An Electric Magnetometer for Traveling.

By

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[Read Sept. 18, 1920]

Introduction.

In almost all the magnetic surveys in the world, the horizontal intensity has been observed by Gauss-Lamont's method. This is an absolute one, and gives very exact result in a fixed observatory. But, it is very difficult to get a precision more than \( 10^{-7} \) (\( \gamma = 0.00001 \) c.g.s.) on field work. This can be seen from the results of comparisons of instruments in the previous survey executed by the Hydrographic Office of Japan. Eliminating the local error, the time effect, and the instrumental corrections, the probable error of a single observation was 187.

The current in a coil of known electrical constant being measured by potentiometric method, the horizontal intensity was often measured by the deflection only.

The investigation herein reported was to make a field instrument of those means.

By modern progress of the standardization of electric units, the accuracies of the standard cell and the standard resistance have reached far above those achieved by Gauss-Lamont's method in traveling survey. And the consistency of the e.m.f. of the standard cell during transportation for a long period has been proved, while the standard resistance is evidently portable and invariable. Replacement of the permanent magnet by an electric current has the following merits.

(1) Time of observation is reduced to a few minute from 20 minutes or more.

(2) Observation is simplified.

(3) Amount of deflection can be changed at will by changing the resistance shunted by the standard cell.
(4) Dispensation of the deflection bar which is quite deformable by transportation.

(5) Dispensation of the half second beat chronometer, which had made traveling very troublesome.

The coil used to measure the horizontal intensity may be used to observe the magnetic declination by the method of Prof. Tanakadate\(^1\).

1. Construction of the coil.

Maxwell’s or Helmholtz’s? The coil proposed usually to give a uniform field is Maxwell’s four or three coils or Helmholtz’s coil. It is very difficult to give the precise form to Maxwell’s coil, and Helmholtz’s coil has been adopted, for the latter gives a sufficiently uniform field as it may be seen from the following.

Size of coil determined by the formulae given by Prof. H. Nagaoka. Prof. H. Nagaoka gave the following formulae for the components of the magnetic field at a point near the center of Helmholtz’s coil.

\[
X = \frac{i}{625\sqrt{5a}} \frac{2304}{x} \left( -1 + 4y^2 - 3x^2 \right) \frac{1}{a^2}
\]

\[
Y = \frac{i}{5\sqrt{5a}} \left( 1 - 18 \left( -8y^2 + 24y^2x^2 - 3x^2 \right) \frac{1}{125a^2} \right)
\]

where

\( i \) = the strength of the current,

\( X \) = the transversal component,

\( Y \) = the axial component,

\( x \) and \( y \) = the transversal and axial distances of the considered point from the center of the coil respectively.

The second term in the brackets in the express-
sion for $Y$ is the increment of the field over that at the center.

According to these formulae, the region, in which the increment of the field is less than $0.00010 + 0.3$ or $3 \times 10^{-4}$ is shown in Fig. 1. The region, in which $X/Y$ is less than $24 \times 10^{-6}$ or deviation of direction of the field from the axial line is less than $5''$, is shown in Fig. 2. From these figures, we may say the region in the sphere, whose radius is less than one-tenth of radius of the coil, can be utilised for the measurement of the horizontal intensity with precision of $10\gamma$ and for determination of the magnetic declination with precision of $5''$ by Prof. Tanakadate's method.

The position of the hanging magnet can be easily adjusted in a region of $2\, \text{mm}$, radius, and the length of the hanging magnet is $3\, \text{mm}$. So, the coil having a diameter greater than $7\, \text{cm}$ will give an ample space for the measurement. The actual coil used has the diameter of $13.2\, \text{cm}$.

Construction of the coil. The general feature of the coil is given in Fig. 3. The frame of the coil is of duralumin, which was cast in Tokyo and was found to be non-magnetic, and was made as to fit to a theodolite of Prof. Tanakadate's magnetometer used in the previous survey of Japan. This coil is used with a mirror magnetometer attached to the theodolite. The pivots $P, P'$ are to be out on $Y$'s of the theodolite. The grooves $G, G'$, in which wires are to be wound, and the pivots were turned on a lathe at the same time. The pivots were bored along its axis. By looking through this boring, the position of the magnet is adjusted.

The diameter of the pivots are equal to those of the astronomical telescope of the theodolite. The inner surface of the groove was painted with enamel lack and heated to get good insulation.

