99. *Flow in a Curved Pipe and Its Stability.*

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It is very difficult to solve the laminar flow of a viscous incompressible fluid in a curved channel directly from the hydrodynamic equations of motion and continuity. If the channel is enclosed by a solid boundary, such as a curved pipe made in the form of a coil, the solution is much more difficult and almost impossible. The writer has intended to make the latter solution by an indirect approximate method assuming the pipe to be a combination of cylindrical annular channels with dimensional rules employed in it, and obtained the following equations, which determine the distribution of velocities over the cross-sectional plane containing the axis of the pipe,

$$\begin{aligned} v &= \frac{x(d-x)}{4\mu(1+am)(1-m+x/R)} \frac{\partial p}{\partial z}, \\ v_0 &= \frac{d^2 - 4R^2(1 - \sqrt{1-m^2})^2}{16\mu(1+am)\sqrt{1-m^2}} \frac{\partial p}{\partial z}, \\ v_m &= \frac{d^2}{32\mu(1+am)} \frac{\partial p}{\partial z}, \end{aligned}$$

where v is the velocity of the flow at a point distant x from the inner edge 0 of the pipe as shown in the annexed figure, v_0 the maximum velocity, v_m the mean velocity over the cross-section perpendicular to the axis of the pipe, d the diameter of the pipe; R the radius of curvature of the axis of the pipe; m the ratio of the radii $d/2R$, $\frac{\partial p}{\partial z}$

the rate of the depression of the pressure along the axis of pipe, μ the coefficient of viscosity of the fluid; and a an empirical constant irrespective of the dimension which is equal to about 1620.

The distribution of the velocity v is as shown by the curve OBA, with the maximum velocity v_0 at the position N distant δ inwards from the centre M of the pipe, δ being calculated by

$$\delta = R - \sqrt{R^2 - d^2/4}.$$

With the same Reynolds numbers and the same mean velocities, the viscous resistance is greater in a curved pipe than in a straight

one by the ratio of $1+am$ to 1, and this is the same thing when the coefficient of viscosity is increased to $(1+am)\mu$ in a curved pipe as compared with μ in a straight one.

Comparing the straight and curved pipes of the same diameters with the same discharge, the depression of the pressure along the pipe line is $1+am$ times greater in a curved pipe than in a straight one, and for the same depression of the pressure the discharge in a curved pipe is $1/(1+am)$ that in a straight one.

It is well-known that in a straight pipe the critical Reynolds number calculated with the mean velocity is about 2000, while in a curved pipe it is very much smaller depending on the value of m , such as 130 for $m=0.0089$ according to the experiment carried by Prof. Gibson, showing that the flow in a curved pipe is more unstable.

All the above relations will easily be obtained if we use the criterion $\frac{v_m d}{\nu}(1+am)$ instead of using the Reynolds number $\frac{v_m d}{\nu}$ as

is usually done. This criterion is obtained from the dimensional rule applied to a curved pipe, ν being the kinematic coefficient of viscosity of the fluid. Denoting this criterion by M , the critical state of the flow will be unified to occur at $M=2000$, and the viscous resistance will be compared directly with the same value of M , for all kinds of pipes whether they are straight or curved.

In all the above formulae, $1+am$ may be replaced by b^m if the exponential form is preferable, b being an empirical constant irrespective of the dimension which is equal to about 15.4^{113} according to Prof. Gibson's experiment above mentioned.