17. *Earthquake Motion with a Constant Acceleration?*

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Prof. A. Imamura¹⁾ has remarked an interesting fact that seismographically recorded magnitude of some slow waves leading earthquake motion is quite inconsistent if it be interpreted as due to sinusoidal translational motions. For an accurate interpretation of such seismograms, it is necessary to introduce, besides the correction for the first throw of seismograph as suggested by Prof. Saem. Nakamura²⁾, another one which I am now going on to discuss.

In the present case the earth's motion is considered to begin rather gradually, as, for example,

$$x = 0 \text{ when } t < 0,$$

and
$$x = x_m \left\{ \sin \left(pt + \frac{\pi}{2} \right) - 1 \right\} \text{ when } t > 0.$$

This motion is supposed to last for a certain interval of time (say π/p), no restriction being made to the subsequent motion.

The equation of motion of the pendulum is given by

$$\frac{d^2y}{dt^2} + 2\varepsilon \frac{dy}{dt} + n^2y + V_0 \frac{d^2x}{dt^2} = 0,$$

using customary notations. Initial conditions of the pendulum are

$$y = 0 \text{ and } \frac{dy}{dt} = 0 \text{ at } t = 0.$$

1) A. IMAMURA, Proc. Imp. Acad. 2 (1926) 129.

2) Saem. NAKAMURA, Journ. Meteor. Soc. Japan, [ii] 2 (1924) No. 4. 109; Proc. Imp. Acad. 3, 32.

The solution is generally given by

$$y = \frac{V_0}{\gamma} e^{-\varepsilon t} \left[-\cos \gamma t \int_0^t e^{\varepsilon t} \sin \gamma t \left(-\frac{d^2 x}{dt^2} \right) dt + \sin \gamma t \int_0^t e^{\varepsilon t} \cos \gamma t \left(-\frac{d^2 x}{dt^2} \right) dt \right],$$

where

$$\gamma^2 = n^2 - \varepsilon^2.$$

In this case, as $\frac{d^2 x}{dt^2} = -p^2 x_m \sin \left(pt + \frac{\pi}{2} \right)$,

$$y = V_0 x_m \frac{p^2}{\sqrt{(n^2 - p^2)^2 + 4\varepsilon^2 p^2}} \left\{ \cos p(t - \tau) - \frac{n}{\gamma} e^{-\varepsilon t} \sin (\gamma t + \tau') \right\},$$

where

$$\tan p\tau = \frac{2\varepsilon p}{n^2 - p^2},$$

$$\tan \tau' = \frac{\gamma}{\varepsilon} \frac{n^2 - p^2}{n^2 + p^2}.$$

Hence, for the reduction of the initial motion, the first maximum value of the expression in the bracket { } (say = z) must be taken into account.

Similarly, if the prescribed motion be a tilting of a sinusoidal type and sets in gradually, such that

$$\phi = 0 \text{ when } t > 0,$$

$$\phi = \phi_m \left\{ \sin \left(pt + \frac{\pi}{2} \right) - 1 \right\} \text{ when } t > 0, \text{ then}$$

the displacement of seismograph is given by

$$y = \phi_m V_0 \frac{g}{\sqrt{(n^2 - p^2)^2 + 4\varepsilon^2 p^2}} \left\{ \cos p(t - \tau) - \frac{n}{\gamma} e^{-\varepsilon t} \sin (\gamma t + \tau') \right\} - \phi_m \frac{V_0 g}{n^2} \left\{ 1 - e^{-\varepsilon t} \sqrt{1 + \left(\frac{\varepsilon}{\gamma} \right)^2} \cos (\gamma t - \tau'') \right\},$$

where g is the acceleration due to gravity,

$$\tan \tau'' = \frac{\varepsilon}{\gamma}$$

and τ and τ' the same expressions as before.

In the next place, if the motion be such as suggested by Prof. Imamura, then

$$y = \phi \frac{V_0 g}{n^2} \left\{ 1 - e^{-\varepsilon t} \sqrt{1 + \left(\frac{\varepsilon}{\gamma} \right)^2} \cos(\gamma t - \tau'') \right\}.$$

Actual numerical examples¹⁾ are given in the annexed table. It will be seen from that table that the type of motion which gives the most consistent results for the data obtained from the different seismographs is that which was proposed by Prof. Imamura, i.e., a constant tilt with sudden commencement.

