55. *Motion of the Stars in Nebulous Matter.*

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This is a brief account of the capture theory of the stars, aimed to explain the origins of the binary stars and of the star clusters. By a spherical cloud, I mean in the following paragraphs, a nebula, bright or dark, which has spherical symmetry. Spherical clouds of the diameter from a few thousand astronomical units to several light years may have occasions to be traversed by the stars.

Let us consider at first the case in which a star passes through a spherical cloud without impact. Then the effective attracting mass, decreases as the star approaches to the centre of the cloud, and increases as it recedes from the centre. Now, we have the well-known relation in the two-body motion,

$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{m},$$

in which a is the semi-major axis of the relative orbit, r , the distance, v , the relative velocity, and m , the sum of the masses. $1/a$ is positive for the elliptic, zero for the parabolic, and negative for the hyperbolic motions. For an *instantaneous* variation of m , we have, therefore,

$$\delta\left(\frac{1}{a}\right) = \frac{v^2}{m^2} \delta m$$

whence $1/a$ decreases when m decreases, and the motion becomes *more hyperbolic*. Contrarily, $1/a$ increases when m increases and the motion becomes *more elliptic*. This variation of $1/a$ is reversible, and therefore, the integrated effect becomes zero, as long as the cloud maintains spherical symmetry.

Now consider the case of an impact. Supposing the velocity of the associated particle is zero, relative to the centre of the cloud, v decreases instantly. Hence, by the relation

$$\delta\left(\frac{1}{a}\right) = -\frac{2v\delta v}{m},$$

$1/a$ increases and consequently the motion becomes *more elliptic*. This variation is secular in character, and therefore, the integrated effect increases with the time, supposing the impacts occur in succession.

Thus, if the original motion of the star were slightly hyperbolic, it may be varied to elliptic by a succession of impacts. This elliptic motion of the star, being continued after the egress, will bring the same star to the same cloud after a revolution, provided that the disturbances by the surrounding stars and clouds are insignificant. The transit of the star is thus *repeated*, decreasing the value of the semi-major axis every time, until at length the star is *completely captured*.

If the cloud had no systematic angular velocity, the impacts of the particles opposing the motion of the star will occur more frequently than those of the particles moving in the same direction. Hence, the motion of the remaining particles after the capture, will in general be of the same sense as the revolution of the star. The angular momentum of the star relative to the cloud, is accordingly shared to the cloud.

The cloud, even if be of a small size, can capture a star by the repeating action, and the remaining matter, being absorbed by and by, will become small in mass compared with the captured star. A star-system, like our *solar system*, may be formed in this way.

If the spherical cloud is large and dense, can capture two or more stars. Let us consider first the case of two stars, and see how their relative motion is varied by the absorption of the cloud.¹⁾

Let G be the centre of gravity of the two stars whose masses are m_1, m_2 . Let the distances from G be r_1, r_2 and the velocities relative to G be v_1, v_2 , respectively. Then, evidently,

$$\frac{r_1}{m_2} = \frac{r_2}{m_1} = \frac{r}{m}, \quad \frac{v_1}{m_2} = \frac{v_2}{m_1} = \frac{v}{m},$$

in which $m = m_1 + m_2$, $r = r_1 + r_2$, $v = v_1 + v_2$.

Suppose a small mass δm , having no velocity relative to G , is absorbed by m_2 , *instantly*. Then by the theorem of the linear momentum,

$$0 = \delta(m_2 v_2) = v_2 \delta m + m_2 \delta v_2, \quad \delta v = \delta v_2$$

This gives δv corresponding to δm .

