49. *Further Studies on the Condensation Hygrometer.*

By Takematu OKADA and Masanobu TAMURA.

(Comm. by T. OKADA, M.I.A., June 12, 1940.)

1. Since we first introduced our newly devised condensation hygrometer at the April meeting of the Imperial Academy, Tokyo, we have made a number of studies on the hygrometer. In the present note will be given some of the results of our investigations.

Before giving the results of our studies we beg to be excused to make here a few corrections of erroneous statements which we have discovered in our previous note. Also we shall give an improved formula for calculating the tension of aqueous vapour from the manometer reading and the temperature at the moment of the condensation of the aqueous vapour in the air chamber of our instrument allowing for a slight change of the volume of air in it. In our previous investigation we entirely neglected the effect of the change of volume of the air. This may be allowable when the air chambers are so large that the volume change under consideration is negligibly small. In actual case however the air chambers having such a large volume are inconvenient in handling, and moreover an unavoidable difficulty arises in keeping the uniformity of the temperature of air in the chambers when cooling the instrument to the dew point of the air contained in it.

2. In a handy form of the condensation hygrometer the air chambers are comparatively small, and therefore some correction terms must be added to the formula for calculating the tension of aqueous vapour, allowing for the change of volume. This can be easily done in the following way. Let the initial pressure, volume, temperature and the vapour tension of the air contained in chamber *B* be denoted by p_0 , V , T_0 and f , and also let the initial pressure, volume and temperature of the dry air which fills chamber *A* by p_0 , V , and T_0 . We cool both chambers by dipping them into cool water or any other cooling mixture as the case may be. Let the temperature of air in chamber *B* be T' when a part of aqueous vapour in it condenses. Then the vapour tension of the air in chamber *B* becomes, F' , the maximum tension corresponding to T' . The volume in chamber *B* and connected tubing including the arm of the manometer *U*-tube decreases by dV_B . The volume in chamber *A* and connected tubing including the arm of the manometer *U*-tube increases by dV_A . Evidently $dV_A = dV_B = dV$ say.

Let the quantity of dry air and aqueous vapour in chamber *B* be λ_B and μ , and that of dry air in chamber *A* be λ_A in the initial state. Then we have

$$pV = (\lambda_B R + \mu R') T_0 \quad (1)$$

$$fV = \mu R' T_0 \quad (2)$$

for chamber *I*, and

$$pV = \lambda_A R T \quad (3)$$

for chamber A. When the condensation has occurred, we have

$$p_B(V-dV)=[\lambda_B R+\mu' R']T' \quad (4)$$

$$F'(V-dV)=\mu' R' T' \quad (5)$$

for chamber B, and

$$p_A(V+dV)=\lambda_A R T' . \quad (6)$$

We have
$$p_A=p_B+\delta p=p_B+\frac{2dV}{S}\frac{\rho}{D} . \quad (7)$$

Here δp is the pressure change indicated by the manometer, S the section of the manometer tube, ρ the density of the manometer liquid and D that of mercury. The pressure is given in mm-Hg.

From (4) and (5) we have

$$p_B(V-dV)=\lambda_B R T'+F'(V-dV) . \quad (8)$$

From (6) and (7) we have

$$(p_B+\delta p)(V+dV)=\lambda_A R T'=pV\frac{T'}{T_0} . \quad (9)$$

From (9) we have

$$p_B=p\frac{V}{V+dV}\frac{T'}{T_0}-\delta p .$$

Putting this into (8) we get

$$\begin{aligned} p\frac{V-dV}{V+dV}V\frac{T'}{T_0}-\delta p(V-dV) &= pV\frac{T'}{T_0}-\mu' R' T' \\ &+F'(V-dV)=pV\frac{T'}{T_0}-fV\frac{T'}{T}+F'(V-dV) . \end{aligned}$$

Now
$$\begin{aligned} p(V-2dV)\frac{T'}{T}-\delta p(V-dV) &= pV\frac{T'}{T}-fV\frac{T'}{T} \\ &+F'(V-dV) \end{aligned}$$

$$fV\frac{T'}{T}=F'(V-dV)+\delta p(V-dV)+2pdV\frac{T'}{T}$$

$$\therefore f=F'\left(1-\frac{dV}{V}\right)\frac{T}{T'}+\delta p\left(1-\frac{dV}{V}\right)\frac{T_0}{T'}+2p\frac{dV}{V} .$$

