39. *On the Instability of Seismometers.*

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Since the advent of seismometers, their behaviour has been carefully studied by several investigators and many innovations have been introduced to them so as to make their records as faithful as possible. However, so far as the author is aware of, these investigations have been limited to the case when an earthquake shakes only in such a direction that a seismometer is intended to record it. For instance, in studying the behaviour of the Ewing horizontal vibrograph which, as is well known, has a heavy bob weight so suspended by a suitable device that it can oscillate slowly around an axis slightly inclined to the vertical,

the shaking is generally assumed to take place only in the direction perpendicular to the plane containing the centre line of the weight and the inclined axis; and from the outset, it is assumed that horizontal vibrations perpendicular to the above do not tamper with the motion of the bob weight.

If there be a chance even in an exceptional case that the latter vibration sets the bob weight into a motion and thereby its intended indifference to external motion fails, the seismic records obtained therewith will cease to be trustworthy. It is the object of the present paper to show that there is an every possibility of the occurrence of such an undesirable phenomenon.

Fig. 1

Now referring to Fig. 1 let the frame A B C D holding a bob weight M be so fixed to a rigid stand that it can oscillate about the axis A C, making a small angle ϕ to the vertical A O; and let I be the moment of inertia of the bob weight (including appendages) of the mass M with respect to the axis A C, l the distance of its centre of mass from the same axis, and θ the angle of oscillation of the central plane C D to the neutral plane C E.

Then the free oscillation of the apparatus is given by

$$I \frac{d^2\theta}{dt^2} + Mg\phi l\theta = 0$$

whence the period of free oscillation is given by

$$T = 2\pi \sqrt{\frac{I}{Mg\phi l}} \quad (1)$$

neglecting the resistance to the oscillation.

Generally ϕ is made as small as possible in order to elongate T , and thereby to make the centre of mass of the bob weight practically stand-still among horizontal vibrations in the direction perpendicular to the plane of the frame (shown by h in the diagram).

If the horizontal vibration perpendicular to the above (shown by H) had no influence on the immobility of the bob weight, the seismometer will be able to discharge its duty faithfully. Now let us study whether or not such horizontal vibrations disturb the desired steadiness of the bob weight.

Let the motion along H be simple harmonic of frequency p (during the time 2π) and its maximum acceleration α . Then, as the bob weight is subjected to this acceleration relative to the pivotting axis, the motion in this case is given by

$$I \frac{d^2\theta}{dt^2} + Mg\phi l\theta + Ma \cos pt \, l\theta = 0,$$

apart from the tilting of the post which is likely to occur in actual cases.

Now put $pt = 2t'$ in this equation, then we have

$$I \left(\frac{p}{2} \right)^2 \frac{d^2\theta}{dt^2} + (Mg\phi l + Ma l \cos 2t')\theta = 0 \quad (2)$$

Again put

$$\frac{\frac{Mg\phi l}{I}}{\left(\frac{p}{2}\right)^2} = p_0 \quad \text{and} \quad \frac{\frac{Mal}{I}}{\left(\frac{p}{2}\right)^2} = 2p_1 \quad (3)$$

Then (2) is transformed into

$$\frac{d^2\theta}{dt^2} + \{p_0 + p_1(e^{2it'} + e^{-2it'})\}\theta = 0,$$

which is a particular form of well-known Hill's equation.

The solution of this equation may be written

$$\theta = \sum_n b_n e^{ict' + 2int'} \quad (4)$$

in which n is any integer, positive or negative, $i = \sqrt{-1}$, and the value of c is given by¹⁾

$$\cos(\pi c) = 1 - 2 \sin^2 \left(\frac{\pi}{2} \sqrt{p_0} \right) \left[1 + \frac{\pi \cot \frac{\pi}{2} \sqrt{p_0}}{4 \sqrt{p_0}} \frac{p_1^2}{1 - p_0} \right] \quad (5)$$

provided p_1 is sufficiently small.

From (4) it will be seen that, in the case when c becomes a complex quantity, the term given by $e^{ict'}$ ceases to be oscillatory; consequently θ increases indefinitely with time, or in other words, the seismometer becomes unstable. Apart from the fact shown by (4), that, in those cases where b_n assumes a finite quantity, the steadiness of the seismometer is more or less impaired, the possibility of the apparatus becoming unstable under circumstances is a very serious matter to be especially attended to.

