13. **Collision between the Two Columnar Objects. III**

Energy Apportionment after the Collision

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Using the same notation and also the same collision theory as in paper (I), it is treated that when the object $M_1$ with a linear velocity $V_0$ collides with the object $M_2$ in a stationary state, the initial kinetic energy $(1/2)M_1V_0^2$ of the object $M_1$ is apportioned between the two objects in conformity with the principle of energy conservation. In this way, when $(c_1/c_2, l_2/l_1) \geq 1$ and $0 \leq (M_1/M_2) \leq 1$, the energy apportionment after the collision becomes as follows:

$$\frac{(R.E.I)}{(1/2)M_1V_0^2} = (1 - 2\alpha_1)^2, \quad \frac{(E.E.I)}{(1/2)M_1V_0^2} = \frac{2}{3} \alpha_1^2,$$

$$\frac{(R.E.II)}{(1/2)M_2V_0^2} = 4 \frac{M_1}{M_2} \alpha_1^2, \quad \frac{(E.E.II)}{(1/2)M_2V_0^2} = 4 \alpha_1 - 4 \left( \frac{7}{6} + \frac{M_1}{M_2} \right) \alpha_1^2,$$

in which $(R.E.I)$ and $(E.E.I)$ are respectively the kinetic and the elastic energies of the object $M_1$ subsequent to the collision, and $(R.E.II)$, $(E.E.II)$ are individually those of the object $M_2$ after the collision. Of course, $\alpha_1$ in the expression (1) becomes as follows:

$$\alpha_1 = \left[ \frac{7}{6} + \frac{M_1}{M_2} \left( 1 + \frac{1}{6} \left( \frac{c_1 l_2}{c_2 l_1} \right)^2 \right) \right]^{-1} \left( \sum_{i=1}^{3} \frac{1}{l^3} \sin \left( 2\pi \frac{c_2 l_2}{c_1 l_1} \right) \right)^{-1}.$$  \hspace{1cm} (2)

The expression (2) shows that, when $M_1/M_2 = 0$, it becomes that $\alpha_1 = 6/7 = 0.85714$. Therefore, in this case,

$$(R.E.I) \approx 0.51019 \times \frac{1}{2} M_1V_0^2, \quad (E.E.I) \approx 0.48979 \times \frac{1}{2} M_1V_0^2,$$

$$(R.E.II) \approx 0, \quad (E.E.II) \approx 0.00001 \times \frac{1}{2} M_1V_0^2.$$  \hspace{1cm} (3)

And moreover, when $M_1/M_2 = 1, \alpha_1$ becomes $3/7 \approx 0.429$. Therefore,

$$(R.E.I) \approx 0.02 \times \frac{1}{2} M_1V_0^2, \quad (E.E.I) \approx 0.12 \times \frac{1}{2} M_1V_0^2,$$

$$(R.E.II) \approx 0.74 \times \frac{1}{2} M_1V_0^2, \quad (E.E.II) \approx 0.12 \times \frac{1}{2} M_1V_0^2.$$  \hspace{1cm} (4)

For instance, in case of $(c_1/c_2, l_2/l_1) = 1$ and $4$, those energies expressed by (1) may be shown separately in Figs. 1, 2, in which the abscissae are $M_1/M_2$.

Next, when $(c_2/c_1, l_1/l_2) \geq 1$ and $0 \leq M_2/M_1 \leq 1$, the energy apportion-
tionment subsequent to the collision differs from (1), and it becomes as follows:

\[
\begin{align*}
\frac{(R.E.I)}{(1/2)M_1V_0^2} & = \left(1 - 2 \frac{M_2}{M_1} \alpha_3\right)^2, \\
\frac{(E.E.I)}{(1/2)M_1V_0^2} & = \frac{4}{M_1} M_2 \alpha_3 - 4 \left(\frac{7}{6} + \frac{M_2}{M_1}\right) \frac{M_2}{M_1} \alpha_3^2, \\
\frac{(R.E.II)}{(1/2)M_1V_0^2} & = \frac{4}{M_1} M_2 \alpha_3^2, \\
\frac{(E.E.II)}{(1/2)M_1V_0^2} & = 2 \frac{M_2}{M_1} \alpha_3^2,
\end{align*}
\]

(5)

in which \(\alpha_3\) is expressed by

\[
\alpha_3 = \left[\frac{7}{6} + \frac{M_2}{M_1} \left(1 + \frac{1}{6} \frac{c_2}{c_1} \frac{l_1}{l_2}\right)^2 \right.
- \frac{1}{2\pi^3} \left(\frac{c_2}{c_1} \frac{l_1}{l_2}\right)^3 \sum_{n=1}^{\infty} \frac{1}{n^3} \sin \left(2\pi \frac{c_2}{c_1} \frac{l_1}{l_2}\right)\left]\right.^{-1}.
\]

(6)

When \(M_2/M_1 = 0\), \(\alpha_3\) becomes \(6/7 (\approx 0.85714)\). Therefore the expression (5) turns into as follows:
\[(R.E._{I}) = \frac{1}{2} M_1 V_o^2 - 3.42856 \times \frac{1}{2} M_2 V_o^2, \quad (E.E._{I}) = 0.00001 \times \frac{1}{2} M_2 V_o^2,\]
\[(R.E._{II}) = 2.93876 \times \frac{1}{2} M_2 V_o^2, \quad (E.E._{II}) = 0.48979 \times \frac{1}{2} M_2 V_o^2.\]  

(7)

And, moreover, when \(M_2/M_1 = 1\), \(\alpha_3\) becomes \(3/7 \approx 0.429\), and
\[(R.E._{I}) = 0.02 \times \frac{1}{2} M_1 V_o^2, \quad (E.E._{I}) = 0.12 \times \frac{1}{2} M_1 V_o^2,\]
\[(R.E._{II}) = 0.74 \times \frac{1}{2} M_1 V_o^2, \quad (E.E._{II}) = 0.12 \times \frac{1}{2} M_1 V_o^2.\]  

(8)

For example, when \(c_2/c_1, l_1/l_2 = 1\) and 4, those energies expressed by (5) are shown in Figs. 3, 4, the abscissae being \(M_2/M_1\).

Comparing the two expressions (3), (7), and also seeing over Figs. 1–4, many important properties of the energy apportionment subsequent to the collision may be obtained, but the full discussions of these properties will be reserved for other occasion.