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The enamelled copper wires of \#24 B. S. standard were wound in the grooves. The wire is wound in 14 layers, and each layer consists from 13 windings. The ratio $13/14 = 0.92857$ gives a good approach to the ratio $\sqrt{31/36} = 0.92813$ proposed by Maxwell. The two wires were wound side by side with a tension of about two kilograms. The double windings consisting from such two wires are very convenient for testing the insulation. These wires are connected in series for the observation. The terminals of the wires were attached into brass pieces on ebonite blocks fixed on the duralumin frame. The well insulated wires of about 2 meters was connected to this brass piece. About 18 gilded paper tapes, used in a chord of a telephone receiver, were carefully stranded over this insulated wire making a cable with the insulated wire and connected to the other brass block, which was attached to the other terminal of the wire of the coil. Thus the return circuit of the gilded paper cancels magnetic effect of the inner wire at external points. About 3 meters of chord twisted from two insulated wires are connected to this cable to make the connection to battery, galvanometer etc.
By means of a commutator, the current to one of coil is reversed, as the fields due to each coil will add or destroy each other at the center to put the hanging magnet in the middle of the coils by the way described afterwards. Another commutator is added to reverse total field at the center.

The dimensions of the coil are:
- Diameter of the copper wire, bare, 0.32 mm. ca.
- Diameter of the copper wire, enamelled, 0.37 mm. ca.
- Mean radius of the coil, 66.2 mm. ca.
- Mean distance of the coils, 66.2 mm. ca.
- Depth of the groove, 6.62 mm. ca.
- Breadth of the groove, 5.20 mm. ca.
- Constant of the coil, calculated from above data, 24.7 c.g.s. per ampere.
- Resistance of wires in series, 33.5 ohms.

Insulation resistances between wires and between wire and frame were all about $5 \times 10^8$ ohms about one year after winding, during which the coil had been placed in various conditions.

The rated current is 10 milliamperes.

The coil may be displaced in axial direction by turning the screw head $S$ (Fig. 3) and pressing it by the spring $R$. The screw head $S$ and the spring $R$ are attached to the $Y$'s by means of metal frames $ff'$.

General procedure (see Fig. 4). Electric current from dry battery $B$ is sent through the coil, which is placed on the $Y$'s of the theodolite of the magnetometer. The strength of the current is controlled by a rheostat $R$ in such a manner, that the galvanometer $G$ bridged to the ends of the 100 or 50 ohms standard resistance $R$, with a standard cadmium cell $S$ in series, gives no deflection. The electrical constant of the coil is determined by comparison with that of the standard coil as shown in the following article.

The field intensity is to be measured by sine method of deflection due to the current in the coil. The deflection is read on horizontal circle of the theodolite.

A observer reads the deflection at an instant, when an assistant, who is controlling the strength of the current, gives a signal of no deflection of the galvanometer. Then the whole current is reversed and the reading of the deflection is taken again.

Adjustment of the coil. A mirror magnetometer is set and adjusted in usual manner. The pivots of the coil are put on the $Y$'s. By looking through the holes in the pivots, and adjusting the centering screws and the top screw of the hanging magnet, the center of the magnet is brought on the axial line of the pivots. Then, the axis is brought nearly perpendicular to the magnetic meridian. Both coils are connected in series and the current is sent in such a way that the field at the center is zero. The coil is displaced in axial direction by the adjusting screw head ($S$ in Fig. 3) to such a position that the mirror magnet does not deflect by making and breaking the circuit.

Uniformity of the field. This is confirmed by keeping the current constant the deflection did not change when the coil was displaced axially by two turns of the screw head $S$ and the hanging magnet was displaced vertically about 2 mm. In this case, the change of deflection of $1'$ corresponded to the change of $6\gamma$. For the adjustment of the position of the coil, one eighth turn of the screw head is enough to fix the position of the coil, and a half millimeter of excentricity of the magnet from the axis can easily be detected. From these, it can be seen that the field used for observation is sufficiently uniform.

Observation and the calculation. The deflections for direct and reverse currents are read. Correcting for temperature coefficients of
electrical constant of the coil, of the standard resistance and of e.m.f. of the standard cell, the horizontal intensity of the earth field is computed by following way.

Let us put

\[ K \] for electrical constant of the coil at 20°C,

\[ k \] for temperature coefficient of the same,

\[ t_0 \] for temperature of coil,

\[ E_{t_0} \] for e.m.f. of the standard coil at \( t_0 \),

\[ t_r \] for temperature of the standard cell,

\[ R_{t_r} \] for standard resistance at \( t_r^0 \) C.

\[ \varphi \] for angle of deflection.

