104. *A Note on the Theory of Electrical Phenomena in Imperfect Dielectric Substances.*

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1. *Local Transient.* Let η be the specific conductivity, K the inductive capacity and G the electric intensity (the dot below the letter indicates a space vector), then in a space where

$$A = \frac{\eta}{K} = \text{constant} \quad (1)$$

is everywhere constant, the local transient may be expressed by the well known equation

$$\text{div } KG = \{\text{div } KG\}_{t=0} e^{-4\pi A t} \quad (2)$$

and

$$\tau = \frac{1}{4\pi A} = \frac{K}{4\pi\eta} \quad (3)$$

is the relaxation time of the medium.

2. *Stationary State.* When the stationary state is reached, we have by Eq. (2)

$$\text{div } KG = 0 \quad (4)$$

and from Eq. (1) (4), we have also

$$\text{div } \eta G = 0 \quad (5)$$

From these two equations we can show that the solution of the electric field is unique for a suitable boundary condition as a conduction electric current field.

3. *Perfect Conductors in Conducting Dielectric Medium.* If l perfect conductors are placed in conducting dielectric medium satisfying Eq. (1), the law of superposition holds. The charge Q of the conductor and the conduction electric current I_c from the conductor to the dielectric

medium may be expressed as linear functions of the potential V of the conductors.

$$Q_k = q_{k1} V_1 + q_{k2} V_2 + \dots + q_{kk} V_k + \dots + q_{kl} V_l \quad (k=1,2,\dots,l) \quad (6)$$

$$I_{ck} = g_{k1} V_1 + g_{k2} V_2 + \dots + g_{lk} V_k + \dots + g_{kl} V_l \quad (k=1,2,\dots,l) \quad (7)$$

and we have here

$$q_{lk} = q_{kl}, \quad g_{lk} = g_{kl}, \quad g = 4\pi Aq \quad (8)$$

4. *Free Discharge.* When the conductors are left free to discharge, the law of discharge is the same for all conductors and may be expressed by the following equation.

$$Q_k = (Q_k)_{t=0} e^{-4\pi A t} \quad (k=1,2,\dots,l) \quad (9)$$

When the potential of the conductors are given, there is no transient phenomena.

5. *Periodic Steady State.* When the potential of the conductors is harmonic

$$\dot{V}_k = \sqrt{2} \dot{V}_k e^{j\omega t} \quad (k=1,2,\dots,l)$$

(the dot above the letter indicates a complex number) the complex effective value of the current flowing from the conductor to the conducting dielectric medium is

$$\dot{I}_k = (4\pi A + j\omega)(q_{k1} \dot{V}_1 + q_{k2} \dot{V}_2 + \dots + q_{kk} \dot{V}_k + \dots + q_{kl} \dot{V}_l) \quad (k=1,2,\dots,l) \quad (10)$$

the average electrostatic energy $\bar{U}$ stored in the dielectrics is

$$\bar{U} = \frac{1}{2} \sum_{k=1}^l q_{kk} V_k^2 + \frac{1}{2} \sum_{(kh)}^{\frac{1}{2}l(l-1)} q_{kh} (\dot{V}_h \bar{\dot{V}}_k + \bar{\dot{V}}_h \dot{V}_k) \quad (11)$$

The vector power $\dot{P}$ given to the dielectric medium is

$$\dot{P} = (8\pi A - j2\omega)\bar{U} = 2\omega\left(\frac{1}{\omega\tau} - j\right)\bar{U} \quad (12)$$

6. *Dielectric Hysteresis.* If we take

$$(\dot{K}) = (\dot{K}_{xx}, \dot{K}_{yy}, \dot{K}_{zz}, \dot{K}_{xy}, \dot{K}_{yz}, \dot{K}_{zx})$$

as the complex tensor whose tensor vector product with the complex effective value of electric intensity $\dot{G}$ is the complex effective value of electric displacement, the vector power $\dot{P}_d$ given to the electric displacement field is

$$\begin{aligned}\dot{P}_d &= \left(\left[\dot{G}, \frac{j\omega}{4\pi} \dot{D} \right]_p \right) = -\frac{j\omega}{4\pi} ((\dot{G}, (\dot{K}) \dot{G})_p) \\ &= -\frac{j\omega}{4\pi} ((\text{tens} [\dot{G} \dot{G}]_p, (\dot{K})_p))\end{aligned}\quad (13)$$

The complex average electrostatic energy density $\dot{U}$ is

$$\dot{U} = \frac{1}{8\pi} ((\dot{G}, \dot{D})_p) = \frac{1}{8\pi} ((\text{tens} [\dot{G} \dot{G}]_p, (\dot{K})_p)) \quad (14)$$

where $[]_p$ represents vector power product. From Eq. (13) (14)

$$\dot{P}_d = -j2\omega \dot{U} \quad (15)$$

7. *Apparent Complex Inductive Capacity.* In a simple case when

$$\left. \begin{aligned} \dot{K}_{xy} = \dot{K}_{yz} = \dot{K}_{zx} &= 0 \\ \dot{K} = \dot{K}_{xx} = \dot{K}_{yy} = \dot{K}_{zz} \end{aligned} \right\} \quad (16)$$

the apparent complex inductive capacity $\dot{K}'$ is

$$\dot{K}' = \left(1 - j \frac{4\pi \dot{A}}{\omega} \right) \dot{K} \quad (17)$$

where

$$\dot{A} = \frac{\eta}{\dot{K}} = \text{constant} \quad (18)$$

8. *Stationary State in General Conducting Dielectric Medium.* When the condition given by Eq. (1) no longer holds, the electric field is determined by the condition

$$\text{div } \epsilon \nabla V = 0 \quad (19)$$

and the space density ρ of the true electric charge is

$$\rho = -\frac{\eta}{4\pi} \left(\nabla V, \nabla \frac{1}{A} \right) \quad (20)$$

9. *Periodic Steady State in General Conducting Dielectric Medium.* The fundamental equation which determines the electric field in periodic steady state may be shown to be

$$\text{div } \epsilon \nabla V = \frac{\eta}{j - \omega\tau} (\nabla V, \nabla \omega\tau) \quad (21)$$

which generates to the ordinary Laplace's equation when $\omega\tau$ is very small or very large.

10. *Permanent Polarization and the Fixed True Electric Charge.* The permanent polarization may be distinguished from the fixed true electric charge of Eq. (20) by the determination of the tensor ellipsoid

$$r = \frac{n}{\sqrt{\sigma''_{on}}} \quad (22)$$

where r is the radius vector of the tensor ellipsoid, n the unit vector in an arbitrary direction, σ''_{on} the resultant surface charge of a thin disc whose plane is perpendicular to n .

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