69. *Formation of Ellipsoidal Geoid.*

(*Second Communication*)

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In the former communication¹⁾ concerning the occurrence of volcanic eruptions and earthquakes in four sectors of the ellipsoidal geoid, the solution of the problem how such figure was formed has not been touched. The transfiguration of the earth from a spheroid to an ellipsoid with axes slightly different from each other can be easily explained by discussing the consequences of the wandering of the pole from the quarternary up to the present time.

Suppose the axis of earth's rotation moves from P to P' in the same meridian plane PCP' ; then the equatorial plane $QOQO'$ for the pole P passes to $Q'OQ'O'$ for P' . The line of intersection of these planes cuts the surface at O and O' , which are *fixed*, so long as the pole remains in the same meridian, and Q moves to Q' in the same plane. A small deviation from the meridian gives fluctuations in

Fig. 1.

the position of O and O' , which mark the mean position. With P as pole and C as the centre of the spheroid, the semi-major axis $= CQ = CO = CO'$, but with P' as pole the spheroid is deformed on account of the rotation, and $CQ < CO$, becoming a semi-minor axis of a central section of the transfigured spheroid, while $CO = CO'$ is always the semi-major axis, if the deformation of the earth's figure follows hydrostatic equilibrium, so that the equatorial plane cuts it in a circle of constant radius CO . With the ellipticity of $1/297$ as at present, the variation of the central radius vector CQ near the equator amounts to 2.5 km,

1) Nagaoka: Proc. 9 (1933), 207.

when the pole moves 20° . The semi-plastic nature of the earth's crust allows slow adjustment to hydrostatic equilibrium, provided the motion of the pole is sufficiently slow. Owing to glassy nature of the substratum, such adjustment can cause fracture or extrusion of magma, leading to earthquakes and volcanic eruptions. As the plastic yielding increases with time, it is more pronounced in places which remain in the same state than in those where the action is only temporary. The equatorial elevation is therefore greater at O and O' , where the action is stationary, than at Q, Q' , where it is only transient. Thus if the earth's axis of rotation wanders in a meridian plane, the imperfect elasticity of the material near the surface causes antipodal points O and O' , which remain stationary on the equatorial plane during the wandering, protrude more than in any other places, which lie on the equator during the motion of the pole. At intermediate points between the equatorial limits during the motion of the pole, the elevation is greater the nearer to the stationary points O and O' , the longer they remain in the same latitude. The same holds for points in the sectors $OPO'P'O$.

The plastic deformation increases with time; the elevation at O and O' reaches an asymptotic value compatible with the rotation of the earth. As the pole describes a course PP' , the equatorial elevation once reached by Q disappears, and the point is depressed on receding from the equator. All other points, excepting O and O' , suffer similar deformation, and are constantly depressed or elevated according to the motion of the pole; in other words, the surface of the earth is transfigured. The rotation of the earth, however, tends to keep hydrostatic equilibrium, and if the distribution of density in different strata be homogeneous, the shape of the earth finally becomes a spheroid. On account of the plasticity of the materials composing the crust, it cannot immediately follow the stress imposed on it by the rotation of the earth, so that it is not shaped into a spheroid: the plastic nature is thus somewhat antagonistic to hydrostatic adjustment. By displacing the pole along the meridian, *OO' always remains to be the major axis of the equatorial section*, while the minor axis moves about in the meridian plane.

According to Köppen and Wegener,¹⁾ the pole at the beginning of the quarternary period lay roughly at 60° W, 120° E, and described a straight course still it reached the present position, from which it is

1) Köppen a. Wegener: *Die Klimate der geologischen Vorzeit*, Berlin, 1924,

beginning to change its direction. In applying the above discussion to the figure of the earth, we have therefore to remember that O lies at 30° E in Central Africa, O' at 150° W in the Mid-Pacific, the meridian through QQ' (60° W) passes near the Andes range, and that through QQ' (120° E) traverses Java and China. The hatched portion in Fig. 2 shows the region travelled by the equator from the quarternary to the present period.

Fig. 2.

The time-lag of the crust to recovery from the strained state to the unstrained can be easily understood by considering the gradual upheaval of the Baltic region, continuing even to the present day, after it was depressed by the load of a glacier several hundred metres high. As the recovery takes a long time, the reverse would also be the case; when the stress is applied to the crust, the strain cannot attain its ultimate value only after the lapse of many millenniums. Such process may be considered as going on in the vicinities of the meridians QQ' . Thus the points corresponding to O and O' can be considered to be in stable equilibrium, as they remained stationary during the displacement of the axis, but at points in the meridian $QQ'QQ'$ the strain is varying with time, so that in localities not far from it, and especially near the equator, the crust is liable to lose stability. The frequent occurrence of earthquakes and volcanic eruptions near the meridian arc has already been discussed in the former communication. The antipodal character of these terrestrial disturbances is evident, as the old equator in the eastern hemisphere lay north of

the present, and in the western hemisphere south of it. The formation of numerous islands in Indonesia and in West Indies and the Antilles may be looked upon *partly as due to imperfect accommodation of strain to the stress* imposed upon the crust by the rotation of the earth about axis, which is changing from time to time.

Referred to geocentric latitude θ , the radius vector from the earth's centre to the surface is given by $r = b/\sqrt{1 - e^2 \cos^2 \theta}$, where $2b$ denotes the polar axis and e the eccentricity; from this, the difference of the radius vector from the equatorial semi-axis Δr is calculated as follows:—

θ	5°	10°	15°	20°	30°	40°	50°	60°	70°	80°	90°
Δr	0.2	0.6	1.4	2.5	5.3	8.8	12.5	16.0	18.8	20.7	21.3

This Δr can be roughly looked upon as the amount of elevation or depression necessary for hydrostatic adjustment in the meridian of the wandering pole. The process is not at all easy, as the matter to be filled in the space elevated by transfiguration is to be taken from the surrounding, and for depression the matter must be similarly removed. The congestion thus produced by the deficit and the excess of matter leads to earth movements, which continue for a long period on account of plasticity. Probably *the adjustment takes place for the most part by the yielding of subterranean magma*, which is more pliable than the surface rocks. As the quaternary period was comparatively short (estimated at a million years) in comparison with other geological ages, the adjustment is not yet completed in the present stage, so that the crust in these meridians in the vicinity of the equator is still insufficient to reach the required hydrostatic equilibrium. The gravity survey of Indonesia is of great importance for revealing the character of geomorphic transformation, which seems to be still going on in these regions, as shown by interesting measurements of Vening-Meinesz in the surrounding seas. The consequence is that the geoid near the equator at 60° W and 120° E is still depressed than at 30° E and 150° W, where the equatorial points remained almost stationary during the quaternary period; thus the geoid is approximately an ellipsoid with three different axes, of which the two are nearly equal, differing only by 0.2 km.

The subsidences of sea level near Samoan Islands, St. Helena and other places, recently confirmed, are perhaps caused by processes leading to final adjustment of the geoid to a spheroid. There is, however, great probability of the geoid not being a spheroid, long before the quaternary period; the present discussion applies equally to other geological ages with due modifications.