98. An Intermediate Orbit of the Ninth Satellite of Jupiter

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1. Introduction. An intermediate orbit of the ninth satellite of Jupiter (J-IX) is calculated by the numerical method due to E. W. Brown and D. Brouwer.1-2 J-IX is the outermost and retrograde satellite of Jupiter. On account of the large solar perturbation, any known analytical method can not be applied to the present case, and we adopt the numerical one. In this method the true longitude reckoned wholly in the osculating plane, \( \nu \), is adopted as an independent variable, which has been proved to have several advantages.3) But the chief disadvantage—somewhat greater complexity in the development of the disturbing function—is remedied by its numerical character.

2. Notation. \( r = \) distance of the satellite from Jupiter. \( i = \) inclination of the osculating plane of its orbit to the plane of reference. \( I' = 1 - \cos i \). \( \theta = \) longitude of the ascending node of this plane on the plane of reference reckoned from a line fixed in the latter plane. \( v = \) jovicentric longitude of the satellite reckoned along the plane of reference to the node and then along the osculating plane, in the direction of the motion of the satellite.

\[
v = v - \int (1 - \cos i) d\theta.
\]

\[
D = \frac{d}{dv} \mu = \text{sum of masses of Jupiter and the satellite.}
\]

\( r' = \) distance of the Sun from the baricenter of the Jupiter-satellite system. \( \nu' = \) longitude of the Sun as seen from Jupiter reckoned in the direction of the motion of the satellite. The orbit of Jupiter is taken to be an ellipse in the plane of reference. \( \mu' = \) mass of the Sun. \( n, n' = \) mean motions of the satellite about Jupiter and of Jupiter about the Sun respectively, reckoned in the sense of the motion of the satellite, so that \( n' < 0 \). \( a, a': \) \( n^2 a^3 = \mu, \) \( n'^2 a'^3 = m' + \mu \). \( q = aq^*, \) \( \left( \frac{\mu}{q^*} \right)^{1/2} = r' \frac{d\nu}{dt} \).

\[
u = \frac{a}{r}, \quad R = aR^*, \quad \mu R^* = \text{disturbing function due to the Sun.}
\]

\( e, e' = \) eccentricities of the orbit of the satellite and of that of the Sun. \( -\zeta, -\zeta' = \) mean longitudes at epoch and of the apse of the Sun's orbit about Jupiter, reckoned in the same sense as the other angles.

\[
2\xi = 2v \left( 1 - \frac{n'}{n} \right) [1 + (I'D\theta)_{0,0}] + \text{const., where} \ (I'D\theta)_{0,0} \ \text{is the constant term of} \ I'D\theta \ \text{when it is expressed as the sum of periodic terms with arguments depending on} \ v.
\]
3. Equations of motion. We define the intermediate orbit by neglecting certain portions of the disturbing function. The adopted value of $R$ for the intermediate orbit is

$$R = k_1 \left( \frac{1}{3} - \frac{1}{2} \Gamma^2 \right) + k_2 \left( 1 - \frac{1}{2} \Gamma^2 \right) \cos (2\nu - 2n't + 2\varepsilon').$$

With this value of $R$, the equations of motion become

$$Dq = \left( 1 - \frac{1}{2} \Gamma^2 \right) \frac{4k_2q^2}{u^3} \sin (2\nu - 2n't + 2\varepsilon'),$$

$$D^2u + u - q = - \frac{2k_1q}{u^3} \left( 1 - \frac{1}{2} \Gamma^2 \right)$$

$$- \left( 1 - \frac{1}{2} \Gamma^2 \right) \frac{2k_2q}{u^3} \cos (2\nu - 2n't + 2\varepsilon') + \frac{1}{2} \frac{Dq}{Du},$$

$$nDt = q^\frac{1}{2} u^{-\frac{3}{2}}$$

$$\Gamma = \gamma q^\frac{1}{2}, \quad v = v + D^{-1}(\Gamma'D\theta),$$

where $\gamma = \text{constant of integration},$

$$k_1 = \frac{3}{4} \frac{n'^2}{n^2} \frac{m'}{m' + \mu} (1 - e'^2)^{-\frac{3}{2}}$$

$$k_2 = \frac{3}{4} \frac{n'^2}{n^2} \frac{m'}{m' + \mu} \left( 1 - \frac{5}{2} e^2 + \frac{13}{16} e^4 - \frac{35}{288} e^6 \right).$$

We see, from the preceding equations, that $u, q, \Gamma$ are of the form $a_{0,0} + \sum_{j,i} a_{j,i} \cos (2\hat{\xi}j + il),$ $t, \theta, v$ are of the form $a\nu + \beta + \sum_{j,i} b_{j,i} \sin (2\hat{\xi}j + il),$ where $j = 0, 1, 2, \ldots; \ i = 0, \pm 1, \pm 2, \ldots,$ and $a, \beta, a_{j,i}, b_{j,i}$ are constants.

