131. Studies on the Right- and Left-Handedness of Spikelets in Einkorn Wheats. III. Concordance Proportion, Mean Concordance Proportion and Their Relation *

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Introduction

In the first report of this series (Kihara et al., 1951), the proportion(%) of regular antidromic sequence of spikelets (C) and the value of potency expressed by probit (Y) were used to analyse the R/L-character of wheat ears. These values were calculated at every position of the spikelets. The rate of decrease of the potency (Y) at any point of the ear was expressed by the general formula:

\[- \frac{dY_x}{dx} = \alpha Y_x + \beta,\]

where x represents the position of the spikelet in question and \(Y_x\) is the value of Y in that position, while \(\alpha\) and \(\beta\) are constants which are characteristic of a given genotype.

The use of the values of C, \(Y_x\) and \(\frac{dY_x}{dx}\), however, was frequently found to be inconvenient in genetic analysis. Therefore, the writer would like to propose a more convenient measure for the study of the R/L-character, namely the mean concordance proportion (\(\bar{C}\)).

Concordance Proportion (C) and Mean Concordance Proportion (\(\bar{C}\))

If a variate \(X_i\) is assumed to have the value 1 in case of a concordant spikelet and the value 0 in that of a discordant one, \(\sum_{i=1}^{n} X_i\) represents the number of concordant spikelets in a given spikelet position of \(n\) ears. Then the concordance proportion of the \(k\)th spikelet position \(C_k\) is

\[\frac{\sum_{i=1}^{n} X_i}{n}\]

and its expected standard error \(\sigma_{C_k}\) is

\[\sqrt{\frac{V_{C_k}}{n}} = \sqrt{\frac{C_k(1 - C_k)}{n}}\]

The \(C\)'s of 20 spikelet positions in *Triticum monococcum var. flavescens* are shown in Table I and the curves of \(C\)'s for several species of *Triticum* are represented in Fig. 1.

*) Contributions from the Laboratory of Genetics, Kyoto University, No. 256.
Next, the mean concordance proportion will be considered. The value is expressed as

\[ \bar{C} = \frac{\sum C_k}{m} \]

where \( C_k \) is the value of \( C \) at the \( k \)th spikelet position and \( m \) is the number of spikelet positions. The sampling variance can be obtained as follows:

\[ \text{Var}(\bar{C}) = \frac{1}{m^2} (\sum C_k^2 + \sum C_k C_{k'}) - \frac{(\sum C_k)^2}{m^2} \]

where \( C_{k'} \) is the value of \( C \) at the \( i \)th spikelet position. The sampling variance can be obtained as follows:

\[ \text{Var}(\bar{C}) = \frac{1}{m^2} (\sum C_k^2 + \sum C_k C_{k'}) - \frac{(\sum C_k)^2}{m^2} \]

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\[ \text{Var}(\bar{C}) = \frac{1}{m^2} (\sum C_k^2 + \sum C_k C_{k'}) - \frac{(\sum C_k)^2}{m^2} \]
where $V_{c_k}$ represents the variance of $C_k$ and the $W$'s the covariance between $C_k$ and $C_l$. The number of terms of covariance equals to $\binom{m}{2}$, and if $m$ is 20, it is $\binom{20}{2} = 190$. Since the calculation of this variance is very tedious, a new method of calculation has been devised.

\[ V = \sum_{k=1}^{m} C_k = \frac{\sum P_k}{n} = \frac{\sum S X_{ik}}{mn} \]

\[ \bar{C} = \sum_{k=1}^{m} C_k / m \]

\[ V_C = \frac{1}{m^2} \left[ \sum_{k=1}^{m} V_{ck} + 2 \sum_{k=1}^{m} W_{ck} C_k \right] \]

Fig. 2. Diagram of wheat ears for the explanation of $C_k$, $\bar{C}$, $V_{ck}$ and $W_{ck} C_k$

If $X_{ik}$ will be considered as representing $X_i$ at the $k$th spikelet position, $\bar{C}$ equals to

\[ \bar{C} = \frac{1}{m} \sum_{k=1}^{m} X_{ik} / n \]

and the variance is expected as follows:

\[ V_C = V(\sum_{k=1}^{m} X_{ik}) = \frac{V(\sum_{k=1}^{m} X_{ik})^2}{nm^2} = \frac{E(\sum X_{ik}^2) - E(\sum X_{ik})^2}{nm^2} \]

