27. New Observing Programme for the International Latitude Service.

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As it is known, the variation of latitude at any station consists of two variations, namely, variation due to the change of the axis of rotation and the non-polar variation whose cause is still unknown.

The latter variation, so called z , seems to be very complicated and it contains general and local characters. My statistical investigations during long years on this line have concluded that it consists of:

- 1) $\sin(n\odot - m\alpha + A)$, where n, m are integer numbers, among which $\sin(\odot - \alpha + A)$ and $\sin(2\odot - \alpha + B)$ are predominant;
- 2) $\cos 2\odot$;
- 3) several periods beside annuals, most of them exist also in the polar motion itself.

Not only z but also the declination errors are equally complicated. It was found that in the recent international observations they are entangled with several periods other than annual which are contained in the polar motion. The same fact was found early in the year about 1892 by German observers from their own observations. And also the declination errors are different for different stations, which will probably be due to the local anomaly of refraction.

Thus it may be concluded that the analysis of so many terms above mentioned is impossible from the materials obtained by the present international programme; consequently, the exact values of x, y of the polar motion can not be obtained.

However, this difficulty may be much overcome by changing the observing programme. For the purpose, I have newly planned to observe three groups in one night, so that the observing time of the mean of the middle group on the mean date of one combination is just midnight throughout year. To fill up the above condition, 16 groups each of which extends over one and half hour and contains 6 star-pairs are chosen. Consequently, each combination of three successive groups is observed during about 23 days.

The mathematical reason is as follows. Let the general equation of the latitude variation for any group mean α at time t be

$$\Delta\varphi_{\alpha,t} = P_t + N_t + a \sin(\odot_t - \alpha + A) + b \sin(2\odot_t - \alpha + B) + \Delta\delta_\alpha \quad (\text{I})$$

where $\Delta\varphi_{\alpha,t}$ is the observed value of group mean minus the mean latitude of the station; P_t , the variation due to the pure polar motion; N_t , the local variation depending on time only; $a \sin(\odot_t - \alpha + A)$ and $b \sin(2\odot_t - \alpha + B)$, two principal terms of z ; and the last one $\Delta\delta_\alpha$ is the declination error. Here it is noted that $\Delta\varphi_{\alpha,t}$ is corrected by all exact values of the instrumental constants.

1) To find $a \sin (\odot - a + A)$.

By the equation (I) we have the following three equations for the simultaneous observations of three groups whose mean right ascensions are $\alpha_1, \alpha_2, \alpha_3$ at time t_1 ,

$$\Delta\varphi_{1,t_1} = P_{t_1} + N_{t_1} + a \sin (\odot_{t_1} - \alpha_1 + A) + b \sin (2\odot_{t_1} - \alpha_1 + B) + \Delta\delta_1 \quad (\text{II}, 1)$$

$$\Delta\varphi_{2,t_1} = P_{t_1} + N_{t_1} + a \sin (\odot_{t_1} - \alpha_2 + A) + b \sin (2\odot_{t_1} - \alpha_2 + B) + \Delta\delta_2 \quad (\text{II}, 2)$$

$$\Delta\varphi_{3,t_1} = P_{t_1} + N_{t_1} + a \sin (\odot_{t_1} - \alpha_3 + A) + b \sin (2\odot_{t_1} - \alpha_3 + B) + \Delta\delta_3 \quad (\text{II}, 3)$$

Form the differences of (II, 1)–(II, 2) and (II, 2)–(II, 3), we get the following two equations which contain only the diurnal variations,

$$\begin{aligned} \Delta\varphi_{1,t_1} - \Delta\varphi_{2,t_1} = & +2a \sin 11.25 \cos \left(\odot_{t_1} - \frac{\alpha_1 + \alpha_2}{2} + A \right) \\ & + 2b \sin 11.25 \cos \left(2\odot_{t_1} - \frac{\alpha_1 + \alpha_2}{2} + B \right) + \Delta\delta_1 - \Delta\delta_2 \end{aligned} \quad (\text{III}, 1)$$

$$\begin{aligned} \Delta\varphi_{2,t_1} - \Delta\varphi_{3,t_1} = & +2a \sin 11.25 \cos \left(\odot_{t_1} - \frac{\alpha_2 + \alpha_3}{2} + A \right) \\ & + 2b \sin 11.25 \cos \left(2\odot_{t_1} - \frac{\alpha_2 + \alpha_3}{2} + B \right) + \Delta\delta_2 - \Delta\delta_3 \end{aligned} \quad (\text{III}, 2)$$

