Dissociation of Molecules in the Carbon Stars.

By Yoshio Fujita.

(Read December 18, 1937.)

Introduction.

In our previous paper\(^{(1)}\) we have discussed the molecular abundance in the hydrogen stars from the point of view of dissociative equilibrium. Assuming the abundance of each element in the atmosphere, we have calculated the number of molecules as a function of the electron pressure, the surface gravity and the temperature at the depth \(\tau = \frac{2}{3}\) of the stellar atmosphere and applied it to the problem of the intensity variation of molecular bands in spectra of the hydrogen stars. In this calculation we have used the formulae for the electron pressure and the surface gravity deduced empirically by H. N. Russell.

As we mentioned in the previous paper, we consider a similar problem in the carbon stars, taking the physical character of these stars into account. Here by a carbon star we mean a star in the atmosphere of which the carbon atoms are more abundant than in an ordinary star, the abundance of other elements being the same. In the present paper it is intended at first to give theoretically the electron pressure and the surface gravity of the carbon stars as a function of luminosity, mass and temperature. Then by applying these formulae to our calculations, we attempt to explain the scattering of observational points in Shane's diagram\(^{(2)}\) in which the band intensities are plotted against the spectral types.

The materials here treated are diatomic molecules which are remarkable in the carbon stars.

I. Electron Pressure and Surface Gravity.

We start with the equation of equilibrium for the gas pressure in the stellar atmospheres. We shall obtain an equation which expresses the electron pressure as a function of the temperature and the surface gravity.

Now the surface gravity of a star is given as a function of its mass and radius, and its luminosity as a function of its energy flow.

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and radius. On the other hand the energy flow can be expressed by the effective temperature of radiation. Hence if we eliminate the radius of the star in these equations, we obtain the surface gravity as a function of mass, luminosity and temperature. Inserting this in the equation of equilibrium, we shall also obtain the electron pressure as a function of mass, luminosity and temperature. Thus our aim will be attained.

Neglecting the radiation pressure, we have as the equation of equilibrium for the gas pressure \( p \) at a depth \( h \) below the surface

\[
dp = g \rho \, dh.
\]

Multiplying by the mass absorption coefficient \( \kappa \), and by denoting

\[
d\tau = \kappa \rho \, dh
\]

We shall adopt Chandrasekhar’s formula\(^{(1)}\) \( \kappa = \alpha \bar{x} P / T^n \), where \( P \) is the electron pressure, \( \bar{x} \) the fraction of the ionized atoms, \( \alpha \) a constant, and \( P = \frac{\bar{x}}{1 + \bar{x}} p \).

If we neglect the variation of \( \bar{x} \) with \( \tau \),

\[
dP = \frac{\bar{x}}{1 + \bar{x}} \, dp.
\]

Substituting for \( \kappa \), (1) gives

\[
(1 + \bar{x}) \, \alpha P \, dP = g T^n \left(1 + \frac{3}{2} \tau \right) d\tau,
\]

where

\[
T^i = T^n \left(1 + \frac{3}{2} \tau \right).
\]

Integrating this we get

\[
\frac{1}{2} \alpha (1 + \bar{x}) P \, \int_0^\tau \left(1 + \frac{3}{2} \tau \right)^{\frac{n}{2}} \, d\tau.
\]

Suppose \( \bar{x} \) negligibly small compared with 1. This is possible for the temperature is low in the stars of types R and N. Then

\[
\frac{P^i}{\alpha} = \frac{2 g}{57} T^n \int_0^\tau \left(1 + \frac{3}{2} \tau \right)^{\frac{n}{2}} \, d\tau
\]

\[
= \frac{32}{57} g T^n \left[ \left(1 + \frac{3}{2} \tau \right)^{\frac{n}{2}} - 1 \right] = \frac{32}{57} g T^n \left[ \left( \frac{T}{T_0} \right)^{\frac{n}{2}} - 1 \right].
\]

\(^{(1)}\) Chandrasekhar: M.N., \textit{92} (1931), 150.

\(^{(2)}\) \( T_n \) means the effective temperature at the boundary.
The surface gravity and the luminosity of a star are given by

$$g = \frac{GM}{r^2},$$

(5)

and

$$H = \frac{L}{4\pi r^2},$$

(6)

where $M$ is its mass, $r$ its radius, $H$ the energy flow.

The energy flow is related with the effective temperature of radiation by

$$a c T^4 = H(2 + 3\tau),$$

(7)

where $a$, coefficient of Stefan's law and $c$, velocity of light.

