.93		" "	" 116 337
7	1888 May 9.00	222.18-16.70		" "	" 132 49
8	1890 Sept. 20.90	7.94+10.93		" "	" 129 333
9	1891 Nov. 7.97	57.39+18.65		" "	" "
10	1895 Mar. 19.98	173.10- 5.92		Rossard, Toulouse	B.A. 15 32
10'	" " 30.90	171.53- 5.02		Rambaud, ^{a)} Algier	A.N. 138 179
11	1896 May 9.04	234.17-18.23		Villiger, Munich	" 144 153
11'	" " 30.99	230.61-16.70		Borrelly, Marseille	B.A. 13 440
12	1897 July 22.02	313.50- 6.02		Rossard, Toulouse	" 16 329
12'	" " 24.95	313.04- 6.09		Villiger, Munich	A.N. 145 215
13	1898 Oct. 8.92	13.78+11.91		Borrelly, Marseille	B.A. 16 141
13'	" " 17.96	12.43+11.12		Villiger, Heidelberg	A.N. 150 229
14	1905 July 28.99	321.89- 3.35		Palisa, Vienna	" 171 289
14'	" Aug. 22.94	317.99- 4.57		Valentiner, Heidelberg	" " 353
15	1906 Oct. 24.89	18.74+12.59		Baranow, Kasan	" 174 123
15'	" " 26.14	18.56+12.49		Porter, Cincinnati	" 175 123
16	1907 Nov. 28.06	66.93+18.42		Metcalf, Taunton	" 177 95
17	1910 Mar. 4.07	139.72+ 5.27		" "	" 184 237
17'	" " 10.04	139.00+ 5.63		" "	" "
18	1912 May 18.89	255.65-18.53		Wood, Johannesburg	" 192 228
18'	" June 9.86	252.01-17.41		" "	" " 411
19	1913 Aug. 7.91	330.53- 0.53		Neujmin, Simeis	" 195 447
19'	" " 29.86	327.17- 1.84		Palisa, Vienna	" 198 171
20	1914 Sept. 26.94	28.73+16.00		M. Wolf, Heidelberg	" 199 224
21	1915 Dec. 4.87	69.94+18.18		Demetrescu, Bucarest	" 202 54
21'	" " 6.86	69.61+18.10		" "	" "
22	1918 Mar. 6.89	144.00+ 3.90		Gonnessiat, ^{a)} Algier	J.O. 2 140
23	1921 Aug. 9.01	339.14+ 2.29		Reinmuth, Heidelberg	A.N. 232 339
24	1922 Oct. 26.20	30.79+15.35		G.H. Peters, Williams Bay	A.J. 37 5
24'	" " 28.20	30.48+15.20		" "	" "
25	1926 Feb. 6.96	152.46+ 0.81		Jekhowsky, ^{b)} Algier	J.O. 10 26
26	1927 Apr. 6.92	198.84-13.34		" "	" " 159
27	1928 June 20.01	270.30-15.97		Jackson, ^{c)} Johannesburg	A.N. 235 351
28	1929 Aug. 12.02	345.50+ 4.43		Gonnessiat, ^{d)} Algier	J.O. 13 48
28'	" Sept. 28.89	338.86+ 1.08		Schwassmann, ^{e)} Bergedorf	B.Z. 81
29	1930 Oct. 3.13	39.75+17.91		Arend, ^{f)} Uccle	A.N. 240 304
30	1931 Nov. 10.07	83.71+18.79		Boyer, Algier	J.O. 16 9
31	1934 Feb. 10.94	157.94- 1.11		Reinmuth, Heidelberg	A.N. 251 252
31'	" " 18.99	156.75- 0.70		Volta, Pino Torinese	" 252 404

a) and Sy. b) Croisé and Andrès. c) and Wood.
d) and Filippoff. e) and Wachmann. f) and Delporte.

The coordinates α and δ are referred to the mean equator and equinox for the beginning of each year.

