The Non Holonomic Representation of Projective Spaces.

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T. Y. Thomas\(^{(1)}\) showed that it is possible to reduce the theory of projectively connected \(n\)-dimensional manifold \(P_n\) to the theory of affinely connected manifold \(A_{n+1}\) involving one additional dimension.

Adopting this introduction of one additional dimension, J. H. C. Whitehead\(^{(2)}\) studied the properties of the affine space \(A_{n+1}\) which may be taken to represent the projectively connected manifold.

In the present paper, we shall take a non holonomic hypersurface \(A_{n+1}^*\) in such a space \(A_{n+1}\) to represent a projectively connected manifold \(P_n\) and shall consider the paths and the projective normal parameter on each path introduced by J. H. C. Whitehead\(^{(3)}\) and studied recently by L. Berwald,\(^{(4)}\) J. Haantjes\(^{(5)}\) and the present author.\(^{(6)}\)

I. Properties of \(A_{n+1}\).

Following J. H. C. Whitehead, we consider an affinely connected manifold \(A_{n+1}\) referred to a coordinate system \(x^1(\lambda, \mu, \nu, \ldots = 0, 1, 2, \ldots, n)\) whose coefficients of affine connection are \(\Pi^i_{\mu\nu}\) and a vector field \(\xi^i\) in this space satisfying the following conditions:

\[
\begin{align*}
(1'1) & & \Pi^i_{\mu\nu} = \Pi^i_{\nu\mu} , \\
(1'2) & & \xi^{\lambda}_i \cdot x = \delta^\lambda_i , \\
(1'3) & & \Pi^i_{\mu\nu} \xi^\nu = 0 ,
\end{align*}
\]

where

\[
(1'4) \quad \xi^{\lambda}_i \cdot x = \frac{\partial \xi^\lambda}{\partial x^i} + \xi^a \Pi^a_{\mu\nu} ,
\]

and

\[
(1'5) \quad \Pi^i_{\mu\nu} = \frac{\partial \Pi^i_{\lambda\nu}}{\partial x^\mu} - \frac{\partial \Pi^i_{\nu\lambda}}{\partial x^\mu} + \Pi^a_{\mu\nu} \Pi^i_{a\lambda} - \Pi^i_{\mu\lambda} \Pi^\mu_{a\nu} .
\]


\(\text{(3)}\) J. H. C. Whitehead: loc. cit.


The geometrical meaning of the equations (1.1) is evident, that is, the connection is symmetric or without torsion.

The second condition $\xi^\alpha, = \delta^\alpha_\alpha$ means that in the tangent affine space associated with the point $M$ of the manifold, the point \( M - \xi^\alpha, \hat{e}_\alpha \) forming "repère naturel" of E. Cartan, is invariant with respect to the groupe of holonomy. For we have

\[
d(M - \xi^\alpha, \hat{e}_\alpha) = dM - \xi^\alpha, dx^\alpha \hat{e}_\alpha = du^\alpha \hat{e}_\alpha - du^\alpha \hat{e}_\alpha = 0,
\]

this point corresponds to the ideal point introduced by J. H. C. Whitehead\(^{(1)}\).

From these considerations, it follows that the curves generated by the vector $\xi^\alpha$, that is to say, the rays are paths.

When the first and the second conditions are satisfied, the third gives us a necessary and sufficient condition that the vector $\xi^\alpha$ generates a continuous groupe of affine collineation.

The conditions (1.1), (1.2) and (1.3) represent also that there exist totally geodesic two dimensional surfaces generated by the rays. We shall call such surfaces planes.

In an $A_{n+1}$ satisfying the three conditions (1.1), (1.2) and (1.3) the points of the projective space may be represented by the rays, and the geodesics by the planes.

In the next discussion, we shall confine ourselves to the coordinate systems with respect to which the vector $\xi^\alpha$ has the components \(1, 0, \ldots, 0\), say

\[
\xi^\alpha = \delta^\alpha_0
\]

then the transformations of coordinates carrying such a system to another take the following form :

\[
\begin{align*}
& (1.7) & \bar{x}^0 = x^0 + \psi(x^1, x^2, \ldots, x^n), \\
& & \bar{x}^i = \bar{x}^i(x^1, x^2, \ldots, x^n), \quad (i, j, k, \ldots. = 1, 2, \ldots, n).
\end{align*}
\]

In these coordinate systems, the conditions (1.1), (1.2) and (1.3) may be expressed as follows :

\[
\begin{align*}
& (1.8) & \Pi^\alpha_\mu = \Pi^\alpha_\nu, \\
& (1.9) & \Pi^\alpha_\mu = \delta^\alpha_\mu, \\
& (1.10) & \Pi^\alpha_\mu \nu = 0,
\end{align*}
\]

2. Non holonomic spaces $A^{n+1}_{m+1}$

Consider a non holonomic space $A^{n+1}_{m+1}$ defined by

\[(2'1)\]

\[dx^0 + \varphi_0 dx^0 = 0,
\]
or

\[(2'2)\]

\[\varphi_\lambda dx^\lambda = 0,
\]

where \[(2'3)\]

\[\varphi_0 = 1.
\]

and the $\varphi_\lambda$ do not contain the variable $x^0$.