At $\tau = \frac{2}{3}$, we have

$$acT^4 = 4H.$$  

(8)

Eliminating $r$ and $H$ in (5), (6) and (8), we have

$$g = \pi a \frac{cGM}{L} T^4.$$  

(9)

Using (9), (4) can be written

$$P = \sqrt{\frac{32}{57}} \pi \sqrt{\frac{g}{\alpha} \frac{T^9}{T_1^4} \sqrt{1 - \left(\frac{T}{T_1}\right)^9}}.$$  

(10)

Numerical values of the constants in the equations (9) and (10) are given below

$$a = 7.64 \times 10^{-15}, \quad \alpha = 5.30 \times 10^{-21}.$$  

Then (9) and (10) take the following forms

$$g = 4.79 \times 10^{-11} \frac{M}{L}, \quad P = 1.285 \times 10^{-31} \sqrt{\frac{M}{L}} \sqrt{1 - \left(\frac{1}{2}\right)^9} T^4.$$  

Writing in logarithmic forms, we have

$$\log g = 11.68 + \log \frac{M}{L} + 4 \log T.$$  

(11)

$$\log P = 16.67 + \frac{1}{2} \log \frac{M}{L} + \frac{19}{4} \log T.$$  

(12)

(1) $T$ in this expression is equal to $T_{eff}$, the effective temperature of the star.
II. Luminosity.

It is necessary to know the numerical values of \( L \) and \( M \). If its absolute magnitude and effective temperature are given, the luminosity of a star is easily obtained.

The carbon stars are almost of R-N giant types of high, low and intermediate luminosity. No dwarf carbon stars have been observed. We consider, here, the upper and the lower limit of the luminosity of these stars, and call the giant of highest luminosity the giant "A" and that of lowest luminosity the giant "B"\(^{(2)}\). The numerical values of these extreme cases are given below from the table of Miss Payne's "The Stars of High Luminosity"\(^{(2)}\).

**TABLE 1.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;A&quot;</td>
<td>-35</td>
<td>2000°</td>
<td>( \oplus \times 20 )</td>
</tr>
<tr>
<td>&quot;B&quot;</td>
<td>+17</td>
<td>4000°</td>
<td>( \oplus \times 2 )</td>
</tr>
</tbody>
</table>

From these data, the numerical values of \( L \) are obtained as is shown in Eddington's "Internal Constitution of the Stars"\(^{(3)}\).

**TABLE II.**

<table>
<thead>
<tr>
<th>Giant</th>
<th>( L )</th>
<th>( M )</th>
<th>( \log \frac{M}{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;A&quot;</td>
<td>( 5.50 \times 10^8 )</td>
<td>( 5.97 \times 10^4 )</td>
<td>5.86</td>
</tr>
<tr>
<td>&quot;B&quot;</td>
<td>( 1.51 \times 10^5 )</td>
<td>( 3.97 \times 10^3 )</td>
<td>2.93</td>
</tr>
</tbody>
</table>

\[
\log \frac{M}{L} = 5.86 \text{ (giant "A")}, \quad \log \frac{M}{L} = 2.52 \text{ (giant "B")} \tag{13}
\]

III. Numerical Values of \( \log g \) and \( \log P \).

Inserting the numerical values of \( \log \frac{M}{L} \) (13) in (11) and (12), we have

\[
\begin{align*}
\log g &= 15.54 + 4 \log T \text{ (giant "A")} \\
\log g &= 12.20 + 4 \log T \text{ (giant "B")}
\end{align*} \tag{14}
\]

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(1) Our giant "A" is called by Payne a supergiant.

(2) Payne: The Stars of High Luminosity (1930), 188.

(3) Eddington: Internal Constitution of the Stars (1926), 139.
Calculated values of \( \log g \) and \( \log P \) in the temperature range 1500° and 4500° are given in the next Table III and are plotted in Fig. 1. The important role of these formulae in the calculation of the molecular abundance of the carbon stars will be explained in a later paragraph.

### TABLE III.

<table>
<thead>
<tr>
<th>( T )</th>
<th>( \log g ) (g. &quot;A&quot;)</th>
<th>( \log g ) (g. &quot;B&quot;)</th>
<th>( \log P ) (g. &quot;A&quot;)</th>
<th>( \log P ) (g. &quot;B&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500°</td>
<td>3.22</td>
<td>0.88</td>
<td>3.65</td>
<td>2.98</td>
</tr>
<tr>
<td>1800</td>
<td>3.54</td>
<td>1.20</td>
<td>3.93</td>
<td>3.36</td>
</tr>
<tr>
<td>2000</td>
<td>3.74</td>
<td>1.40</td>
<td>3.27</td>
<td>1.60</td>
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<tr>
<td>2300</td>
<td>3.98</td>
<td>1.64</td>
<td>3.56</td>
<td>1.89</td>
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<tr>
<td>2500</td>
<td>4.10</td>
<td>1.76</td>
<td>3.70</td>
<td>0.63</td>
</tr>
<tr>
<td>2800</td>
<td>4.30</td>
<td>1.96</td>
<td>2.94</td>
<td>0.27</td>
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<td>3000</td>
<td>4.43</td>
<td>2.08</td>
<td>1.88</td>
<td>0.41</td>
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<td>3300</td>
<td>4.58</td>
<td>2.24</td>
<td>1.37</td>
<td>0.60</td>
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<td>2.36</td>
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<td>3800</td>
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<td>0.88</td>
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<td>1.70</td>
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<td>4300</td>
<td>5.06</td>
<td>2.72</td>
<td>1.84</td>
<td>1.17</td>
</tr>
<tr>
<td>4500°</td>
<td>5.14</td>
<td>2.80</td>
<td>1.93</td>
<td>1.26</td>
</tr>
</tbody>
</table>