Since
$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{m},$$

we have
$$\frac{\delta a}{a^2} = \frac{\delta v^2}{m} - \frac{v^2}{m^2} \delta m = -\left(\frac{2m_1}{m_2} + 1\right) \frac{v^2}{m^2} \delta m,$$

1) The case of the planet is treated by Tisserand in *Mécanique céleste*, Tome IV, Chapt. XIII, and by Poincaré in *Hypothèses cosmogoniques*, Chapt. VI.

whence a decreases as m increases, and δa is given by

$$\frac{\delta a}{a} = -\left(\frac{2m_1}{m_2} + 1\right)\left(\frac{2a}{r} - 1\right)\frac{\delta m}{m}.$$

If θ is the longitude in the orbit, the angular momentum of the system is, relative to G ,

$$H = (m_1 r_1^2 + m_2 r_2^2) \frac{d\theta}{dt} = \frac{m_1 m_2}{m} r^2 \frac{d\theta}{dt}.$$

But, we have in the relative motion of two bodies,

$$r^2 \frac{d\theta}{dt} = \sqrt{ma(1-e^2)},$$

where e is the common eccentricity of the orbits. Since δm is supposed to have no velocity, the variation of H is zero. Therefore,

$$0 = \frac{2\delta H}{H} = \frac{2\delta m}{m_2} - \frac{\delta m}{m} + \frac{\delta a}{a} - \frac{2e\delta e}{1-e^2}.$$

Or, by the equation of δa ,

$$\frac{e\delta e}{1-e^2} = -\left(\frac{2m_1}{m_2} + 1\right)\left(\frac{a}{r} - 1\right)\frac{\delta m}{m}.$$

This result can be extended to the case in which m_1 , m_2 continuously increase. Thus, if

$$\frac{dm}{dt} = \text{const.}$$

then

$$\frac{de}{dt} = 0,$$

since the mean of a/r is zero, with respect to the time t . If the particles are distributed in the orbit at equal space intervals, then, the

mean value of $\left(\frac{a}{r} - 1\right)\frac{dm}{dt}$

becomes positive, as the velocity is great when r is small, and consequently,

$$\frac{de}{dt} < 0.$$

This effect will become more conspicuous, if the particles are more densely distributed near the centre of gravity.

In applying these conclusions to the actual cases of binary stars, the following effects must also be taken into consideration :

(a) Attraction of the cloud mass ;

(b) Individual velocities of the cloud particles.

With regard to (a), the outer layers do not affect the motion of the stars, as long as the cloud maintains spherical symmetry. The effect of

the inner mass may be very complicated if it is dense enough; but as this effect is conservative after all (whereas that of the impact is progressive in character), may be neglected in the average motion. The effect (b) may be neglected if the cloud has no angular momentum. If it has a component in the direct sense, the velocities in the orbit will be equalized and the eccentricity will decrease earlier than the major axis. If it has a component in the opposite sense the major axis will decrease earlier.

In short, *the motion of the binary stars in the spherical cloud is characterized with simultaneous decreases of the semi-major axis and the eccentricity of the relative orbit, together with increases of the masses.* If the cloud which has captured the stars, be either nearly exhausted by the stars, or almost swept away by other cloud, the stars will make appearance and may be observed as a real binary system.

Herewith, it may be interesting to note that if the two masses vary by the same ratio, we have from the equation of δa

$$\frac{\delta a}{a} = -3 \left(\frac{2a}{r} - 1 \right) \frac{\delta m}{m}.$$

This equation gives in the mean, the relations,

$$m^{\alpha} a = \text{const.}, \quad \alpha > 3; \quad m^{\beta} T = \text{const.}, \quad \beta > 5;$$

where T is the period of revolution. The corresponding relations for the case of the mass-radiation¹⁾ are

$$ma = \text{const.}, \quad m^2 T = \text{const.}$$

Both a and T vary *more rapidly*²⁾ in our case, for a variation of m .

It goes without saying that, in a long duration of time, a spherical cloud of huge extension can capture a great number of stars, large or small. Such a cloud will be steadily absorbed by the captured stars, and will, at length, be almost exhausted. The aggregation of stars in such state may be a *globular cluster*. If the original cloud was irregular, as those actually observable near the galactic plane, the aggregation will not be compact in form, and may be an *open cluster*. A globular cluster once formed and swept by other cloud may also be an open cluster.

The capture theory of the stars is naturally connected with other problems of the stellar evolution. I hope to discuss those problems of wider range in another paper.

1) Jeans: *Astronomy and Cosmogony*, Second Ed., p. 301.

2) See Eddington's *The Internal Constitution of the Stars*, p. 312.