Saliency torque and V-curve of permanent-magnet-excited synchronous motor

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Abstract: The permanent-magnet-excited synchronous motor is expanding its usage considerably. It has strong and peculiar saliency of magnetic poles, which causes peculiar torque characteristics. The conventional theories, such as the two reaction theory and the two axis theory are based on the d, q axis method. Although they have been resorted to for a long time, their derivation of the circuit equation is not logical and it seems that they involve approximation and even errors. The author applied the spiral vector method (SV method) to analysis of PMSM and derived the new circuit equations of PMSM, whose solutions revealed new aspect of performances of PMSM.1-7 This paper reports further developments of SV theory of PMSM; one of them is a new V-curve of PMSM. It is quite different from the V-curve of the conventional theories, and is much more useful.

Key words: Permanent magnet synchronous motor; PMSM; reluctance torque; V-curve; saliency torque; SV method; torque control; automobile transmission.

Introduction. The permanent-magnet-excited synchronous motor (PMSM) is expanding its usage in driving robots, machine tools and automobiles, etc. Most of them have strong and peculiar saliency, which makes analysis of PMSM complicated and difficult. Their torque-current characteristics are quite different from field winding excited conventional synchronous machine. The salient-pole synchronous machine has been analyzed by the d, q axis method, making use of direct axis reactance and quadrature axis reactance. But it seems that even the steady state equation is not logically derived. And so the analysis involves not small errors. The author applied the spiral vector method to analysis of PMSM and derived a new circuit equation, which does not involve variable transformation. Its solution revealed new aspect of PMSM performance, for example new torque-current characteristics.5-7 This paper reports further analytical developments of PMSM, whose gravity center is located at the new V-curve of PMSM. The new V-curve is quite different from that of the conventional theories of the synchronous machine and is very useful in determination of performances of PMSM.

Excitation type and inductance of the armature winding. The synchronous machine has generally saliency. Fig. 1 shows the field winding excited synchronous machine.

The armature reaction self-inductances vary with double frequency, as the rotor rotates and are given by

\[ L_a = L_a + L_a \cos(2\theta), \quad \theta = \omega t + \varphi_d \]
\[ L_b = L_a + L_a \cos(2\theta - \frac{2\pi}{3}) \]
\[ L_c = L_a + L_a \cos(2\theta + \frac{2\pi}{3}) \]

Their mutual inductances are

\[ M_{ab} = M + M_a \cos(2\theta - \frac{2\pi}{3}) \]
\[ M_{bc} = M + M_a \cos(2\theta) \]
\[ M_{ca} = M + M_a \cos(2\theta + \frac{2\pi}{3}) \]

When the machine is excited with permanent magnets, two types of installation are possible, as shown in Fig. 2.

Permeability of permanent magnets is very small, close to that of air. Therefore in the surface mounted type of Fig. 2(a) we have \( L_a = 0 \) and in the embedded-magnet type of Fig. 2(b) we have \( L_a < 0 \). We call \( L_a \) saliency inductance, and when \( L_a < 0 \), we call it negative saliency. Thus we have Table I.

SV method analysis of synchronous machine. Steady state three phase currents are
\[ i_a = \sqrt{2} \left| I_1 \right| \cos(\omega t + \varphi_1) \]
\[ = \text{real} \left( |I_1| e^{j(\omega t + \varphi_1)} \right) \]
\[ i_b = \sqrt{2} \left| I_1 \right| \cos(\omega t + \varphi_1 - \frac{2\pi}{3}) \]
\[ = \text{real} \left( |I_1| e^{j(\omega t + \varphi_1 - \frac{2\pi}{3})} \right) \]
\[ i_c = \sqrt{2} \left| I_1 \right| \cos(\omega t + \varphi_1 + \frac{2\pi}{3}) \]
\[ = \text{real} \left( |I_1| e^{j(\omega t + \varphi_1 + \frac{2\pi}{3})} \right) \]  

Magnetic flux linkage of phase a winding due to armature reaction is

\[ \lambda_{ga} = L_a i_a + M_{ab} i_b + M_{ac} i_c. \]  

Inserting eqs. [1], [2] and [3], eq. [4] becomes

\[ \lambda_{ga} = \frac{3}{2} L_v \sqrt{2} |I_1| e^{j(\omega t + \varphi_1)} \]
\[ + \frac{3}{2} L_s \sqrt{2} |I_1| e^{j(\omega t + \varphi_1 + 2\varphi_d - 2\varphi_1)} \]
\[ = \frac{3}{2} L_s \sqrt{2} I_a + \frac{3}{2} L_v I_1 e^{j(2\varphi_d - 2\varphi_1)}. \]  