### IV. Survey of the Problem.

Using the notation of our previous paper, we consider the case of an element \( A \) combining with other more abundant elements \( B_1, B_2, B_3, \ldots \). Let \( y_A \) denote the total number of \( A \) atoms in a unit volume and \( k \) Boltzmann's constant. Then we have

\[
\begin{align*}
\mu_{bi} &= \frac{P}{kT}, \\
\mu_A &= \frac{P}{kT}
\end{align*}
\]

where \( p \) is the total pressure.

The numbers \( n_{ABi} \) and \( n_A \) will now satisfy the equations

\[
\sum_i n_{ABi} + n_A = y_A, \quad \text{(17)}
\]

\[
n_A n_{ABi} = K_{ABi} n_{ABi}. \quad \text{(18)}
\]

Eliminating \( n_{ABi} \) from these equations (16) and (18) we have

\[
n_A = \frac{K_{ABi} kT}{\mu_{bi} p} n_{ABi}.
\]
Fig. 1. log $g$ and log $P$ for the giant "A" and the giant "B".

(Curves may be drawn corresponding to any intermediate values of $\log \frac{M}{L}$.)

Heavy line .......... $\log g$ (g. "A")
Thin line ........... $\log g$ (g. "B")
Dotted heavy line .... $\log P$ (g. "A")
Dotted thin line ...... $\log P$ (g. "B")

$$\sum_i n_{ABi} \frac{K_{ABi}}{\mu_{Bi}} \frac{kT}{p} = \mu_A \frac{p}{kT}.$$  

Summation of $n_{ABi}$ over $i$ gives by (18)

$$\sum_i n_{ABi} = \sum_i \frac{n_{AHi} n_{Bi}}{K_{ABi}} - \sum_i \frac{n_{Hi} \mu_{Bi}}{K_{ABi}} \frac{kT}{p} = K_{ABi} \frac{n_{ABi}}{\mu_{Bi}} \sum_i \frac{\mu_{Bi}}{K_{ABi}}.$$  

$$n_{ABi} \left( \frac{K_{ABi}}{\mu_{Bi}} \sum_i \frac{\mu_{Hi}}{K_{ABi}} + \frac{kT}{\mu_{Bi}} \right) = \mu_A \frac{p}{kT}.$$  

$$n_{ABi} = \frac{\mu_A \mu_{Bi}}{K_{ABi}} \frac{p}{kT} \frac{1}{\sum_i \frac{\mu_{Bi}}{K_{ABi}} + \frac{kT}{p}}.$$  

Put

$$\sum_i \frac{\mu_{Hi}}{K_{ABi}} = F_A.$$  

Then we have

$$n_{ABi} = \frac{\mu_A \mu_{Bi}}{K_{ABi}} \frac{p}{kT} \frac{1}{F_A + \frac{kT}{p}}.$$  

(19)
The equation of hydrostatic equilibrium is
\[ dp = g \frac{p}{RT} \, dh, \]  
(20)
where \( R \) is the gas constant of the material.

The total number \( N_{AB} \) is given in the following expression.

\[ N_{AB} = \int \frac{n_{AB}}{p} \, dp. \]

By (20) we have
\[ N_{AB} = \int \frac{n_{AB}}{p} \, dp. \]

Hence, by (19)
\[ N_{AB} = \frac{\mu_A \mu_B}{K_{AB}} \frac{R}{k} \frac{1}{g} \int_0^\infty \frac{dp}{F_A + \frac{kT}{p}}. \]

\[ N_{AB} = \frac{\mu_A \mu_B}{K_{AB}} \frac{R}{k} \frac{1}{g} \frac{1}{F_A} \ln \left( 1 + \frac{F_A p}{F_A + \frac{kT}{p}} \right). \]  
(21)

For such molecules as \( A_2 \) we have
\[ N_{A_2} = \frac{\mu_A^2}{K_{A_2}} \frac{R T}{g} \frac{1}{F_A} \ln \left( 1 + \frac{F_A p}{F_A + \frac{kT}{p}} \right). \]  
(22)

These two formulae correspond to the formulae (8) and (11) of our previous paper.

