105. On the Damaged Fan-Shaft.

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1. The damage of shaft to be reported in the following refers to coal-mine-fans of a horizontal, double-suction type. The shaft is supported by two end-bearings, being loaded with a runner at the middle of the span, and it is connected with the motor-shaft through a flexible coupling. The characteristic feature of the damage as will be stated presently is not limited to a particular fan, but the present note is specially concerned with a 1500 HP-fan, in which the first damage (complete rupture) occurred after running for five years since erection. Again, the second shaft failed in three years. A sketch of this damaged shaft calls attention to the notable feature that many transverse cracks were found at various points in a wide range on the *both* sides of the runner (A similar thing was observed also in the shaft of another plant to be referred to subsequently). As the shaft of mild steel was constantly exposed to the current of wet air containing the mine-gas, the corrosion fatigue was thought fatal by the experts. The third shaft was therefore provided with metal-sheaths for protecting the surface from the corrosive action. This provision proved good, but the shaft being attacked partly at the terminal of the sheath during a period of three years and a half, it was replaced by the fourth one, which is now in use. The second and third shafts were of the same size as the first, while the present shaft is made greater.

So far the history in brief as the author is aware! To clarify the cause of the damage it was able by the courtesy of the authorities concerned to use the data necessary for the calculations, of which the result is here laid down as a theoretical view though not conclusive since the experimental research on an old shaft is no more possible.

2. With regard to the shaft under consideration, the vibration problem was taken up by applying an approximate formula for the diameter:

$$d = d_0(1 + cx^2)^{-1/4}, \quad c > 0, \quad (1)$$

x being the distance of the cross section from the centre of the span; it is measured in a proportion of the half length l so that d varies from d_0 at $x=0$ to $d_0(1+c)^{-1/4}$ at $x=1$. The above formula is not strictly correct, as the actual profile is stepped at places, and moreover it is slightly unsymmetrical about the centre. Still,

(1) is correct enough for estimating the natural frequencies of the lateral vibrations. The numerical data read: $2l=600$ cm (measured between the bearing-centres), $d_0=43.5$ cm, and $c=3.4$; the weight of the shaft (length $2l$)=5042 kg, and the weight of the runner=8300 kg.

Beginning with the mode 1, Fig. 1, the frequency—strictly a higher bound—is found to be 11.6 cycles per sec., that is, nearly so high as twice the speed of the fan; on record (1940–51) is the speed from 345 to 443 r. p. m. Owing to the lack of the elasticity-constant of the flexible coupling, only a rough calculation of the torsional vibration can be made; so far as the calculation is made, the frequency seems to be not greatly different from the fan-speed. These circumstances make one think of the so-called secondary critical speed. Making this point thus an item of the experiment, the strain-measurement with wire-strain-gauge was carried out by the maker this February on the third shaft just before the removal. No considerable stress was observed, however, in the range of speed up to 400 r. p. m. Higher velocity could not be attained at that time because of the electrical condition.

Next comes the lateral vibration of the mode 3, Fig. 3, into question. The frequency of the third mode is nearly 12.7 times the first-mode-frequency; so it is equal to $11.6 \times 12.7 = 147.3$ cycles per sec. As the runner has twenty sets of the blades round the periphery, so many number of impulses may occur during a revolution. The forced vibration of this cause being assumed, the resonance speed of the shaft is $147.3 \times 60/20 = 441.9$ r. p. m. This speed is within the range stated above, though it is near the peak. So the third-mode-vibration may excite imagination, but the stress caused thereby *cannot* be taken as the *direct* cause of the damage. The reason is this: In the mode 3 there exists on each side of the centre a point of no curvature and therefore of no stress. In case of the third-mode-stress being really responsible for the damage, the above condition ought to be shown by the existence of a safe zone free from cracks. This inference has not been proved on examining the distribution of the cracks. Two shafts were examined in detail; one is the second shaft of the present fan, and the other is a larger shaft taken out from another fan, 3200 HP, of a similar type; the both shafts had been used in two different plants to which the informations as to the distribution of the cracks are due.

3. It appears from the above (and what follows) that the vibrations have to be viewed in another aspect. Owing to a small clearance existing between the journal and the bearing surface, the shaft has a certain freedom of motion. The condition for the shaft-end

being held in position requires that the inertia force is less than the bearing-reaction or it is same to say that the amount of the shearing force at the end-section of the shaft is less than the weight of the runner and shaft shared by the corresponding bearing. At an instant when the condition becomes critical, if such a state occurs, the mode of the vibrations, as seen in Figs. 1-4, loses ground, and it gives place, if the journal gets quite free from the bed, to a type of motion with more or less free end-conditions. Then, it may happen that the shaft *knocks* on the bed causing the impact stress in itself. Whether such a critical state may occur without being accompanied by an important amount of the vibratory stress has to be found.

