20. On the Limb Effect of Faint Fraunhofer Lines.

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Abstract. Limb center variations of some fifty faint Fraunhofer lines were measured for $\cos \theta = 1.0, 0.8, 0.6, 0.4, 0.3$, and 0.2 , and it was found that, statistically, lines with η decreasing more rapidly with depth show stronger limb strengthening as compared with those with η of slower decrease. Limb center variations were calculated for three types of η variation at $\lambda 5100 \text{ \AA}$, and they were compared with those measured by Adam.

In a previous paper¹⁾ the present author suggested that the general tendency of faint Fraunhofer lines to strengthen towards limb may primarily be explained in terms of the general tendency of η to decrease with increasing depth, and that each line must show a limb effect characterized by its η . Some investigators²⁾³⁾⁴⁾ in this field, however, agree to attribute the main cause to the anomalous temperature distribution such as proposed by Plaskett. If the latter explanation be correct, lines with approximately equal intensities and wave lengths should show similar limb effects. While on the former assumption, lines do not show similar limb effects, but have to be classified according to the run of η along the depth. It is the purpose of the present paper to test observationally which assumption is more plausible.

Solar spectra were obtained by the grating spectrograph of the tower telescope of the Tokyo Astronomical Observatory at Mitaka. In order to minimize the scattered light, condenser lens was diaphragmed off down to 20 cm. in diameter. Solar image of some 13 cm. in diameter was guided relative to the spectrograph slit with an accuracy of $\pm 0.2 \text{ mm}$. The slit width used was 0.04 mm . throughout. In order to get as high a resolving power as possible, the third order spectrum was studied. The theoretical resolving power of the third order amounts to 225,000, while the linear dispersion is about $0.4 \text{ \AA/mm}$. The wavelength region selected was $\lambda\lambda 4830\text{--}4880$. Fuji Panchromatic plates were used. On each plate one solar spectrum and one wedge spectrum were photographed

1) Tokyo Astr. Bull., 2nd Ser, No. 12, 1948.

2) M. G. Adam: M. N., **100**, 595, 1940.

3) D. S. Evans: M. N., **107**, 433, 1947.

4) Cf. R. O. Redman: M. M., **103**, 173, 1943.

with equal exposure times which vary from plate to plate ranging from 2 to 5 minutes according to the position on the disk. Wedge spectrum was photographed with the same spectrograph as the solar spectrum, by illuminating the widened slit with the central part of the solar disk. Plates were taken for $\cos \theta = 1.0, 0.8, 0.6, 0.4, 0.3$, and 0.2 . All plates were secured in two hours in the morning of November 2, 1949. In order to minimize the distortion of the solar image due to the deformation of mirrors, the coelostat mirror was protected from sunlight by the dome as often as possible during the observation. All plates were brush developed in fine grain developer.

The neutral filter wedge was calibrated by another spectrograph of moderate size by the method of perforated screen.

Since no self-registering microphotometer was then available at our observatory, the measurement was carried out with a wedge microphotometer of Fabry-Buisson type constructed by the Tokyo Kogaku Optical Company Ltd. With suitable attachments the displacement of the plate was made to be read with an accuracy of $\pm 2\mu$, and the illuminated area as small as 0.01×1 mm., by replacing the circular hole by a narrow slit. Fifty points were measured per millimeter on the plate. The continuous background was estimated on tracings covering the wavelength range of the order of 20 Å. The conversion of density into intensity was done in the usual way. Equivalent widths of about fifty lines were measured with a planimeter.

Our measures of the center spectrum are in good agreement with those of Allen⁵⁾, except that there is a slight difference which may well be explained by applying the correction for the scattering due to the ghost to our measurement.

Comparison of our limb center ratios of equivalent widths with those of other observers²⁾³⁾⁷⁾ is illustrated in Fig. 1, where the ratios between $\cos \theta = 0.3$ and 1.0 are plotted. Agreement is very satisfactory in view of the fact that the wavelength region of our measurement lies between those of Allen and Adam (in the green). The decrease of the tendency of the limb strengthening towards longer wave lengths is obviously associated with limb darkening and strongly supports the view that these faint lines are formed by "absorption", not by "scattering" like the wing of strong Fraunhofer lines⁶⁾.

