63. *Effect of an Irregular Succession of Impulses upon a Simple Vibrating System— Its Bearing upon Seismometry.*

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In 1917, one of the authors published a paper¹⁾ on the theory of motion of a simple vibrating system excited by a series of a large number of impulses succeeding one upon another with irregular intervals. Recently, we carried out some experiments regarding the above problem and the results obtained were compared with those to be expected from the above theory.

A physical pendulum was subjected to impact of two opposite streams of shots impinging upon the targets fixed to the pendulum rod near its lower end, the impulse due to the shots being in the direction of swing. The irregular motion of the pendulum thus excited was recorded on a smoked paper covering a revolving drum. A liquid damper was provided for by means of which the damping ratio could be varied through a wide range. The ratio α was defined as the ratio of the successive swings in alternate directions. Besides, the average number n of the shots impinging on each side per unit time and also the average velocity v_0 of impact were varied. The dependency of the mean kinetic energy I_0 of the vibrating system upon these quantities n , v_0 and α or $\log \alpha$ was investigated.

If the equation of motion is

$$m\ddot{x} + f\dot{x} + kx = 0, \quad \alpha = \frac{f}{m}, \quad \beta = \frac{k}{m} - \frac{f^2}{m^2},$$

the mean kinetic energy I_0 is given by

$$I_0 = \frac{a^2 Q^2}{4 m a t_0},$$

where a is the amplitude of impulses, t_0 the mean interval of the successive pulses and Q a function of β above defined. In the present case, the above may be written

$$I_0 = C' \frac{v_0^2 n}{\log \alpha},$$

1) T. Terada, Proc. Tok. Math.-Phys. Soc., [ii] 9 (1917), 142.

C' being a constant proportional to the square of the mass M of a single shot and inversely proportional to the pendulum mass m .

For evaluating the mean kinetic energy from the instrumental record of the motion, the method of "mean variability"¹⁾ initiated by Dr. S. Ono was found most convenient. Thus, a glass plate ruled with a number of equidistant parallel lines with a constant interval was placed upon the record, with these lines parallel to the time axis, and the number of the points of intersection, R , of the graph with these lines was counted. Taking the mean number $\bar{R}$ for unit time, it may be shown that I_0 is proportional to $(\bar{R})^2$.

When n is of the order of several hundreds to thousands the agreement between the experiment and theory is found to fail, a fact which may be explained by the increase of the effective values of the vibrating mass as well as of the damping with increasing n , as in this case the pendulum is surrounded with a dense atmosphere of shots. Thus, the denominator of the above expression for I_0 must be replaced by an expression of the form $[m + \phi(n)] \log [x + f(n)]$.

In the case, however, when n is of the order of 10-20, the simple theoretical relation is found to hold approximately.

The general characters of the motion of the pendulum show a striking resemblance to those of the records of earthquakes or micro-seismic waves obtained with ordinary seismographs (Fig. 1). The same may be said for the records of the secondary undulations of tides in bays, or of the "seiches" of lakes.

The ordinary seismographs are some or other kinds of oscillating system as is here considered. The building in which it is installed may be another such system. Again, the part of the earth crust, say a "block," where the station is situated may have its own proper mode of vibration. It is, therefore, difficult, if not impossible, to tell whether the motion transmitted as earthquake waves is in reality a succession of pulses or not, as long as the seismograms are the only data. The question here raised will not be meaningless in connection with some fundamental problems of seismology and seismometry.

The details of the experiments and discussions will appear shortly in Vol. 5 of the Bulletin of the Earthquake Research Institute, Tokyo Imperial University.

1) S. Ono, Jap. Journ. Astr. Geoph., 3 (1925), 11.

Part of preliminary phases of Alaska Earthquake, Sept. 11, 1899, obs. in Tokyo.

Part of the last phase of principal portion of the same.

Part of pulsatory motions obs. Tokyo Seism. Inst., Nov. 17-18, 1900.

$$M=0.153; n=310; \kappa=1.7.$$

$$M=0.024; n=1550; \kappa=1.02.$$

$$M=0.153; n=310; \kappa=1.02.$$

$$M=1.25; n=9.1; \kappa=2.5.$$

$$M=1.25; n=127; \kappa=10.$$

Fig. 1. M =Mass of a single shot in gr.
 n =Number of shots colliding on one side per sec.
 κ =Ratio of damping of pendulum.