The Effect of Magnetic Field on the Absorption of $\lambda$- and $\gamma$- Rays.

By

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1. Considering the discrepancies between the experimental results and the J. J. Thomson's theory of $\lambda$- and $\gamma$- ray absorption, Compton (1) recently proposed the view that electron must be thin flexible ring-shaped and have radius comparable with the wave-length of the hardest $\gamma$- ray, i.e. $(1.85 \pm 0.5) \times 10^{-10}$ cm, which reminds us Parson's magneton and Kelvin's vortex-atom.

But shortly after, Schott (2) published another theory from the standpoint of classical dynamics which explains the phenomena fairly satisfactorily. According to this theory, high frequency radiation consists of undamped unpolarized wave, and atom, of sensibly point charge electron rings.

Our knowledge relating to atomic structure is still very poor, yet we may await probably more to the atomic model of electronic rings.

The author thought that it might be possible to decide between the arrangements of electronic orbits of para-, dia- and ferro-magnetic atoms in magnetic field by the observations of scattering which always occurs in absorption, and this theoretical investigation was undertaken.

2. It was shown by Schott, scattering coefficient per unit solid angle $s$ in the direction which makes an angle $\theta$ with the incident beam by single electronic orbit with $n$ electrons neglecting multiple scatterings is

$$ s = \frac{e^2 n}{2 \epsilon_0 m^2} \left( 1 + \cos ^2 \theta \right) \sum_{i=0}^{n} \frac{\pi i}{\pi} J_0 \left( \frac{6 \pi r}{\ell} \sin \frac{\pi i}{n} \sin \gamma \sin \frac{1}{2} \theta \right), \quad (1) $$

where $\gamma$ is the angle between the axis of the electronic ring and the

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external bisector of the angle $\theta$; $e$, elementary charge expressed in electrostatic unit; $m$, mass of electron; $c$, velocity of light; and $\frac{v}{L}$, ratio of the radius of the ring to the wave-length.

3. Mass scattering coefficient $\sigma$ is therefore

$$\sigma = \frac{e^4}{2c^4 m^2} \left( 1 + \cos^2 \theta \right) \cdot \sum_{\ell=0}^{\infty} \sum_{i=0}^{\infty} J_{2\ell} \left( \frac{8\pi e}{L} \sin \frac{\pi i}{n} \sin \frac{\pi \sin \gamma}{2} \theta \right) \frac{1}{A \cdot 1 \times 10^{-3}}.$$  \hspace{1cm} (2)

where $\Sigma$ denotes the summation extending to the whole orbits of an atom and $A$ atomic weight.

4. Now let us consider the effect when strong magnetic field is applied parallel to the incident beam at low temperature and the scattering is observed along this direction—longitudinal effect.

In this case $\gamma = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ for magnetic atoms.

For the forward scattering $\theta = 0$,

$$\sigma_{\text{fs}} = \frac{e^4}{c^4 m^2} \cdot \frac{\Sigma n^2}{A \times 1 \times 10^{-3}}.$$  \hspace{1cm} (3)

For the backward scattering $\theta = \pi$,

$$\sigma_{\text{bs}} = \frac{e^4}{c^4 m^2} \cdot \frac{\Sigma n^2}{A \times 1 \times 10^{-3}} \cdot \frac{1}{A \times 1 \times 10^{-3}}.$$  \hspace{1cm} (4)

5. Next let us imagine the other effect when intense magnetic field is applied perpendicular to the incident beam and the scattering is measured along the direction of incidence—transverse effect.

In this case $\gamma = 0$ for magnetic atoms.

If $\theta = 0$,

$$\sigma_{\text{tr}} = \frac{e^4}{c^4 m^2} \cdot \frac{\Sigma n^2}{A \times 1 \times 10^{-3}}.$$  \hspace{1cm} (5)

And if $\theta = \pi$,

$$\sigma_{\text{br}} = \frac{e^4}{c^4 m^2} \cdot \frac{\Sigma n^2}{A \times 1 \times 10^{-3}}.$$  \hspace{1cm} (6)

No dissymmetry.