As for the atomic abundance in number of the elements relative to the hydrogen atom in the carbon stars, we take the numerical values tabulated in the next. The molecules here treated are \( \text{CH, C}_2, \text{NH, CN, N}_2, \text{OH, CO, NO and O}_2. \)

V. Shane’s Spectrographic Observations.

The band spectra of the carbon stars have been completely summarized and discussed by Shane\textsuperscript{(1)} from the standpoint of his spectrographic observations. Let us summarize some important results of his paper.

The cyanogen bands (4216, 3883, 3590) increase in strength from \( R_0 \), have a maximum at \( R_5 \) and disappear at about \( N_4 \). The 4606

\textsuperscript{(1)} C. D. Shane: Lick Obs. Bull. 13 (1928), 123.
cyanogen band shows a little different behavior—increasing from R0 to R8 and remaining constant thereafter.

The Swan band at 4737 increases in strength from R0 to R5, decreases from R5 to R8, and then increases again through the N classes.

The G band increases from R0 to a maximum of unusual strength in R3 and then fades almost to nothing at about N3.

Figures 2 and 3 illustrate graphically the variation of intensity of molecular bands with spectral class. As we see in these figures, it is noticeable that the scattering of the points is remarkable, particularly where the intensity of the carbon bands are concerned. About this fact, his argument is that it is difficult in R-N stars to arrange their spectra accurately in a one-dimensional system. If they could be arranged in a one-dimensional system.
dimensional system, the problem of spectral classification would be much simplified. In such a case, a smooth curve would be obtained if the intensity of any spectral feature were plotted against spectral class and we could infer the complete dependence of stellar spectra on a single variable. This is indicated by the main branch of the spectral sequence, in which temperature is undoubtedly a controlling factor, while pressure, density and chemical composition probably may exert some influence. But it is doubtful that temperature plays so important a rôle in the branch of the carbon stars. He says in his conclusion that the physical interpretation of the R-N spectrum is evidently a problem of some complexity.

VI. Interpretation of the Spectral Sequence in the Carbon Stars.

We calculated the number of molecules as in our previous paper, applying our new for-
mulae for the electron pressure and the surface gravity obtained in the previous paragraph. These formulae in which the luminosity of the stars is taken into account, play an important rôle for the interpretation of scattering of the observational points in Shane's diagram (Fig. 2 and 3). It is concerned with the redness of the stars under consideration. The results of our calculations are illustrated graphically in Figs. 4, 5 and 6. We shall, here, give some remarkable notices concerning these figures.

CN band: As is shown in Fig. 7, we can easily understand the agreement with observations of its intensity variation along the spectral sequence, by assuming the maximum intensity of this band at $R_5^{(1)}$.

And if we plot Shane's results in this figure, by bringing the maximum of our values and that of the observational data into coincidence, some scattered observational data agree with these two curves—curves of two different sequences of the electron pressure and the surface gravity—with fair accuracy. These two curves correspond to the sequence of the giant "A" and of the giant "B", as was shown already in (13). Therefore, if we take intermediate values of $\frac{M}{L}$ between the giant "A" and the giant "B" for the calculation of $\log g$ and $\log P$, then we can obtain similar curves corresponding to each sequence of values of the electron pressure and the surface gravity and all the scattering points may lie on some of these curves.

C$_2$ band: It is also shown in this figure that the theoretical intensity maximum agrees well with observational intensity maxima. And, moreover, the conspicuous scattering of the observational data are

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(1) This assumption is plausible. Thus we can arrange the spectral types on the temperature scale.
Dissociation of Molecules in the Carbon Stars.

Other curves by our calculations (Figs. 4, 5 and 6) can not be applied here, because of the lack of observational data, but they agree well with Russell's curves (1) which are shown in Fig. 8.

It is realized thus that the scattering of the carbon stars are explained by the difference of the physical conditions, in other words, different sequences of the electron pressure and the surface gravity which indicate the difference of the luminosity in these stars.

**Conclusion.**

In the carbon stars in which their redness may have some influence on the interpretation of their physical character, we tried to calculate the molecular abundance and to discuss the intensity variation of the molecular bands. We applied new formulae for the electron pressure and the surface gravity, taking the luminosity of R and N stars into account. Then we can explain the scattering of CN and C₂ band intensities in these stars as due to the difference in the sequences of luminosity, mass and temperature.

Department of Astronomy of the Tokyo Imperial University.

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