(c.f. $E$: mathematical expectation)

\[ \therefore m^2 n V_C = E(\sum X_{ik}^2) + E(2 \sum_{k=1}^{m} \sum_{i=k}^{m} X_{ik} X_{il}) - (E(\sum X_{ik}))^2 \]

\[ = \sum E(X_{ik}^2) + 2 \sum \sum E(X_{ik} X_{il}) - (\sum E(X_{ik}))^2 \]  \hspace{1cm} (A)

On the one hand $X_{ik} = X_{ik}$, because $X = 1$ or 0, then

\[ E(X_{ik}) = S X_{ik} / n = S X_{ik} / n \]

\[ \sum E(X_{ik}) = S \sum X_{ik} / n \]  \hspace{1cm} (B)

on the other hand

\[ 2 \sum \sum X_{ik} X_{il} = (\sum X_{ik})^2 - \sum X_{ik}^2, \]

\[ 2 \sum \sum E(X_{ik} X_{il}) = S (\sum X_{ik})^2 - \sum X_{ik}^2 \]  \hspace{1cm} (C)

If the values of (B) and (C) are put into formula (A),

\[ m^2 n^2 V_C = S(\sum X_{ik})^2 - (S \sum X_{ik})^2 / n \]

\[ \therefore V_C = \frac{1}{m^2 n^2} S(\sum X_{ik})^2 - \frac{1}{n} \bar{C}^2. \]
When the value of $P_i = \sum_{k=1}^{m} X_k / m$ (Fig. 2) is used,

$$V_C = \frac{1}{n^2} \sum_{i=1}^{n} S P_i^2 - \frac{1}{n} \bar{C}^2.$$  

When it is assumed that the sampled ears are independent of each other in antidromic potency, this $V_C$ can be obtained directly from the asterisked part in the above calculation.

The values of $\bar{C}$ of several species of *Triticum* are shown in Table II with their variances and standard errors.

### Table II

Mean concordance proportions, their variances and standard errors

<table>
<thead>
<tr>
<th>Species</th>
<th>Number of ears</th>
<th>Variance</th>
<th>S.E. %</th>
<th>$\bar{C}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>T. monococcum var. vulgare</td>
<td>29</td>
<td>0.0001511</td>
<td>1.23</td>
<td>94.48</td>
</tr>
<tr>
<td>T. mono. var. flavescens</td>
<td>100</td>
<td>0.0001301</td>
<td>1.14</td>
<td>81.11</td>
</tr>
<tr>
<td>T. mono var. vulgare (mutant “early”)</td>
<td>29</td>
<td>0.0015773</td>
<td>3.97</td>
<td>80.16</td>
</tr>
<tr>
<td>T. aegilopoides var. boeoticum</td>
<td>108</td>
<td>0.0000934</td>
<td>0.97</td>
<td>87.54</td>
</tr>
<tr>
<td>T. turgidum var. nigrobarbatum</td>
<td>18</td>
<td>0.0007069</td>
<td>2.66</td>
<td>56.55</td>
</tr>
<tr>
<td>T. Timopheevi</td>
<td>64</td>
<td>0.0002591</td>
<td>1.61</td>
<td>50.78</td>
</tr>
</tbody>
</table>

**Discussion**

The expected variance of concordance proportion is that of a binomial distribution, and then if $C$ is 70% the standard error equals to $\sqrt{0.21 / n}$. In order to keep the error under 2%, the number of samples ($n$) must be more than 525 ears. Thus the determination of the $C$ value having less than 2% of standard error requires much work, while that of the $\bar{C}$ value having the same accuracy needs usually no more than 50 ears. (Cf. Table II, in which the value $\bar{C}$ of mutant “early” of *T. monococcum* is an exception.)

In addition to this, as the number of ears sampled from an individual is restricted to about ten, it becomes necessary to obtain a measure with a smaller error with as few ears as possible. It is particularly so, when a population consisting of segregated genotypes is investigated. The mean concordance proportion can satisfy these requirements.

It should, however, be noted that the analysis of the mode of decreasing potency of antidromy with the ascending position of the spikelets is quite important to clarify the mechanism of its determination, and for this purpose $C$ does not lose its merits.

Consequently, the writer concludes that the mean concordance
proportion is a more efficient statistics than the concordance proportion in the analysis of the genetic system governing the R/L-character, especially in small samples.

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References