Since $dT=T_0-T'$, we have

$$\frac{T}{T'}=\frac{T}{T_0-dT}=1+\frac{dT}{T_0} .$$

Hence
$$f=F'\left(1-\frac{dV}{V}+\frac{dT}{T_0}\right)+\delta p\left(1-\frac{dV}{V}+\frac{dT}{T_0}\right)+2p\frac{dV}{V}$$

or
$$f=F'+\delta p+2P\frac{dV}{V}+f\left(\frac{dT}{T_0}-\frac{dV}{V}\right) . \quad (10)$$

3. Thus far we have not taken into account the effect of the thermal expansion of the manometer liquid and the solid parts of the instrument. On account of the symmetrical construction, the thermal expansion of the air chambers and connecting tubings does not effect the final result of the measurements made with our new hygrometer. The thermal expansion of the manometer liquid however has a slight influence in the change of the volume and density of the liquid itself. Let the coefficient of the cubical expansion of the manometer liquid in glass be α within the range of $(T_0 - T')$, then the volume expansion corresponding to the manometer reading h mm is

$$\pi r^2 \alpha \times \frac{h}{2} \times \Delta T \times 0.1 \text{ cm}^3,$$

where r is the internal radius of the manometer tube.

In a specimen of our condensation hygrometer the volume of each of air chambers is 209 cm^3 . The internal diameter of the manometer tube is 3.2 mm, hence its radius is 0.16 cm. Therefore the change of volume corresponding to the manometer reading h mm is

$$dV = 3.14 \times (0.16)^2 \times \frac{h}{2} \times \frac{1}{10} \text{ cm}^3 = 0.004 h \text{ cm}^3.$$

$$\text{Hence } \frac{dV}{V} = \frac{0.004}{209} h = 0.0000192 h. \quad (11)$$

The pressure change δp corresponding to the manometer reading h mm is

$$\delta p = \frac{0.89}{13.57} \times h = 0.0656 h \text{ mm}, \quad (12)$$

where the density of the manometer liquid is 0.89 at 15°C and that of mercury 13.57 at 0°C . In our previous note¹⁾ the density of manometer liquid was given as 0.9.

4. Next we shall examine the effect of the thermal expansion of the manometer liquid on the variation of the pressure observed by it due to the change of the density of liquid. The density ρ of the liquid is

$$\rho = \frac{\rho_0}{(1 + \alpha t)}.$$

$$\text{In our case } \rho_0 = 0.901, \quad \alpha = 0.72 \times 10^{-3}, \quad t = T',$$

$$\therefore \rho = 0.901 / (1 + 0.72 \times 10^{-3} \times T').$$

Hence the pressure change δp corresponding to the manometric height h mm is

$$p = 13.57 \frac{0.901 h}{(1 + 0.7 \times 10^{-3} t)} \text{ mm}. \quad (B)$$

As in the preceding example we take a case in which h is 30 mm, then the effect of the change of density of the manometer liquid on the pressure change observed with the manometer amounts to $0.34 h$

1) Proc. 16 (1940), 143.

for $T' = 30^\circ \text{C}$, in the calculated value of the tension of aqueous vapour.

5. We have made an improvement to the construction of our hygrometer. We have dispensed with the thermometers mounted into the air chambers for observing the temperature of air contained in them, since we have found that the ordinary thermometers do not rapidly follow the change of the temperature of air owing to their large thermal capacity. We have therefore assumed the temperature of air in the chambers is the same as that of water into which the chambers are dipped. By using a thermo-junction of very small heat capacity it was ascertained that the temperature of the air in both chambers attains to that of the cooling medium in about two minutes.

In using our hygrometer we have found that the following order of operations is best one. Firstly the index of the cock N is made to point upward, and air is drawn into B through E , and into A through F , by means of a suction pump which is connected to D and D' . F is filled with drying chemicals, therefore the air which passes through F is almost deprived of aqueous vapour. Secondly, the index of the cock N is turned toward right, and the communication with the outside air is closed. The air in A and B is in communication with the manometer. Then A is filled with dry air, and B with natural air of the same temperature and pressure. Thirdly, A and B are dipped into cool water or cooling medium. After a few minutes the manometer shows a sudden fall of the pressure in B . This shows a part of aqueous vapour in B becomes condensed. The reading h of manometer and the temperature T' of water at the moment of condensation are observed.