Hence the intermediate orbit contains all terms depending on the eccentricity and the ratio of the mean motions together with such portions of the disturbing function as can be included qualitatively, and we obtain a quasi-periodic orbit depending on the arguments, $l$ (elliptic terms) and $2\hat{\xi}$ (variations).

4. Definitions of the constants. a) $\left( \frac{u}{q} \right)_{o,0} = e = 0.275000$ (adopted value). This is the definition of the eccentricity in the disturbed motion and furnishes the boundary condition for determining $Dl.$

b) When $R$ does not contain $\theta,$ we have immediately $\Gamma'q^{-\frac{1}{2}} = \text{const.} = \gamma,$ so that $\cos i = 1 - \gamma q^{\frac{1}{2}}.$ In each stage of the approximation, $(q^{\frac{1}{2}})_{0,0}$ is determined and the value of $\gamma$ is, then, found so that the mean value of $i$ may be equal to the assigned one $(24'150'').$ But, for convenience, we put $\gamma = (1 - \cos 24'150'')/1 - (0.275000)^2.$ The value of $\gamma$ is then fixed from the beginning and the process of the calculation is much simplified. In the final approximation the mean value of $i$ is
found to be 24°8'50". We may suppose that the adopted mean value of $i$ is 24°8'50" instead of 24°1'50" if necessary (cf. Section 5).

c) $\left( n \frac{dt}{dv} \right)_{0,0} = 1$. This boundary condition determines the correction to $q_{0,0}$ in each stage of our approximation: $(q_{0,0})_{(n+1)\text{th app.}} = (q_{0,0})_{n\text{th app.}} + \delta q$.

d) In the undisturbed elliptic motion, $l D\theta = 0$ and $v = \nu$. Also in the second and higher approximations, when the equation for $v$ is integrated, no constant is added.

e) All other angular constants in the equations for $\theta$ and $t$ are retained in a literal form throughout the solution until a comparison with observation is undertaken.

5. The adopted numerical values of the constants. a) The constants for the satellite. The values of the constants for the satellite are taken from a paper by S. B. Nicholson. He has computed in this paper the jovi-centric rectilinear co-ordinates of J-IX which represent satisfactorily all the positions observed from 1938 to 1943 by mechanical integration with the co-ordinates and velocities on September 28,0, 1940, U. T., as the constants of integration. The mean values of the elements of the satellite with their approximate ranges are as follows:

\[ T = \text{Jan. 0, 1943} \pm 75 \text{ days}, \quad e = 0.275 \pm 0.15, \]

\[ \omega = 103^\circ \pm 3^\circ.7 (t-1940) \pm 24^\circ, \quad a = 0.1585 \pm 0.008 \text{ A. U.}, \]

\[ \Omega = 61^\circ + 4^\circ.44 (t-1940) \pm 7^\circ, \quad P = 758 \pm 25 \text{ days}, \]

\[ i = 180^\circ - 23^\circ \pm 5^\circ (i: \text{with reference to ecliptic}), \]

where the observations for the interval from 1914 to 1918 are also taken into account.

As Nicholson points out, the solar perturbation changes the orbit so rapidly that the ordinary osculating elements can describe the motion of the satellite only for a short time and the preceding values of the elements are probably of little physical significance. One must expect changes of at least several per cent, when the results are compared with observations.

b) The constants for Jupiter. For self-consistency, these values are taken from The American Ephemeris and Nautical Almanac (1940) by reducing them to the values on September 28 by interpolation. We have:

\[ e' = 0.0484039, \quad i' = 1^\circ 18'23'', \quad \Omega' = 99^\circ 51'0''5, \quad \frac{m'_J}{m'} = 1047.355 \text{ (Newcomb)}, \]

\[ -n' = 299''.128 \text{ per mean solar day}. \]

c) The inclination of the orbit with reference to the ecliptic is reduced to that with reference to the orbital plane of Jupiter; we then obtain $i = 24^\circ 1'50''$ in our notation.

The value of $\frac{n'}{n}$ is calculated from the values of $P, n'$; we have
\[ \frac{n'}{n} = -0.174953 \]. With these values we have: \( k_1 = 0.0230154 \), \( k_2 = 0.0228003 \), \( \gamma = 0.0833299 \).