There is, however, another point to be taken into consideration. That is the comparative study of the periods which the same seismic wave gives to the different registers. In the actual examples, they seem, according to Prof. Imamura, to be equal to, or, at least, not to differ so much from each other, which is also favourable to his assumption. However it may be, it seems to me that his assumption is rather unnatural, when we inquire how such a constant tilt with sudden commencement is actually produced. Consequently, I propose here another type of motion, which is, on the other hand, a special case of that represented by the following equation :

$$\frac{d^2 y}{dt^2} + 2\varepsilon \frac{dy}{dt} + n^2 y + V_0(a - g\phi) = 0,$$

where ϕ is a constant tilt and a a translational acceleration. If we take the case $\phi = 0$, i.e., one when the pendulum is only subjected to a constant horizontal acceleration, then the condition that the resistered periods should be equal to each other is satisfied, provided that the proper period of the pendulum is sufficiently longer than the time interval between the first and the second motion of the earth crust. Further, as may be seen from the last column of the annexed table, the accelerations coming out from the different seismographs are pretty near each other, so that we may say that the other condition is also satisfied by this type of motion. Of course, there may be many other cases in which the motion can be considered to consist of sinusoidal ones; such a difference of types of motion would furnish an important clue for elucidating the mechanism of the occurrence of earthquakes.

Finally, a remark should be added that the above stated examples of motions cannot favourably be interpreted as sinusoidal tilt, though numerical results obtained in this respect are here omitted.

1) The same as investigated by Prof. Imamura.

TABLE.

Earthquake.	Instrum. Constants.			Obs. motion.		As sinusoidal translation.		With correction of initial motion.		As constant tilt during $t = \frac{T_p}{2}$			As translation with constant acc. during $t = \frac{T_p}{2}$	
	T_n	ϵ	V_0	T_p	A	V	A	κV	A	$\frac{V_0 g}{n^2}$	$\kappa' \frac{V_0 g}{n^2}$	ψ	$\kappa' \frac{V_0}{n^2}$	Acc. mm/sec ²
EW Comp. Sept. 1, 1923. No. 1.	33 ^s	.008	1.5	12 ^s 93 ^{mm}		1.73	53.7 ^{mm}	2.42	38.4 ^{mm}	40.6 × 10 ⁴	23.0 × 10 ⁴	4.04 × 10 ⁻⁴	23.5	3.95
	10	.11	2.0	„ 31		3.36	9.23	1.21	25.6	4.96 „	7.37 „	4.21 „	7.52	4.12
	4	.37	1.0	„ 2.5		0.123	20.3	.139	18.4	.397 „	.438 „	5.70 „ *	.447	5.58*
EW Comp. Sept. 2, 1923. No. 3.	31	.015	1.5	6.4 10		1.56	6.42	1.96	5.10	35.7 „	7.14 „	1.40 „	7.28	1.37
	9	.12	2.0	„ 6.5		3.63	1.79	1.96	3.32	4.02 „	3.98 „	1.63 „	4.06	1.60
EW Comp. Dec. 29, 1924. No. 4.	65	.036	15	31 1.7		17.7	.096	16.2	.105	1.57 × 10 ⁷	1.01 „	1.68 × 10 ⁻⁷	1.03 × 10 ³	1.65 × 10 ⁻³
	19	.11	120	„ 2.1		32.7	.064	21.2	.099	1.078 „	1.16 „	1.81 „	1.18 „	1.78 „
NS Comp. „	54	.005	20	„ 4.8		29.8	.161	33.65	.143	1.445 „	1.63 „	2.95 „	1.66 „	2.89 „
	19	.14	120	„ 3.8		29.7	.128	18.4	.205	1.08 „	1.175 „	3.24 „	1.20 „	3.17 „

$$\text{N. B. } V = V_0 \frac{p^2}{\sqrt{(n^2 - p^2)^2 + 4\epsilon^2 p^2}}$$

$$\kappa = \left\{ \cos p(t - \tau) - \frac{n}{\gamma} e^{-\epsilon t} \sin(\gamma t + \tau') \right\} \text{ first max.}$$

$$\kappa' = \left\{ 1 - e^{-\epsilon t} \sqrt{1 - \left(\frac{\epsilon}{\gamma}\right)^2} \cos(\gamma t - \tau'') \right\} t = \frac{T_p}{2}.$$

The inconsistency of the data marked with * may possibly be due to the shortness of the pendulum period; the expression κ' takes its maximum value (≈ 1.6) at 2.4s after the commencement of the initial motion which is earlier than the commencement of the second motion (6s in this case). Introducing this correction, the data come out as 3.94×10^{-4} and 3.85 respectively, thus becoming more consistent.