6. For ring electron, we may put $n = \infty$ and substitute $\frac{e}{n}$ for $e$. 

\( \frac{e^i}{n} \text{ for } m, \phi \text{ for } \frac{\pi i}{n}, \frac{d\phi}{\pi} \text{ for } \frac{1}{n}, \frac{2 \pi}{n} \text{ for } \int_{n}^{\infty} \text{ in (2)}. \) Denote atomic number by \( N. \)

\[
\sigma' = \frac{e^i N}{2 \pi e^m} (1 + \cos^2 \theta) \cdot \int_{0}^{\pi} J_0 \left( \frac{4 \pi r}{\lambda} \sin \phi \sin \gamma \sin \frac{1}{2} \theta \right) d\phi
\]

\[
\quad \times \frac{1}{A \times 1 \times 1 \times 10^{-33}}. \tag{7}
\]

By the well-known integral due to Neumann, we obtain

\[
\int_{0}^{\pi} J_0 \left( \frac{8 \pi r}{\lambda} \sin \phi \sin \gamma \sin \frac{1}{2} \theta \right) d\phi = \pi J_5 \left( \frac{4 \pi r}{\lambda} \sin \gamma \sin \frac{1}{2} \theta \right) \tag{8}
\]

\[
\sigma' = \frac{e^i N}{2 \pi e^m} (1 + \cos^2 \theta) \cdot \int_{0}^{\pi} J_0 \left( \frac{4 \pi r}{\lambda} \sin \gamma \sin \frac{1}{2} \theta \right) d\phi
\]

\[
\quad \times \frac{1}{A \times 1 \times 1 \times 10^{-33}}. \tag{9}
\]

7. \[
\sigma_{\lambda \rho} = \frac{e^i}{e^m c} \cdot \frac{N}{A \times 1 \times 1 \times 10^{-33}} \tag{10}
\]

\[
\sigma_{\lambda \lambda} = \frac{e^i}{e^m c^2} \cdot \frac{N}{A \times 1 \times 1 \times 10^{-23}} \tag{11}
\]

8. \[
\sigma_{\lambda \rho} = \frac{e^i}{e^m c} \cdot \frac{N}{A \times 1 \times 1 \times 10^{-33}} \tag{12}
\]

\[
\sigma_{\lambda \lambda} = \frac{e^i}{e^m c^2} \cdot \frac{N}{A \times 1 \times 1 \times 10^{-23}} \tag{13}
\]

No asymmetry as before.

9. We shall deal with transverse effect more fully taking into consideration the probability that the axis of the rings lies between \( \gamma \) and \( \gamma + d\gamma \).

The magnetic moment of atom is \( \frac{e^i m r^2 v_0}{2}, \) \( v_0 \) being the angular velocity of rotation of electrons in an orbit.

\(^{(1)}\) K. Honda, Magnetism and Matter, p. 298.
\[ \sigma = \frac{2 \pi \kappa}{N^1} \int_0^1 \frac{e^t}{2 e^{m^2}} \left( 1 + \cos^2 \theta \right) \cdot \frac{1}{A \times 1.64 \times 10^{-\kappa}} \times \sum_{i=0}^{n-1} \int_0^1 \left( \frac{8 \pi r}{\lambda} \sin \frac{\pi i}{n} \sin \frac{1}{2} \theta \sin \gamma \right) \times \frac{\varepsilon \sin^2 r}{2} H \cos \gamma \frac{1}{r^2 + \varepsilon \sin \gamma} d\gamma, \]

\[ N^1 = 2 \pi \kappa \int_0^1 \frac{e^t}{2} H \cos \gamma \frac{1}{r^2 + \gamma} \sin \gamma d\gamma, \quad (14) \]

\[ H \text{ being field strength and } \kappa, \varepsilon \text{ certain constants.} \]

\[ J_0 (\kappa_1 \sin \gamma) = J_0^2 \left( \frac{\kappa_1}{2} \right) + 2 \sum_{s=1}^{\infty} J_0^2 \left( \frac{\kappa_1}{2} \right) \cos 2s \gamma, \]

\[ \int_0^\infty \varepsilon \kappa_2 \cos \gamma \sin 2s \gamma d\gamma = 8s \sum_{s=1}^{\infty} I_{2n+1}(\kappa_1) \frac{1}{4s^2 - (2n+1)^2} ; \quad (16) \]

