12. Collision between the Two Columnar Objects. II

Kinematics after the Collision

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Using the same notation and the same analytical method as in paper (I), the velocities of x -direction of both objects (M_1, M_2) subsequent to the collision are obtained as follows:

For the case when $(c_1/c_2 \cdot l_2/l_1) \geq 1$ and $0 \leq M_1/M_2 \leq 1$,

$$\frac{dU_1}{dt} = V_0(1-2\alpha_1), \quad \frac{dU_2}{dt} = V_0 \cdot 2\alpha_1 \frac{M_1}{M_2}. \quad (1)$$

And further, when $(c_2/c_1 \cdot l_1/l_2) \geq 1$ and $0 \leq M_2/M_1 \leq 1$,

$$\frac{dU_1}{dt} = V_0 \left(1 - 2 \frac{M_2}{M_1} \alpha_3\right), \quad \frac{dU_2}{dt} = V_0 \cdot 2\alpha_3. \quad (2)$$

In the equations (1), (2), α_1 and α_3 are respectively shown by the following expressions:

$$\alpha_1 = \left[\frac{7}{6} + \frac{M_1}{M_2} \left\{ 1 + \frac{1}{6} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^2 - \frac{1}{2\pi^3} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^3 \sum_{\nu=1}^{\infty} \frac{1}{\nu^3} \sin \left(2\nu\pi \frac{c_2}{c_1} \frac{l_1}{l_2} \right) \right\} \right]^{-1}, \quad (3)$$

$$\alpha_3 = \left[\frac{7}{6} + \frac{M_2}{M_1} \left\{ 1 + \frac{1}{6} \left(\frac{c_2}{c_1} \frac{l_1}{l_2} \right)^2 - \frac{1}{2\pi^3} \left(\frac{c_2}{c_1} \frac{l_1}{l_2} \right)^3 \sum_{\nu=1}^{\infty} \frac{1}{\nu^3} \sin \left(2\nu\pi \frac{c_1}{c_2} \frac{l_2}{l_1} \right) \right\} \right]^{-1}. \quad (4)$$

For obtaining a concrete understanding of the characteristics of these velocities, the two graphs in Figs. 1 and 2 are obtained from (1) and (2) respectively.

As it may be seen from Fig. 1 that, in case of $(c_1/c_2 \cdot l_2/l_1) \geq 1$ and $0 \leq M_1/M_2 \leq 1$, the velocity dU_1/dt becomes zero or negative or equal to the velocity dU_2/dt in conformity with the values of (M_1/M_2) and $(c_1/c_2 \cdot l_2/l_1)$. Concerning these matters, of course, the analytical treatments may be possible, but they will be done in other occasion.

In the next place, the repulsion coefficient e between the two objects may be obtained from (1) and (3) as follows:

$$e \doteq \frac{\left[\frac{5}{6} + \left\{ 1 - \frac{1}{6} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^2 + \frac{1}{2\pi^3} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^3 \sin \left(2\pi \frac{c_2}{c_1} \frac{l_1}{l_2} \right) \right\} \frac{M_1}{M_2} \right]}{\left[\frac{7}{6} + \left\{ 1 + \frac{1}{6} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^2 - \frac{1}{2\pi^3} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^3 \sin \left(2\pi \frac{c_2}{c_1} \frac{l_1}{l_2} \right) \right\} \frac{M_1}{M_2} \right]}. \quad (5)$$

Fig. 1

Now, the collision in case of $e=1$ is commonly called a perfectly elastic collision, but the expression (5) shows that, in case of $e=1$, collision cannot be existent, because of the fact that the following relation must hold good in case of $e=1$.

$$\frac{M_1}{M_2} = - \frac{1}{\left(\frac{c_1}{c_2} \frac{l_2}{l_1}\right)^2 \left\{ 1 - \frac{3}{\pi^3} \left(\frac{c_1}{c_2} \frac{l_2}{l_1}\right) \sin \left(2\pi \frac{c_2}{c_1} \frac{l_1}{l_2} \right) \right\}}. \quad (6)$$

For example, for the case when $(c_1/c_2 \cdot l_2/l_1)=1$, the expression (6) shows that the value of (M_1/M_2) becomes equal to -1 . Therefore, it can be seen that a perfectly elastic collision corresponding to the case $e=1$ is unable to be existent in the present collision problem.

In the next place, the collision in case of $e=0$ is called ordinarily a non-elastic collision. For example, in case of $(M_1/M_2) \doteq 0.275$, and $(c_1/c_2 \cdot l_2/l_1)=5$, as shown in Fig. 1, both the velocities dU_1/dt and dU_2/dt become equal to each other, and moreover this fact corresponds to $e=0$. And, therefore, this fact needs not necessarily be called a non-elastic collision.

As concerns coming to a stop of the object M_1 subsequent to the collision, the following relation may hold in a good approximation.

Fig. 2

$$\frac{M_1}{M_2} \doteq \frac{1}{\left[6 + \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^2 - \frac{3}{\pi^3} \left(\frac{c_1}{c_2} \frac{l_2}{l_1} \right)^3 \sin \left(2\pi \frac{c_2}{c_1} \frac{l_1}{l_2} \right) \right]}. \quad (7)$$

Then, taking $(c_2/c_1 \cdot l_1/l_2)$ in abscissa, M_1/M_2 in ordinate, it is able to obtain the curve of the condition of stop $(dU_1/dt)=0$, and both regions of $(dU_1/dt)>0$ and $(dU_1/dt)<0$. In this case, of course, the relation $0 \leq (c_1/c_2 \cdot l_2/l_1) \leq 1$ (or $\infty \geq (c_2/c_1 \cdot l_1/l_2) \geq 1$) holds good in both objects.

Next, the relation $(c_1/c_2 \cdot l_2/l_1) \geq 3.91$ ($\doteq 4.0$) is the condition, in which both the velocities subsequent to the collision become equal to each other when $(c_1/c_2 \cdot l_2/l_1) \geq 1$, $0 \leq (M_1/M_2) \leq 1$. The condition above-mentioned is, of course, obtained under the permissible assumption that $\sum_{\nu=1}^{\infty} 1/\nu^3 \cdot \sin(2\nu\pi(c_2/c_1 \cdot l_1/l_2)) \doteq \sin(2\pi(c_2/c_1 \cdot l_1/l_2))$, and moreover, $\sin(2\pi(c_2/c_1 \cdot l_1/l_2)) \doteq 2\pi(c_2/c_1 \cdot l_1/l_2)$.

And, therefore, for the case when $4.0 > (c_1/c_2 \cdot l_2/l_1) \geq 1$ and $0 \leq (M_1/M_2) \leq 1$, there is no appearance of $dU_1/dt = dU_2/dt$ between the two objects. Further, it should be added that, in case of $M_1/M_2 = 0$, both velocities become $dU_1/dt = -0.714V_0$ and $dU_2/dt = 0$ respectively, in spite of the value of $(c_1/c_2 \cdot l_2/l_1)$.