6. Method of solution. The method adopted here is one of successive approximations based on the fact that the constants \( k_1, k_2 \) are small. In the first approximation we neglect \( k_1, k_2 \); the motion is then elliptic. Compared with the method of equations of variations, our adopted method which is combined with harmonic analysis\(^5\) has several advantages: we can avoid accidental errors by referring to the results of the previous stage of the approximation, or else few checks are available in our calculation; errors do not accumulate but disperse in the succeeding stage of the approximation; harmonic analysis carries its own checks.

The values of the variables are found in the order of \( q, u, t, \Gamma^\prime D\theta, v, \Gamma' \). \( Dl \) is determined when the equation for \( u \) is integrated. \( \delta q \) is obtained from \( nDt \) and \( (\Gamma^\prime D\theta)_{0,0} \); \( D2\xi \) from \( (\Gamma^\prime D\theta)_{0,0} \). It is found that one approximation to \( t \) is sufficient with two for \( q, u \) and that \( \Gamma^\prime D\theta, v, \) and \( \Gamma' \) have to be computed anew even less frequently. For \( q \) and \( u \), two successive approximations give a gain in the accuracy of about one decimal place, while the change of \( \Gamma^\prime D\theta \) during the course of the work is 5% at most.

7. The small divisors. There occur small divisors in connection with the argument \( 6\xi - 7l \) when we integrate the equations for \( q, t, \theta \) (0.0468), and in connection with the arguments \( 6\xi - 6l, 6\xi - 8l \) when we integrate the equation for \( u \) (0.0926, 0.0942 respectively). This difficulty is due to the fact that the period of the satellite is \( \frac{1}{5.7} \) of that of Jupiter; it is then independent of the particular method of the reduction. Since we carry out the calculations to six places at most, we estimate before our integration that the corresponding coefficients are zero within the limits of accuracy: the two or three units actually present in the last place are probably due to arithmetical accumulation. Such coefficients are made zero in order to avoid spurious results and are denoted by \([0]\) in the table. It is noticed that, if we make the calculations to eight places or more, this difficulty may become inevitable and we may be led to interminable calculations not being unable to obtain any self-consistent solution by this method.

8. The mean motions of the peri Jove and of the node. With the adopted value of \( R \) (cf. Section 3), we obtain:

\[ l = 0.998520 v + \text{const.}, \quad \theta = -0.021864 v + \text{const.} + \text{periodic terms}, \]
\[ v = 0.998112 v + \text{periodic terms}. \]

Then
\[ l = 1.000409 v + \text{const.} + \text{periodic terms}, \]
\[ \theta = -0.021905 v + \text{const.} + \text{periodic terms}. \]
Final results for the intermediate orbit (10th approximation)

\[
q = \sum q_{j,4} \cos (2\pi j + \delta)
\]

\[
\alpha/r = u = \sum u_{j,4} \cos (2\pi j + \delta)
\]

\[
v = -0.998112 v + \sum v_{j,4} \sin (2\pi j + \delta)
\]

\[
n' = \frac{-0.174623 (v - \pi) + \sum n_{j,4} \sin (2\pi j + \delta)}{\cos i - r \sin \delta - \sum \Gamma_{j,4} \cos (2\pi j + \delta)}
\]

\[
v = -0.021384 v + \theta_{4} + \sum \theta_{j,4} \sin (2\pi j + \delta)
\]
The mean motions of the perijove and of the node are \(-0.000409\ n\) and \(-0.021905\ n\) respectively.

The value of the mean motion of the perijove depends on the values of \(Dl\) and \((\Gamma D\theta)_{h,0}\). The value of \((\Gamma D\theta)_{h,0}\) is finally found to be \(-0.001888\), but nearly the same value is obtained already in the third approximation; the value of \(Dl\), however, approaches its final one less rapidly. The value of the mean motion of the perijove, therefore, varies rather considerably during the course of the work, and is also fairly sensitive to any change in the constants for the satellite.

With the value \(n=173.37\) per year, we have:
- the mean motion of the perijove = \(-0^\circ.08\) per year \((-0^\circ.7\) per year),
- the mean motion of the node = \(-3^\circ.80\) per year \((-4^\circ.44\) per year),
where the values written in the parentheses are due to Nicholson.

9. Concluding remarks. Generally in any iteration method, even if the difference between any two successive approximations can be neglected in the range of the “round-off” error, we must expect much larger errors in the final results when the speed of convergency is slow. In the present case, as is stated, two successive approximations give one additional decimal place, and the errors in the final results are probably about twice as large as the round-off error, namely, \(2.5\times10^{-7}\), or \(10^{-6}\). The results of the final approximation (10th app.) are listed in the table.

The calculation of the complete orbit which contains the additional portions depending on the inclination, the first power of the ratio of the parallaxes, and the first power of the eccentricity of Jupiter is now being undertaken.

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References