Figs. 1 and 3

Figs. 2 and 4

To consider the point just stated, let us denote by B the flexural rigidity EI and by ρ_1 the mass per unit length of a uniform shaft, which will be taken for the general survey of the question; then the constant α in the expression for the frequency, namely, $p = \alpha^2 l^{-2} \sqrt{B/\rho_1}$, is a root of transcendental equation containing a parameter:

$G = M/4\rho_1 l$, M the mass of the runner, for the symmetric modes, Figs. 1 and 3; or

$H = J/4\rho_1 l^3$, J the moment of inertia of the runner with respect to a diametral axis through the centre, for the anti-symmetric modes, Figs. 2 and 4.

For the present $J = 8730.10^4 g^{-1} \text{ kg cm sec}^2$; using this and other values (M , ρ_1 , and l), we obtain the following values of α^2 :

Order of Mode	1	2	3	4
Coefficient α^2	1.19	3.76	16.45	16.11

With a very great value of G or H $\alpha^2 = 15.42$ for the modes 3 and

4; further G and H may alter considerably with slight increase of α^2 .

Here we see the possibility of the mode 4 being likely forced as well as the mode 3; however, when it exists, the relation to the damage seems to be also indirect by the same reason as was stated with regard to the latter mode.

For each mode the bending stress σ at any section and the shearing force S at an end-section are found except for an indeterminate factor, which is eliminated from the two equations with the result:

$$S = \sigma \frac{2I}{d} \frac{\alpha}{l} N, \quad (2)$$

where N is a numerical coefficient to be determined for each mode, being a function of the distance of the section acted on by σ . Although N is not a constant, S depends largely on α and is greater in the modes 3 and 4 than in 1 and 2. So restricting our view to the former two modes, we take a maximum stress at an intermediate point as the criterion of the strength, since the stress at the middle section is modified (reduced) in the actual case by the boss of the runner. Then, $N=1.027$ and 1.035 for the modes 3 and 4, respectively. With $l=300$ cm and $d=36.9$ cm (equivalent to the weight of the actual shaft—not to the frequency) we find that $S=68\sigma$ kg, σ in kg/cm², approximately for the two modes. Taking, for example, $\sigma=100$ kg/cm², S equals 6800 kg, while the bearing-reaction is $\frac{1}{2}(8300+5042)=6671$ kg plus something due to the overhung parts of the shaft. The above result may serve for controlling the calculation to be made by taking the cross-sectional variation into account.

4. An approximate calculation of the shearing force is made in continuation of Art. 2, namely, by expressing the lateral deflection of the mode 3 in the form:

$$y = c_1 \cos \frac{\pi}{2} x + c_3 \cos \frac{3\pi}{2} x,$$

x being measured from the centre as before. The shearing force at an end $x=1$ is found from

$$l^{-3}(By_{xx})_x = p^2 l \int_0^1 \rho_1 y dx + p^2 \frac{1}{2} My_1,$$

writing y_1 for y at x_1 , that is, the centre of one of the contacting surfaces of the runner-boss. After integrating, the amount of the force is

$$S = 6.355 c_3 \frac{\pi^4}{(2l)^4} B_0 l, \quad B_0 = EI_0 = B \text{ at } x=0. \quad (3)$$

Except for the centre the stress is greatest somewhere near the section $x=0.7$. The stress there is thought to be little less than a maximum; it is

$$\sigma = 0.09315 \, c_3 \frac{\pi^4}{l^2} E d_0. \quad (4)$$

From (3) and (4)

$$S = 4.265 \frac{I_0}{d_0 l} \sigma. \quad (5)$$

With $I_0/(d_0 l) = 13.47 \text{ cm}^2$ we have $S = 57.4 \, \sigma \text{ kg}$, σ in kg/cm^2 . This is smaller than the foregoing result by 20 per cent, showing the effect of tapering—a favourable effect for reducing the vibratory force.

For a definite amount of S corresponding to the critical condition σ is thus greater than in the uniform shaft. Still, the difference is of no consequence; it is likely unimportant for the damage, suggesting that the *impact* at the bearing may bring about something. It can be imagined that the impact causes bending of which the moment increases simply with the distance from the end, and that a stressless zone as was made a topic in Art. 2 need not exist. This remark will probably apply also to the mode 4. Whether the above inference may hit the point to be grasped ought really to be examined by experiments, which cannot be looked for, however; the question will be laid open until further experience may decide the matter.