Now let us examine if there exists any peculiarity characteristic to an individual line. Unfortunately, however, our meas-

5) C. W. Allen: Mem. Commonwealth Solar Obs., **5**, 1934.

6) M. G. Adam: M. N., **98**, 112, 1938.

7) C. W. Allen: M. N., **109**, 343, 1949.

8) Z. Suemoto: Publ. A. S. Japan, **1**, 78, 1949.

urement is not accurate enough for the definitive discussion of the present problem. We must, therefore, content ourselves with an analysis of somewhat statistical nature.

Fig. 1. Observed and calculated limb center ratios of equivalent widths.
Measured by the present author.
---- Calculated curves near λ 5100 Å.

In order to eliminate the errors of measurement, let us introduce $\bar{C}$ and $\bar{L}$ defined by

$$\bar{C} = [W(1.0) + W(0.8) + W(0.6)]/3,$$

$$\bar{L} = [W(0.4) + W(0.3) + W(0.2)]/3,$$

where W 's denote the equivalent widths. In the diagram, in which $\log \bar{L}/\bar{C}$ are plotted against $\log C$ the deviation δ of the point from the mean curve measures the degree of the limb strengthening. If the tendency of limb strengthening is to be explained in terms of η variation, δ 's of various lines must correlate with their types of η variation. As a measure of the η variation we assigne to each line a quantity¹⁾

$$J = |\log (N_i/N_a)|_{\tau=0.1}^{\tau=1.5},$$

where N_a is the total number of atoms of an element a per cubic centimeter, and N_i is the number of atoms in the lower state of the line under consideration. Arithmetically smaller J corresponds to a steeper decrease of η with depth. J 's of measured lines were

Fig. 2. Correlation between δ and A .

computed by applying Saha's and Boltzmann's formulae. Correlation between δ and A is shown in Fig. 2. Although the scattering in the diagram is considerable, there is no doubt as to the reality of the negative correlation between them. In other words, it is established that a line with η decreasing more rapidly with depth show stronger limb strengthening as compared with the one with η of slower decrease, in conformity with the theoretical prediction.

If it is accepted that the scattering around the mean curve in the diagram such as Fig. 1 is not due solely to observational errors but is associated with some character of an individual line, explanations based on the anomalous temperature distributions become, at least partly, untenable. Since the characteristic behavior of the line is correlated with the run of η with depth, it is tempting to assume that the general tendency of the limb strengthening of faint lines is also associated with the general tendency of η to decrease with depth.

In order to see the validity of the explanation, theoretical curves for three types of η variation are drawn in Fig. 1 for the wave length near λ 5100 Å, where reliable limb darkening coefficients are available. Depression $r^*(\theta)$ is given by¹⁾

$$r^*(\theta) = \int_0^\infty \frac{\beta + 2\gamma\tau + 2\gamma \cos \theta}{1 + \beta \cos \theta + 2\gamma \cos^2 \theta} \cdot \eta \cdot \exp\{-(\tau + \tau_v) \sec \theta\} d\tau,$$

where

$$\tau_v = \int_0^\infty \eta d\tau,$$

which is an exact solution of the equation of transfer for $\varepsilon=1$, and is free from any assumption concerning η . Limb darkening coefficients β and γ of the first and second power are $+3.31$ and -0.27 , respectively. In computing equivalent widths, the contour of the absorption coefficient was assumed to be of pure Doppler with half-width $0.040 \text{ \AA}$ ⁹⁾. Although the theoretical curves are slightly lower than the mean curve of Adam at $\lambda 5100 \text{ \AA}$, general agreement seems to be satisfactory, and, moreover, the scattering due to the difference in η type is also well represented. More detailed comparison will be insignificant at present, since measurements of different observers do not give fully consistent results. It is to be added, however, that the assumption of pure Doppler contour for the absorption coefficient is not altogether justified. In case when the contour has an appreciable damping wing, the theoretical curves would lie somewhat higher, especially for stronger lines, since increasing limb strengthening towards smaller depressions would be effective in the widened wing. There is a reason to believe that cross-section for broadening collisions by neutral hydrogen atoms are several times larger for these lines than the one, for example, of D line. Detailed investigation along this line will be made when some observations concerning the contours of faint lines will become available.

9) P. ten Bruggencate u. J. Houtgast: Zs. f. Ap., **20**, 149, 1940.