Here \( \dot{I}_a \) is circular vector of armature current of phase a. Now the circuit equation of phase a is

\[ V_a = R_1 I_a + L_s \dot{I}_a + p \lambda_{ga} + p(\ddot{e}^{\text{m} \lambda}). \]  

This equation is valid to all three phases only difference being phase angles of \( 2\pi/3 \). Subscript a is changed to 1. And eq. [7] becomes

\[ \dot{V}_1 = R_1 I_1 + \left( I_1 + \frac{3}{2} L_1 \right) \dot{I}_1 \]
\[ + \frac{3}{2} L_s \ddot{e}^{\text{m} \lambda} \left( \dot{I}_1 \left| e^{\text{m} \lambda} \right| e^{j(2\varphi_d - 2\varphi_1)} \right) + \dot{E}_1 \]
\[ = R_1 I_1 + j\omega L_s \dot{I}_1 + j\omega \frac{3}{2} L_1 \cos(2\varphi_d - 2\varphi_1) \dot{I}_1 \]
\[ - j\omega \frac{3}{2} L_s \sin(2\varphi_d - 2\varphi_1) \dot{I}_1 + \dot{E}_1. \]  

Here \( L_s = I_1 + (3/2) L_1 \) is synchronous inductance and \( \dot{E}_1 \) is internal induced voltage given below.

\[ \dot{E}_1 = j\omega L_e e^{j(\omega t + \varphi_d)} = \omega L e^{j(\omega t + \varphi_0)}, \varphi_0 = \frac{\pi}{2} + \varphi_d. \]  

Fig. 1 shows the SV vector diagram of eq. [10], which looks like the conventional phasor diagram of the non-salient pole synchronous machine. However, it is quite different, as you see it in eq. [10].

**Torque of PMSM.** Eq. [11] shows that armature resistance \( R \) is increased by \( (3/2) \omega L_s \sin(2\varphi_0 - 2\varphi_1) \) due to saliency inductance \( L_s \). This means that energy conversion increases by the following amount.

\[ \Delta P = 3\omega \frac{3}{2} L_s \sin(2\varphi_0 - 2\varphi_1) |I_1|^2. \]  

Thus three phase torque increases by
The saliency torque V-curve of PMSM is given by eq. (14).

\[
\Delta T = \frac{9}{4} P L_s \sin (2\varphi_0 - 2\varphi_1) |I_1|^2 \quad [\text{Nm}].
\]

Here \( P \) is the number of poles. Total three-phase torque becomes

\[
T_3 = \frac{P}{2} 3\lambda |I_1| \cos (\varphi_0 - \varphi_1) + \frac{9}{4} P L_s \sin (2\varphi_0 - 2\varphi_1) |I_1|^2 \quad [\text{Nm}].
\]

Here \( \lambda \) is flux from poles. This equation gives torque in terms of armature current \( |I_1| \). Fig. 4 is the torque-current curves for PMSM. Measured and calculated torques are in good agreement. They are determined for DC current. It means all losses are eliminated.

**V-curve of PMSM.** In the field winding excited synchronous machine field winding is varied, and then armature currents also vary. Field current and armature current are drawn in rectangular coordinate. Then the graph gives the V-curve. However, PMSM does not have field current to regulate. So it has no conventional V-curve.

In PMSM running under no load terminal voltages are varied. Then armature currents vary. The voltage and the current give a graph of V-shaped curve, in which one of the variables is different from that of the conventional machine. The conventional V-curve is interesting but does not have much usage in analysis and performance calculation. The new V-curve of PMSM in this paper has much more practical usage in analysis and performance calculation.

SV method gave eq. (10), as the circuit equation of PMSM, which contains saliency inductance \( L_s \). And torque was given by eq. (14).

These are new circuit equation and torque equation, which are missing in the conventional theories.