During a year we have two independent series corresponding to the above two equations each consisting of 16 in number. And the first term in the right hand side is constant throughout each series, and the second term is a sine series whose angles are distributed in the aliquot part. Now summing up all 16 values of each series separately we obtain,

$$\begin{aligned} [\Delta\varphi_{n-1,t_{n-1}} - \Delta\varphi_{n,t_{n-1}}] = \\ + 32a \sin 11.25 \cos \left(\odot_{t_{n-1}} - \frac{\alpha_{n-1} + \alpha_n}{2} + A \right) \end{aligned} \quad (\text{IV}, 1)$$

$$\begin{aligned} [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}] = \\ + 32a \sin 11.25 \cos \left(\odot_{t_{n-1}} - \frac{\alpha_n + \alpha_{n+1}}{2} + A \right) \end{aligned} \quad (\text{IV}, 2)$$

as the second and third terms vanishes by summation. Next take the sum and the differences of the above two equations, then

$$\begin{aligned} [\Delta\varphi_{n-1,t_{n-1}} - \Delta\varphi_{n,t_{n-1}}] + [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}] = \\ + 64a \sin 11.25 \cos 11.25 \cos (\odot_{t_{n-1}} - \alpha_n + A) \end{aligned} \quad (\text{V}, 1)$$

$$\begin{aligned} [\Delta\varphi_{n-1,t_{n-1}} - \Delta\varphi_{n,t_{n-1}}] - [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}] = \\ - 64a \sin 11.25 \sin 11.25 \sin (\odot_{t_{n-1}} - \alpha_n + A) \end{aligned} \quad (\text{V}, 2)$$

By dividing $\frac{(\text{V}, 2)}{(\text{V}, 1)}$, we get

$$\frac{[\Delta\varphi_{n-1,t_{n-1}} - \Delta\varphi_{n,t_{n-1}}] - [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}]}{[\Delta\varphi_{n-1,t_{n-1}} - \Delta\varphi_{n,t_{n-1}}] + [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}]} = -\tan 11^\circ 25' \tan (\odot_{t_{n-1}} - a_n + A) \quad (\text{VI})$$

This is the equation for determining A , where $\odot_{t_{n-1}} - a_n$ is equal to π . Next squaring and summing the equations (V, 1) and (V, 2), we obtain an equation for the amplitude a , viz.

$$\left\{ \frac{[\Delta\varphi_{n-1,t_{n-1}} - \varphi_{n,t_{n-1}}] + [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}]}{32 \sin 22^\circ 5'} \right\}^2 + \left\{ \frac{[\Delta\varphi_{n-1,t_{n-1}} - \Delta\varphi_{n,t_{n-1}}] - [\Delta\varphi_{n,t_{n-1}} - \Delta\varphi_{n+1,t_{n-1}}]}{64 \sin^2 11^\circ 25'} \right\}^2 = a^2 \quad (\text{VII})$$

- 2) To find $b \sin (2\odot - a + B)$.

Now $a \sin (\odot - a + A)$ having been known, the equations (III) are reduced to

$$\Delta\varphi_{1,t_1} - \Delta\varphi_{2,t_1} = +2b \sin 11^\circ 25' \cos \left(2\odot_{t_1} - \frac{a_1 + a_2}{2} + B \right) + \Delta\delta_1 - \Delta\delta_2 \quad (\text{VIII, 1})$$

$$\Delta\varphi_{2,t_1} - \Delta\varphi_{3,t_1} = +2b \sin 11^\circ 25' \cos \left(2\odot_{t_1} - \frac{a_2 + a_3}{2} + B \right) + \Delta\delta_2 - \Delta\delta_3 \quad (\text{VIII, 2})$$