Then it can be seen very easily,

\[ H = \frac{K E_{t_0}}{\sin \varphi R_{t_r}(1 + kt)} \quad \text{or} \quad H = \frac{K E_{20} - E_{t_0}}{\sin \varphi (1 + kt) R_{20} - R_{t_r}}. \]

Putting \( H_0 = K E_{20}/R_{20} \), we have

\[ H = \frac{H_0}{\sin \varphi} \frac{E_{t_0}}{E_{20}} \frac{R_{t_r}}{R_{20}} \frac{1}{1 + kt}. \]

The e.m.f. of the cell and the resistance at 20°C are 1.0183 volt and 101.01 ohms respectively. \( K \) being 24.708 at 20°C. (see the next article). log. \( H_0 \) is 1.37618. Log. \( 1/(1 + kt) \) and log. \( E_{t_0}/E_{20} + \log R_{20}/R_{t_r} \) are tabulated from the results of standardization, temperature being argument.

Accessory Instruments. The theodolits used is that of No. 3 of Tanakadate's magnetometer.

The current is supplied very conveniently from dry battery. But, dry battery can not give a sufficient current to give desired deflection by a single loop of wire of a coil, without rapid deterioration of the battery and rapid variation of the current, which is very inconvenient for observation. According to my experience, a dry battery shows decay of current of \( 0.477 \times 10^{-6} \) amp. minute discharging 10 milliamperes. To give a sufficient deflection with about 10 m.a., we used the coil of a hundred or more turns for traveling purpose.

The theostat for current regulation (R in Fig. 3) was made in the same form as that of Leeds and Northrup's potentiometer.
The galvanometer shall be sensible as far as $0.5 \times 10^{-5}$ volt and should not disturb external magnetic field, and also not be disturbed by the field due to the coil of the magnetometer. Broca galvanometer of high resistance coils was found to be suitable. The needle of the galvanometer was suspended with a phosphor bronze wire, which gave sufficient torsion to change azimuth of the needle at will by its torsion head. To observe its deflection, a small autocollimating telescope was attached to the galvanometer.

Three standard cadmium cells and standard resistance of about 100 ohms were supplied from Tokio Electric Co., and standardized at electrical laboratory of the Department of Communication. The cells and resistance are immersed in a tank of $10 \times 10 \times 12$ cm, filled with transformer oil stirred by a stirrer. So the temperatures of the cells and the resistance are the same always. By pressing a head of rods, one of the cells can be put in circuit by a mercury contact contained in small glass vessel. By pressing heads of other rods, the resistance of 100 ohms or the galvanometer can be put in circuit by the same way (compare Fig. 4).

Results of observations. A few examples of observation made in Hydrographic Office are given here.

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On Aug. 2, the all wires reconnected between fourth and fifth observations. From these results, we see that this instrument gives sufficiently consistent results for field work.

To compare this method with that of Gauss-Lamont, a few simultaneous observations with this instrument and No. 1 magnetometer used in the previous survey were made. The former seems to give about 20° higher than the latter. This point is now under studying.

Observation of declination. To make the observation of declination, another mirror is attached below the usual mirror. The lower one has its plane perpendicular to that of the higher. After the adjustment of the position of the coil, the coil is turned through 90 degrees, and the observing telescope is readjusted to see image of the scale on the lower
mirror. The declination is observed by Prof. Tanakadate's method sending the current to oppose the earth field. But, it may be noted that angle read on the circle gives the magnetic prime vertical instead of the meridian in his case.

3. Standardization or the determination of the electrical constant of the coil.

It is almost impossible to measure exact dimensions of a coil with many windings. So, we made a standard coil of Helmholtz's type of a single turn of wire, the dimension of which is easily measured exactly, and the coil for traveling—shortly say "traveling coil"—was compared with the standard.

Construction of the standard coil. Figs. 5 and 6 show its general construction. A hollow cylinder of white marble of 20 cm. in diameter is turned and ground, and also two thin grooves \(G, G'\) were ground on its cylindrical surface. The distance between the grooves are 10 cm. The four windows \(W_1, W_2, W_3,\) and \(W_4\) were bored. Two metallic covers \(C, C'\) with the pivots \(P, P'\) were turned. Edges of inner flanges of the covers fit to inner surface of the cylinder, thus making coincidence of axes of the pivots and cylinder. These covers are fixed to the cylinder by means of three bolts \(b's.\) The pivots are hollow and to embrace the pivots of the traveling coil.

