78. **Experiments on the Sound Field due to a Conical Horn.**

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Formerly the author calculated the sound field due to a conical horn with a simple source at its vertex and found that if the origin is taken at the vertex of the cone and the polar axis on its axis, the velocity potential ($\phi$) at any point $(r, \theta)$ external to the horn is given by the following expression:

$$
\phi = \frac{e^{ikt(r-r)}}{r} \cdot \frac{2ika}{1-1-ika} \sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{p_n f_n(ika)}{F_n(ika)}
$$

$$
\times \sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{p_n f_n(ikr)}{F_n(ika)} \cdot P_n(\mu),
$$

where $k = \frac{2\pi}{\lambda}$ ($\lambda$ = wave-length),

$$
p_n = \int_{\theta_0}^{1} P_n(\mu) d\mu, \quad \theta_0 = \cos \alpha,
$$

$$
f_n(z) = 1 + \frac{n(n+1)}{2 \cdot n} + \frac{(n-1)n(n+1)(n+2)}{2 \cdot 4 \cdot z^2} + \ldots\ldots,
$$

$$
F_n(z) = (1+z)f_n(z) - zf_n'(z),
$$

and $a$, $2\alpha$ are the length and the angle of the cone respectively.

The theory started with the assumption that on the spherical surface ($r=a$) containing the opening of the conical horn, the motion of air exists in its opening only. The present investigation aims to compare the results of this theory with actual measurements.

As the source of sound, Edelmann's metallic pipe was blown under a constant pressure with the aid of a carefully designed pneumatic tank. Fig. 1 shows the distributions of intensity in different directions, set at various pitches of sound produced by the pipe. As seen from the figure, the metallic pipe is different from a simple source of sound,
the potential of which is given by \( \frac{e^{i(kr-r)}}{r} \), but within a small range of its front portion, it may be treated practically as such.

![Diagram](image)

(a) \( \lambda = 10 \text{ cm.} \), (b) \( \lambda = 16.6 \text{ cm.} \), (c) \( \lambda = 33.2 \text{ cm.} \)

At first, I tried to connect the source with the conical horn at its vertex, but in vain, for the pitch of sound changed considerably by such a connection. Consequently I made use of the reciprocal theorem, and put the measuring apparatus at the vertex of the cone, to receive the sound of the source put at different positions.

The conical horn has an opening of 15 cm. diameter and its angle is sixty degrees (\( a = 30^\circ \)). It is made to rotate horizontally with the vertex as center. The sound received through the cone is conducted by a metallic tube to a Rayleigh disc and its intensity is measured. The experiments were made in a sound-proof chamber.

Fig. 2 shows the results of experiments for \( \lambda = 16.6 \text{ cm.} \). The distances between the vertex and the source (\( r \)) are \( 4\lambda \), \( 6\lambda \) and \( 10\lambda \) respectively. In the same figure, the intensity of sound received is shown in polar diagrams as function of the angle between the axis of the cone and the line connecting the source with the vertex of the
cone. The curves in full and dotted lines are the results of observation and the theory respectively.

Fig. 2.

(a) \( r=4\lambda \)  
(b) \( r=6\lambda \)  
(c) \( r=10\lambda \)

In the case of \( r=4\lambda \), the experimental values are smaller than the theoretical ones in all directions. It is due to a sensible deviation of the source from a simple source, which becomes notable when \( r \) is not large as compared to the length of the cone. When \( r \) is large as compared to the length of the cone, its effect is negligible, as can be seen from the case \( r=10\lambda \), where the experiment coincides with the theory within the scope of experimental errors.

From the above results, we can infer the correctness of the theory and the applicability of the reciprocal theorem.

Similar curves for \( \lambda=33.2 \text{ cm.} \) at \( r=\lambda \) and \( r=2\lambda \) are given in Fig. 3. As seen from the theory, the directive property of a conical horn becomes less pronounced as the wave-length or the distance \( (r) \) increases.
Fig. 3.

(a) $r = \lambda$

(b) $r = 2\lambda$