The corrections of the adopted elements determined from the residuals of the observations are

dM	-0.146	di	0.000
$d\omega$	$+0.120$	$d\varphi$	$+0.044$
$d\Omega$	0.000	$d\mu$	-0.051

by which the final residuals O—C of each observations are determined as follows:

No.	$\Delta\alpha$	$\Delta\delta$	No.	$\Delta\alpha$	$\Delta\delta$
1	-0.04	0.00	15'	0.00	$+0.01$
1'	-3	-1	16	0	0
2	-3	0	17	$+1$	0
2'	-2	0	17'	$+3$	0
3	$+1$	0	18	0	0
3'	-1	-1	18'	$+2$	0
4	$+1$	0	19	$+2$	$+2$
4'	0	0	19'	$+1$	0
5	0	0	20	$+1$	-1
5'	$+1$	$+1$	21	-1	-3
6	$+2$	-1	21'	-1	-3
7	$+1$	$+3$	22	0	0
8	0	$+1$	23	0	0
9	$+2$	$+1$	24	0	0
10	$+1$	0	24'	$+2$	0
10'	$+2$	0	25	$+1$	0
11	$+3$	0	26	-2	$+1$
11'	$+3$	-1	27	-1	0
12	$+1$	0	28	-2	-1
12'	0	$+1$	28'	0	0
13	$+1$	0	29	0	0
13'	0	-1	30	0	0
14	0	0	31	-1	$+1$
14'	0	$+1$	31'	-1	0
15	$+1$	0			

The corrected elements calculated for three different epochs are as follows:

Ep. and Osc. U.T.	1925 Jan. -6.5	1938 July 4.5	1950 July 1.5
Equinox	1925.0	1950.0	1950.0
M	193.589	92.699	278.423
ω	52.294	49.818	49.586
Ω	228.364	228.448	228.351
i	7.851	7.848	7.846
φ	9.208	8.780	8.835
μ	448.871	449.153	448.081

From the results of computations briefly stated in above, various characters of the perturbations can be revealed, among which the varia-

tions of the longitude of perihelion and of the critical argument will be specially mentioned.

As shown by a curve in the accompanying figure, the variation of the longitude of perihelion, $\bar{\omega}$, is approximately represented by a straight line with the formula,

$$\bar{\omega} = 283.5^\circ - 0.134(t - 1900.0).$$

The perihelion moved backward about 10 degrees during the period of 80 years. This fact should be particularly noticed, as the sense of the motion is opposite to the ordinary case. The case of Hyperion, the seventh satellite of Saturn, is similar, in which the perisaturnium moves steadily backward with a considerable rate.

With regard to the minor fluctuations, it is to be noticed that prominent variations occurred at three epochs during the whole interval. These epochs evidently correspond to the heliocentric conjunctions with Jupiter, which occurred in

1888 October,	with the min. distance	1.95 a.u.
1912 March,	"	1.93 "
1935 November,	"	1.90 "

The critical argument, θ , is computed by the formula,

$$\theta = 2l - 3l' + \bar{\omega}$$

in which l and l' are the mean longitudes of Hilda and Jupiter, respectively. The value of θ , aside from the fluctuations due to the approach with Jupiter, reached its maximum in about 1925 and is decreasing. Mean value of the argument is approximately represented by the formula,

$$\theta = 40.02^\circ \sin \{67.4^\circ - 0.869(t - 1900.0)\}$$

showing that the argument is librating with the amplitude 40° and the period 415 years, approximately. Compared with the case of Hyperion, it becomes :

	Hyperion-Titan ¹⁾	Hilda-Jupiter
Commensurability	4 : 3	2 : 3
Libration about	180°	0°
Amplitude	36	40
Period	640 days	415 years
Annual motion of pericentre	$-18^\circ.4$	$-0^\circ.134$

As a consequence of the libration, the heliocentric conjunctions occur near the perihelion of Hilda. Accordingly it does not approach very near to Jupiter. It is true that our burden of the computations was greatly lessened by this circumstance.

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1) W. S. Eichelberger: The orbit of Hyperion; A. J. **11** (1892), 145.
See also Tisserand: *Traité de mécanique céleste*, IV, Chapt. VI.