A circular ring \(R,\) inner diameter of which is slightly larger than that of the pivot of the traveling coil, is secured in each pivot. The traveling coil is to be put in the standard, the pivots of the former being supported in the holes of the rings \(R.\) The traveling coil can be displaced in the standard axially by means of a screw head (\(S\) in Fig. 5) Each coil of the standard is of bare copper wire, which was selected for its uniform diameter and wound on each groove \(G, G'.\) The ends
ELECTRIC MAGNETOMETER.

of wire bent perpendicular to plane of the coil after passing through small holes in two ivory blocks kept by four brass screws. A thin mica plate was inserted between ends of the wires. A pull on the ends of the wire keep it in the groove. The ends of wire were soldered to each end of the cable, which are made of a well insulated wire and gilded paper chord in the same manner to that of the traveling coil. The spare cable quite similarly constructed but short circuited at the end, is put along this cable to test external magnetic effect of the cable.

Electrical constant of the standard coil. By kind permission of Mr. S. Kikkawa, the superintendent of Central Bureau of Weights and Measures, the dimensions of the coil were measured there. I wish to express my sincere thanks to him and the members there.

The distance between wires 10.0034±0.00017 cm. at 15.7 C.

The mean diameters of coil:  
Right: 20.0519±0.0059 
Left: 19.9961±0.0057 at 16.1 C.

The diameters of the two coils are slightly different. So the point, where the fields due to each coil cancels each other is not exactly in the middle of axis. The distances of this point computed from the diameters of the coils are 5.0013 cm. from the right coil and 5.0021 cm. from the left coil.

By these data, we have

The electrical constant of the standard coil, or the field intensity of the rated point for one ampere

End correction. The ends of wires of the standard are bent parallel to the axis of the coil, and the triangular space are formed there. These triangular areas were computed from X-ray photograph of the ends of the wire. The horizontal component due to these areas at the center
of coil is 0.000036 c.g.s. per ampere. Correcting for this, the constant of the coil is

\[ 0.0898771 \text{ c.g.s. per ampere at } 15^\circ 9 \text{ C.} \]

Effect of temperature  
Temperature coefficient of the standard was calculated from its expansion coefficient. As marble is known to be aeolotropic concerning to thermal expansion, external diameters of the coil along quadrantal directions and the distance of wires along different axial lines were measured at different temperatures. The means of temperature coefficients thus obtained were adopted.

- Temperature coefficient of diameter: \(10 \times 10^{-6}\)
- Temperature coefficient of distance: \(12 \times 10^{-6}\)

From the above, the temperature coefficient of the constant of the standard is \(11 \times 10^{-6}\). Then we have

The constant of the standard coil = \(0.089877 [1 - 11 \times 10^{-6}(t-15.9)]\).

Method of comparison. To obtain electrical constant of the traveling coil by comparison with that of the standard, the current ratio of two coils were measured when magnetic fields due to two coils cancel each other. In this case, the magnetic axes of coils and their middle points should coincide separately. The inverse ratio of the currents gives the ratio of electrical constants.

Mounting of the coil (see Figs. 5 and 6). The standard coil embracing the traveling coil is mounted on Y's of a frame of duralumin, specially made for this purpose and attached to the top plate of horizontal circle of the magnetometer of Kew pattern. Thus the standard coil may be turned and its azimuth may be read on the horizontal circle. A mirror magnetometer used in Prof. Tanakadate's magnetometer with two mirrors is placed on the base plate B of the frame. This may be levelled by means of three screws T's and displaced transversally to the axis of the coil by a screw head H. To get rid of magnetic disturbances due to tram cars, a duralumin damper of rectangular form of 1 x 8 mm. is attached below the mirror by means of a thin duralumin wire. This damper is immersed in transformer oil reserved in a vessel V. This devices gave strong damping but did not disturb setting of the magnet. A scale and a telescope were used to observe the magnet. The distance of the scale from the magnet was three meters.

Electric connection (see Fig. 7). A current from accumulator A, of 4 volts were divided into two after passing a mercury cup commutator C. One circuit consisted of the standard coil MCi and
MC, and one ohm standard resistance immersed in oil. The other circuit consisted of the traveling coil TC1 and TC2, a one hundred ohms standard resistance and current regulators R1 and R2. Current

in one of coil in each circuit may be reversed by means of commutator C2 or C3.