Similar equations for the just succeeding epoch t_2 are,

$$\Delta\varphi_{2,t_2} - \Delta\varphi_{3,t_2} = +2b \sin 11^\circ 25' \cos \left(2\odot_{t_2} - \frac{a_2 + a_3}{2} + B \right) + \Delta\delta_2 - \Delta\delta_3 \quad (\text{IX, 1})$$

$$\Delta\varphi_{3,t_2} - \Delta\varphi_{4,t_2} = +2b \sin 11^\circ 25' \cos \left(2\odot_{t_2} - \frac{a_3 + a_4}{2} + B \right) + \Delta\delta_3 - \Delta\delta_4 \quad (\text{IX, 2})$$

Take the difference of the two equations (VIII, 2)-(IX, 1), then

$$(\Delta\varphi_{2,t_1} - \Delta\varphi_{3,t_1}) - (\Delta\varphi_{2,t_2} - \Delta\varphi_{3,t_2}) = +4b \sin 11^\circ 25' \sin (\odot_{t_2} - \odot_{t_1}) \sin \left(\odot_{t_1} + \odot_{t_2} - \frac{a_2 + a_3}{2} + B \right) \quad (\text{X})$$

Forming similar differences for all two consecutive groups simultaneously observed, we have a sine series whose argumental angles are divided into aliquot parts so that easy determinations of b and B can be made.

- 3) To find $\Delta\delta$.

Now two principal terms of z being found, we have in the equations (III) only the remaining two different series of the differences of the declination errors for two consecutive groups. As usual method, by the successive addition of each series of the differences and putting any one of the declination error as zero, we can find very simply two independent values of $\Delta\delta$.

- 4) To find P and N .

Now all terms being known except P and N in the general equa-

tion (I), P and N are to be determined by the well known formula

$$x \cos \lambda + y \sin \lambda + z \quad (\text{XI})$$

The exact separation, however, of P and N is impossible unless N is the same for all stations. Really N which is local variation depending on t only seems to be different for different stations, therefore, the values of x and y determined by the above equation (XI) would contain slight errors due to the different local variations. But if the number of stations are as large as the local variations are to be regarded as a kind of accidental errors, true values of x and y can be obtained and also even the local variation for each station may be found.

New observing programme for the international north and south parallels.

The following programme is made according to the principle already explained above, and by taking $0^{\text{h}}0$ as the beginning of right ascension. Of course, this is not unique, it may be modified anyhow following the same principle. Theoretically the duration of each group in right ascension longer than $1^{\text{h}}5$ is more favorable for the determinations of all diurnal terms of z , but unfortunately the shortness of the length of night during the summer time on both parallels does not allow it.

The greatest advantage of this new programme is that each of the two principle terms of z and the declination errors $\Delta\delta$ may be found from only the observed materials of each station quite independent of those of the other stations, which are impossible by the past and present international programme. The disadvantage is that the shortness of the duration of the observations in a combination will decrease the accuracy of the group mean.

Star Programme		Observing Programme				
Group		Date	Groups			
I	0.0— 1.5	Dec. 20—Jan. 12	V	VI	VII	
II	1.5— 3.0	Jan. 13—Feb. 4	VI	VII	VIII	
III	3.0— 4.5	Feb. 5—Feb. 26	VII	VIII	IX	
IV	4.5— 6.0	Feb. 27—Mar. 21	VIII	IX	X	
V	6.0— 7.5	Mar. 22—April 13	IX	X	XI	
VI	7.5— 9.0	April 14—May 6	X	XI	XII	
VII	9.0—10.5	May 7—May 29	XI	XII	XIII	
VIII	10.5—12.0	May 30—June 20	XII	XIII	XIV	
IX	12.0—13.5	June 21—July 13	XIII	XIV	XV	
X	13.5—15.0	July 14—Aug. 5	XIV	XV	XVI	
XI	15.0—16.5	Aug. 6—Aug. 27	XV	XVI	I	
XII	16.5—18.0	Aug. 28—Sept. 19	XVI	I	II	
XIII	18.0—19.5	Sept. 20—Oct. 12	I	II	III	
XIV	19.5—21.0	Oct. 13—Nov. 4	II	III	IV	
XV	21.0—22.5	Nov. 5—Nov. 27	III	IV	V	
XVI	22.5— 0.0	Nov. 28—Dec. 19	IV	V	VI	

I desire this programme will be adopted by the international latitude service on the next occasion when the necessity of changing the star programme occurs.