6. On May 30, 1940 a set of comparative observations of the tension of aqueous vapour with our condensation hygrometer and Assmann's aspiration psychrometer was made in the observation lawn of the Central Meteorological Observatory, Tokyo. The observations were carried out at a spot not directly exposed to sunshine. The tension of aqueous vapour was computed from the data given in Table I by using Formula (10). The result of the computation is given in Table II. The tension of aqueous vapour is also computed from the data given in Table-III using A. Sprung's well known formula,

$$f = F' - \frac{1}{2}(t-t') \frac{p}{755} . \quad (13)$$

In all of these computations the maximum tension of aqueous vapour was taken from Scheel and Heuse's tables.

TABLE I. Condensation hygrometer.

Hour	$T'(273^\circ + \dots)$	h	F'	δp	dV/V	$p_0(700 + \dots)$	$2p_0 dV/V$
^h ^m	[°]	^{mm}	^{mm-Hg}	^{mm-Hg}			
14 15	5.6	32.2	6.82	2.18	0.00062	57.7	0.94
14 47	6.3	57.8	7.16	3.79	0.00111	57.8	1.11
15 17	6.5	44.7	7.26	2.93	0.00086	57.9	0.86
15 43	6.6	37.7	7.31	2.47	0.00072	57.9	0.72
16 03	6.8	38.4	7.41	2.52	0.00074	58.0	0.74

Let f_a de an approximate value of f calculated by the following formula,

$$f_a = F' + \delta p + 2p_0 \frac{dV}{V} ,$$

then

$$f = f_a + f_a \left(\frac{dT}{T_0} - \frac{dV}{V} \right) .$$

TABLE II. Condensation hygrometer.

Hour	f_a	$T(273^\circ + \dots)$	$T'(273^\circ + \dots)$	ΔT	$\Delta T/T$	$f_a(\frac{\Delta T/T}{-\Delta V/V})$	f
^h ^m ^a	^{mm-Hg}	[°]	[°]	[°]		^{mm-Hg}	^{mm-Hg}
14 15	9.94	26.0	5.6	20.4	0.06822	0.92	10.86
14 47	12.06	24.0	6.3	17.9	0.06024	0.84	12.90
15 17	11.05	23.7	6.5	17.2	0.05797	0.85	11.90
15 43	10.50	23.5	6.6	16.9	0.05700	0.86	11.36
16 03	10.67	23.3	6.8	16.5	0.05569	0.85	11.52

TABLE III. Assmann's aspiration pycrometer.

Hour	t	t'	$t-t'$	$p(700+)$	F'	$\frac{1}{2}(t-t') \frac{p}{755}$	f_s	$f-f_s$
^h ^m	[°]	[°]			^{mm-Hg}		^{mm-Hg}	^{mm-Hg}
14 15	26.6	17.2	8.8	57.7	14.72	4.42	10.30	0.56
14 47	24.2	18.3	5.9	57.8	15.77	2.96	12.81	0.09
15 17	23.7	17.4	6.3	57.9	14.90	3.16	11.71	0.19
15 43	23.5	17.1	6.4	57.9	14.62	3.21	11.41	-0.05
16 03	23.3	17.0	6.3	58.0	14.53	3.16	11.37	0.15

In the above table f_s denotes the value of the tension of aqueous vapour calculated by Sprung's formula. The result of the comparative observations shows us that there still remains some improvement to be done in the construction of our condensation hygrometer, neverthe-

less it also shows that our instrument is not unsuitable for use in the routine of meteorological observations. In conclusion we wish to express our sincere feeling of gratitude to the authorities of the Imperial Academy for the grant, from which the cost of making a specimen of the newly designed hygrometer defrayed. Also we express our sincere thanks to Drs. H. Honda and Y. Miyake of the Central Meteorological Observatory, Tokyo for their kindness in assisting us in course of our investigation.