These standard resistances were compared previously. Their ratio is

$$100 \times \frac{1 + 16.152 \times 10^{-6} + 1.559 \times 10^{-5}(t - 20^\circ)}{1 + 3.729 \times 10^{-5} + 2.12 \times 10^{-6}(t' - 20^\circ)},$$

where t and t' are temperatures of resistances of 100 and 1 ohms respectively. To the ends of the standard resistances, potentiometers made by the Leeds and Northrup Co. were connected. These potentiometers were calibrated separately and compared with each other. One and same standard cell was used in the observation for both potentiometers. The all electrical arrangements of resistance, potentiometers etc., were placed about three meters away from the coils.

The ratio of currents in the two coils may be computed easily from readings of the potentiometers. It may be remarked that the comparison of potential differences was easier and simpler in our case than that of resistances. One reason is that the current in the standard coil is about one ampere, which changes the resistance of the leading wires by its heating effect.

Adjustment and observations. The magnet of the mirror magnetom-
The electric magnetometer was brought on the axial line of the coils by means of the screw head $H$ and the top nut (see Fig. 5). The axis of the coil was brought in magnetic prime vertical approximately. Quite similarly to measurement of horizontal intensity with traveling coil, the middle point of the standard coil was brought to coincidence with the hanging magnet by turning the screw head $S'$ in Fig. 6. Next, the traveling coil was displaced to bring its middle point into the magnet turning the head $S$. Then the horizontal circle was turned through about 90 degrees and the magnetic meridian was read on the horizontal circle by Prof. Tanakadate's method sending a moderate current into the traveling coil. The horizontal circle was turned reversely through just 90 degrees from the magnetic meridian. Then axes of the coils were perpendicular to the magnetic meridian. The centering of the hanging magnet was adjusted again.

An observer looked in the telescope adjusting the current in the traveling coil with the regulator $R$, in Fig. 7, while two assistants were measuring potentials of the terminals of the standard resistances. After about ten or fifteen minutes the currents became steady. The observer controlled the current in the traveling coil to compensate its magnetic effect with that of the standard so to keep the reading of the scale at its zero point. The change of zero point due to disturbances is determined and corrected by readings of the scale just after the breaking of the all currents.

The all currents were reversed by each observation by means of a pole changer $C$ in Fig. 7 to avoid any polarity.

The result of observation was as follow.

May 14, 1920. Temperature 10 ohm resistance 19°8 C.

The ratio of the terminals potential differences

$$\frac{e_r}{e_s} = \frac{0.363778 \pm 0.000013}{0.363765 \pm 0.000009}$$

by 1st series.

$$\frac{e_r}{e_s} = \frac{0.363772 \pm 0.000008}{0.363772 \pm 0.000008}$$

by both series.

Computation of the constant of the traveling coil. Let us put

$r_s$, the standard resistance connected to the standard coil,

$r$, the standard resistance connected to the traveling coil,

$e_s$, potential difference at the terminals of $r_s$,

$e_r$, potential difference at the terminals of $r$,

$i_s$, the current strength through the standard coil,

$i$, the current strength through the traveling coil,


\( k_s \), the constant of the standard coil,

\( k \), the constant of the traveling coil.

Then we have

\[
\frac{k}{es} = \frac{e_s}{c} \frac{r}{r_s}
\]

Putting the values in the foregoing paragraphs for \( k, e_s/e \) and \( r/r_s \), we have

The electrical constant of the traveling coil \( 24.709 \) c.g.s. per ampere at 18°.5C.

The temperature coefficient of duralumin plate, which has same composition and same treatment to the frame of the coil, was found to be 0·0000195. The electrical constant of the traveling coil is

\[
24.708 [1 - 0·000020 (t - 20)], \text{ c.g.s. per ampere.}
\]

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**The Copper Standard Cell.**

**By**

Jūichi Obata.

[Read April 2, 1920.]

Studies of the standard cell are usually restricted to the Clark and Weston standard cells, and although these two cells have been made the subject of a considerable amount of investigations, little attention has been paid to other chemical combinations which consist of more common materials but still possess the same characteristics as the standard cell.

More than twenty years ago McIntosh\(^1\) studied the copper standard cell, i.e. the following chemical combination:

\[
\text{Cu or Cu amalg.} | \text{CuSO}_4 \cdot 5\text{H}_2\text{O} \cdot \text{Hg}_2\text{SO}_4 | \text{Hg}
\]

However, his process of preparing the materials for use in the cell as well as his method of measuring the electromotive-force cannot be said to be satisfactory, considering the present state of accuracy required in this kind of work. Quite recently Öhölm\(^2\) made a thorough study on the same combination, especially from the thermochemical standpoint.

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\(^{1}\) McIntosh, Journ. Physical Chemistry, 2 (1898), p. 185.