It is obvious that, when the apparatus becomes unstable, even a feeble jerk given to the apparatus transversely may cause a conspicuous displacement of the bob weight, making the record utterly untrustworthy.

Now from (5) the reality of the value of c requires that

$$0 < \sin^2 \left(\frac{\pi}{2} \sqrt{p_0} \right) \left[1 + \frac{\pi \cot \frac{\pi}{2} \sqrt{p_0}}{4 \sqrt{p_0}} \frac{p_1^2}{1 - p_0} \right] < 1 \quad (6)$$

1) F. Tisserand, *Traité de mécanique céleste*, t. III.

As it is very difficult to study this inequality in general, the present investigation will be limited to a special case when p_0 is nearly unity.

From (1) and (3), it will be seen that $p_0 = 1$ denotes that the period of earthquake is one half of the natural period of the oscillation of the apparatus. Thus this case corresponds to well known Melde's experiment in acoustics.

Now it can easily be shown that, when $p_0 \doteq 1$, both of the conditions given in (6) are satisfied approximately by

$$p_1^2 < (1 - p_0)^2 \sqrt{p_0}$$

or

$$|p_1| < |(1 - p_0)| \quad (7)$$

Substituting the values of p_0 and p_1 from (3), and writing

$$\frac{Mg\phi l}{I} = \omega^2$$

where ω is the natural frequency (during 2π) of the apparatus, according to (1), we can write (7) as follows:—

$$|a| < 2 \left| \frac{\left(\frac{p}{2}\right)^2}{\omega^2} - 1 \right| g\phi$$

We are now dealing with the case where the ratio of $p/2$ and ω is nearly unity and moreover, as described before, ϕ is in general made very small (1/200 radian or so). Therefore, even under a slight acceleration a in the longitudinal direction, the apparatus may become unstable and the record obtained in such a case be illusory.

To confirm this, the author made an experiment with a horizontal vibrograph of Ewing's type. The apparatus was placed on a platform which executes horizontal oscillations in a fixed direction, the motion being imparted by a heavy compound pendulum connected to the platform through a link. By the way, with such an arrangement the motion may be started all at once and the facsimile of the principal tremor of an earthquake is well obtained. Although the author does not pretend that the equation of motion given above is directly applicable to the present experiment in which the motion is imparted abruptly, yet the nature of the motion induced thereby would not be wholly different. The centre line plane of the apparatus had a slight deviation (say 1/30

radian) to the direction of the oscillation of the platform. The apparatus had the natural period of oscillation of 4.0 secs. and ϕ about $1/90$ radian.

One set of records are shown in Fig. 2 in which A is the oscillation of the platform in actual size, and B the transverse oscillation of the centre of the bob weight magnified two times; the latter was excited by the former. It will be seen that, although the period of the oscillation of the platform was not strictly half of the natural period of the apparatus, yet owing to the probable instability of the apparatus the bob weight oscillated violently to and fro.

Even when the ratio of the two periods is very different from the case here dealt with, the nature of the motion of the bob weight is practically the same, provided the accelerating motion of the platform is sufficiently large. Here a special attention should be paid to the initial phase of the oscillation of the bob weight. The record B clearly shows that at the start the bob weight moves slightly to one direction, as if there were an impulsive motion of the platform in that direction, and then follows a conspicuous motion in the other direction. Fig. 3 shows a record of the *N-S* horizontal component of a strong earthquake which took place in the Kwanto district on Sept. 2nd. 1923.¹⁾ Not only this one, but nearly all records of strong earthquakes show a common feature in the principal tremor that it is started by a hump to one direction and then conspicuous principal motion follows to the other.²⁾ The motions very closely resemble to those observed in the present experiment.

Although the author is not in a position to judge whether such earthquake motions as recorded were actually in existence or merely due to the incompleteness of the instrument, yet the matter may admit of doubt which must be cleared away by further investigations.

Lastly, it may be added that a similar illusive record would be possibly obtained with a horizontal seismometer of inverted pendulum type under the influence of vertical vibrations, and with a vertical seismometer of lever type when it is subjected to the horizontal vibrations in the direction of the axis of the lever. Thus in seismometry there is still much to be improved.

1) A. IMAMURA, Preliminary Note on the Great Earthquake of S.E. Japan on Sept. 1. 1923. Seismological Note No. 6. Imp. Earthquake Inves. Comm. Pl. V.

2) l. c. Pl. I and VI.

Fig. 2 A.

Fig. 2 B.

Fig. 3.