Under no load running torque is zero and eq. (10) gives

\[
\Delta V_1 = Z_d \Delta I_1.
\]

This equation gives the direct axis impedance \( Z_d \) of PMSM. At the bottom of the V-curve we have

\[
V_1 - E_i = Z_q I_1.
\]

Table I. Saliency inductance and excitation type

<table>
<thead>
<tr>
<th>Excitation type</th>
<th>Saliency inductance</th>
<th>Inductances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface PM type</td>
<td>( L_s = 0 )</td>
<td>( L_s = L_a )</td>
</tr>
<tr>
<td>Embedded PM type</td>
<td>( L_s &lt; 0 )</td>
<td>( L_s &lt; L_a )</td>
</tr>
<tr>
<td>Field winding type</td>
<td>( L_s &gt; 0 )</td>
<td>( L_s &gt; L_a )</td>
</tr>
</tbody>
</table>

Table II. Rating and circuit constant of PMSM

| 2.2 kW, 6 poles, 175 V, 50Hz, 0.62 Ω, \( L_s = 8 \text{ mH} \), \( L_a = 21 \text{ mH} \) |

\[
\Delta T = \frac{9}{4} P L_s \sin (2\varphi_0 - 2\varphi_1) |I_1|^2 \quad [\text{Nm}].
\]

Here \( P \) is number of poles. Total three phase torque becomes

\[
T_3 = \frac{P}{2} 3\lambda |I_1| \cos (\varphi_0 - \varphi_1) + \frac{9}{4} P L_s \sin (2\varphi_0 - 2\varphi_1) |I_1|^2 \quad [\text{Nm}].
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This equation gives the direct axis impedance \( Z_d \) of PMSM. At the bottom of the V-curve we have

\[
V_1 - E_i = Z_q I_1.
\]
Here $Z_q$ is quadrature axis impedance. $V_i$ can be measured at terminals immediately after cutting off current $I_i$ at the bottom.

At the bottom of the V-curve power factor is one. Then minimum input power $P_{\text{min}}$, which is equal to mechanical loss + iron loss. Thus we have

$$P_{\text{min}} = 3E_iI_i.$$  \[18\]

In the V-curve input to the PMSM for $I_i = 7.8$ A is

$$P_{\text{in}} = \sqrt{3} V_i I_i \times \text{p.f.} = \sqrt{3} \times 108 \times 7.8 \times 0.11 = 160.2 \text{ (W)}.$$  \[19\]

DC resistance $R_i = 0.62 \ \Omega$, and copper loss is

$$P_{\text{cu}} = 3 \times 0.62 \times 7.8^2 = 113.2 \text{ (W)}.$$  \[20\]

Excitation iron loss + mechanical loss is given by eq. [18], as follows.

$$P_{\text{min}} = \sqrt{3} \times 70.7 \times 0.16 = 19.56 \text{ (W)}.$$  \[21\]

Iron losses due to $I_i$ is

$$P_{\text{air}} = P_{\text{in}} - P_{\text{cu}} - P_{\text{min}} = 160.5 - 113.2 - 19.56 = 27.74 \text{ (W)}.$$  \[22\]

This is named reaction iron loss and gives rise to additional resistance below

$$r_i = P_{\text{air}} / (3 \times I_i^2) = 0.151 \ (\Omega).$$  \[23\]

Inserting these resistances the circuit eq. [10] becomes

$$V_i = (R_i + R)I_i + jX_iI_i - E_i.$$  \[24\]

And the corresponding equivalent circuit becomes Fig. 8. Efficiency is as follows

$$\eta = \frac{3V_i I_i \times \text{p.f.} - (R + R_i + R_i)I_i^2 - 3E_i^2/r_i}{3V_i I_i \times \text{p.f.}}.$$  \[25\]

For $I_i = 7.8$ A of power factor 1, efficiency is 87.8%.

**Control of PMSM.** High performance control of control motors is torque control. Current equation of eq. [10] and torque eq. [14] are sufficient and adequate for it. There are several ways of torque control. The most appropriate one would be as follows. Shaft torque is given by the driven load. Then eq. [14] gives motor current $I_i$. Then choose phase angle $\alpha - \phi = 0$. Then eq. [14] gives that torque is proportional to $I_i$, and largest for a given $I_i$ in the stable zone.

Eq. [10] determines the terminal voltage $V_i$ for
current $I_p$. Thus terminal voltage $V_1$ and current $I_i$ were determined for torque control of PMSM. They are control input for voltage-fed and current-fed controls of PMSM respectively.

**Conclusion.** SV method derived the new circuit equations of PMSM, which provided the new V-curve. It is very useful in determining circuit constants and losses of PMSM.

**References**


![Equivalent circuit of PMSM including losses.](image)