\[ \int_0^\infty J_0 (\kappa_2 \sin \gamma) \varepsilon \kappa_3 \cos \gamma \sin \gamma d\gamma = J_0 \left( \frac{\kappa_2}{2} \right) \]

\[ \times \int_0^\infty \varepsilon \kappa_3 \cos \gamma \sin \gamma d\gamma + 2 \sum_{s=1}^{\infty} J_0^2 \left( \frac{\kappa_2}{2} \right) \int_0^\infty \varepsilon \kappa_3 \cos \gamma \sin \gamma \cos 2s \gamma d\gamma \]

\[ = J_0 \left( \frac{\kappa_2}{2} \right) \frac{2}{\kappa_2} \sin \kappa_2 + 2 \sum_{s=1}^{\infty} J_0^2 \left( \frac{\kappa_2}{2} \right) \]

\[ \times \left\{ \frac{2}{\kappa_2} \sin \kappa_2 - \frac{16s^2}{\kappa_2} \sum_{s=1}^{\infty} \frac{I_{2n+1}(\kappa_2)}{4s^2 - (2n+1)^2} \right\} . \quad (17) \]

\[ \sigma = \kappa_1 \left[ \sum_{i=0}^{n-1} \sum_{s=1}^{\infty} J_0^2 \left( \frac{\kappa_1}{2} \right) + 2 \sum_{s=1}^{\infty} J_0^2 \left( \frac{\kappa_2}{2} \right) \right] \]

\[ \times \left\{ 1 - \frac{8s^2}{8s^2 \sin \kappa_1} \sum_{s=1}^{\infty} \frac{I_{2n+1}(\kappa_1)}{4s^2 - (2n+1)^2} \right\} \quad (18) \]

where \( \kappa_1, \kappa_2 \) and \( \kappa_3 \) stand for \( \frac{e^t}{2 e^{m^2}} \left( 1 + \cos^2 \theta \right) \cdot \frac{1}{A \times 1.64 \times 10^{-\kappa}}, \]

\[ \frac{8 \pi r}{\lambda} \sin \frac{\pi i}{n} \sin \frac{1}{2} \theta \text{ and } \frac{e^t}{2} H \cos \gamma \frac{1}{r^2 + \varepsilon \sin \gamma} \text{ respectively.} \]

If \( \theta = 0, \]

\[ \kappa_1 = \frac{e^t}{2 e^{m^2}} \cdot \frac{1}{A \times 1.64 \times 10^{-\kappa}}, \kappa_2 = 0. \quad (19) \]
If \( \theta = \pi \),

\[
\kappa_1 = \frac{e^4}{e^4 m^4} \cdot \frac{1}{A \times 1.61 \times 10^{-21}}, \quad \kappa_2 = \frac{8 \pi^2}{\lambda} \sin \frac{\pi q}{n}.
\]  

(20)

10. If we put \( \kappa_1 = 0 \) in the above, the axes are distributed uniformly in all directions, we get the case of diamagnetic substances according to Langevin's theory\(^{1}\).

The expressions for \( \sigma \) and \( \sigma' \) are oscillatory with respect to \( \lambda \), except \( \sigma_1 \) and \( \sigma'_1 \).

\[
\sigma = \frac{e^4}{e^4 m^4} \cdot \frac{N}{A \times 1.61 \times 10^{-21}}, \quad \sigma' = \frac{e^4}{e^4 m^4} \cdot \frac{N}{A \times 1.61 \times 10^{-21}}.
\]  

(21)

11. The above-mentioned effects are of course very slight, but we are capable of studying the manner of arrangement of atoms above and below the critical temperature, distinguishing between miscellaneous theories of magnetism and perhaps obtaining the idea of intermolecular field.

Fluorescent absorption is now under discussion along the same line of thought by the author.

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**On the Theory of Mirage.**

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Tait, in his theory of mirage\(^{2}\), supposing the refractive index of air to be a given function of height, determined the path of the ray. But in the actual cases, what we know is the apparent position of the object and from which we must deduce the refractive index of air; from such studies, we shall have developed the theory of mirage.

\(^{1}\) Langevin, Ann. d. Sci. nat., phys. [5], 17, 51, p. 78

\(^{2}\) Tait, Proc. R. Soc. 120, p. 227