In their note ([1], [2]), Y. Arai and K. Iséki discuss on some theses of equivalential calculus introduced by S. Leśniewski (see, [3]).

The equivalential calculus satisfies the following two fundamental axioms:

\[ \text{E1} \quad EEEprEEqpEeqr, \]
\[ \text{E2} \quad EEpqEeqrEEeqqr, \]

where \( E \) is the truth functor in the calculus (see, [4]).

In his paper [2], Prof. K. Iséki has given a new axiom set and has proved that the equivalential calculus is characterized by it, using some metatheorems. His results are read as below:

**Lemma 1.** The equivalential calculus is characterized by

1. \( Epp \)
2. \( EEpgEqp \)
3. \( EEpgEEqrEpr \).

**Lemma 2.** The above axiom set is equivalent to the single axiom \( EEpqEEeqrEeqr \) (see, [2]).

In this paper, we shall also give a new axiom set of the equivalential calculus and prove that its set characterizes the equivalential calculus.

We use the two rules of inference, i.e., substitution and detachment: \( \alpha \) and \( E\alpha\beta \) imply \( \beta \).

First we shall prove the following

**Theorem 1.** The following axiom set, i.e.,

1. \( EEpqEeqrEEeqrEeqr \)
2. \( EEpqEeqp \)

implies the axiom set, i.e.,

1. \( Epp \)
2. \( EEpqEeqp \)
3. \( EEpqEEeqrEeqr \).

For the proof we shall use proof lines by J. Lukasiewicz.

**Proof.** From the axioms 1 and 2, i.e.,

1. \( EEpqEeqrEEeqrEeqr \)
2. \( EEpqEeqp \)

we deduce the following theses:

1. \( p/Eeqr \) \( \vdash \) \( C2\leftarrow 3 \),
2. \( EEeqrEeqrEeqr \).
Next we shall give the proof of the following theorem.

Theorem 2. The single axiom of the equivalential calculus, i.e.,
1 EEpqEEprErq
implies the following axiom set, i.e., EEpEqrEEsqEsEpr, EEpqEeq.

For the proof we use some results from the axiom EEpqEEprErq (for the details and prooflines, see [5]).

Lemma 3. The axiom EEpqEEprErq implies
2 EEpEqrErq,
3 EEpqEEprErq,
4 EEpqEersErEqs.

If we put B for CcqrCcprCp, C for CcpcqrCqCpr, I for Cpp, then the system BCI is equivalent to I together with
CCpCqrCCsqCsCpr (see [2]). Hence, if I and CCpCqrCCsqCsCpr are independent, then it is clearly seen that $E_{pp}$ and $EEpEqrEEsqEsEpr$ are independent. Further $CCpqCqp$ is not a thesis in the system BCI. Hence $EEpqEqp$ is independent from $E_{pp}$ and $EEpEqrEEsqEsEpr$.

References