The condition that the $\varphi_\lambda$ do not contain the variable $x^0$ may be expressed by

\[(2'4)\]

\[\varphi_\lambda = 0,
\]

where \[(2'5)\]

\[\varphi_\lambda = \frac{\partial \varphi_\lambda}{\partial x^\mu} - \frac{\partial \varphi_\mu}{\partial x^\lambda}.
\]

In a general system of coordinates, \[(2'3)\] and \[(2'4)\] take the following forms respectively

\[(2'6)\]

\[\varphi_\lambda^\mu = 1,
\]

and \[(2'7)\]

\[\varphi_\lambda ; \mu^\mu = 0.
\]

Differentiating \[(2'6)\] covariantly, we get

\[(2'8)\]

\[\varphi_\lambda ; \mu^\mu + \varphi_\lambda ; \mu^\mu = 0,
\]

but, on the other hand, we have

\[(2'9)\]

\[\varphi_\lambda ; \mu - \varphi_\mu ; \lambda = \varphi_\lambda ; \mu,
\]

consequently, we have from \[(2'7)\], \[(2'8)\] and \[(2'9)\],

\[(2'10)\]

\[\varphi_\lambda ; \mu^\mu + \varphi_\lambda = 0.
\]

The equations \[(2'10)\] give us the following geometrical interpretation.

If a vector $\eta^\lambda$ is of $A^{n+1}_{m+1}$, say

\[(2'11)\]

\[\varphi_\lambda \eta^\lambda = 0,
\]

its associate direction $\eta^\lambda ; \mu^\mu$ with respect to $\xi^\lambda$ is also contained in the $A^{n+1}_{m+1}$.

For, from \[(2'11)\], we have

\[(2'12)\]

\[\varphi_\lambda ; \mu \eta^\lambda + \varphi_\lambda \eta^\lambda ; \mu^\mu = 0,
\]

\[\varphi_\lambda \eta^\lambda + \varphi_\lambda \eta^\lambda ; \mu^\mu = 0,
\]

\[\varphi_\lambda \eta^\lambda ; \mu^\mu = 0.
\]
We shall define the unit affinor for the non holonomic space $A_{n+1}$ by

\begin{equation}
B^\mu_\lambda = A^\mu_\lambda - \xi^\lambda \phi_\mu, \tag{2.13}
\end{equation}

where

\begin{equation}
A^\mu_\lambda = \delta^\mu_\lambda. \tag{2.14}
\end{equation}

If $V^\lambda$ and $W_\lambda$ are respectively the contravariant and covariant vectors of $A_{n+1}$, we have

\[ B^\mu_\lambda V^\lambda = V^\mu \text{ and } B^\mu_\lambda W_\lambda = W_\mu. \]

In general, using the quantities

\begin{equation}
B^\mu_\lambda = \delta^\mu_\lambda - \xi^\lambda \phi_\mu, \tag{2.15}
\end{equation}

\begin{equation}
B^\mu_\lambda = \delta^\mu_\lambda, \tag{2.16}
\end{equation}

satisfying

\[ B^\mu_\lambda B^\rho_\lambda = \delta^\mu_\rho, \quad B^\mu_\lambda B^\rho_\mu = \xi^\lambda \phi_\mu, \]

we can define the $A_{n+1}$-components of the quantities of $A_{n+1}$. For example,$^6$

\begin{align*}
A_{n+1} & \quad \quad A_{n+1} \\
V^\mu & \rightarrow B^\mu_\lambda V^\lambda, \\
W_\lambda & \rightarrow B^\mu_\lambda W_\lambda, \\
B^\mu_\lambda \psi^\nu & \leftarrow \psi^\nu, \\
B^\mu_\lambda \omega^\nu & \leftarrow \omega^\nu.
\end{align*}

We shall now define the induced affine connection in $A_{n+1}$ by

\begin{equation}
\Gamma^i_{jk} = B^i_\mu B^\mu_\nu \pi^\mu_{jk} - B^i_\mu B^\mu_\nu B^\nu_i, \tag{2.17}
\end{equation}

where

\begin{equation}
B^i_\mu = \frac{\partial B^i_\mu}{\partial x^\nu}(=0). \tag{2.18}
\end{equation}

Substituting (2.15) and (2.16) in the (2.17) and taking account of (1.8), (1.9) and (1.10), we have

\begin{equation}
\Gamma^i_{jk} = B^i_\mu \phi^\mu_\nu \pi^\nu_{jk} - B^i_\mu \phi^\mu_\nu B^\nu_i. \tag{2.19}
\end{equation}

We shall moreover calculate the components of a tensor which will be used later on.

\[ B^i_\mu \phi^\mu_\nu = B^i_\mu B^\mu_\nu \phi^\nu_\mu = -(\pi^\mu_{jk} - \phi^\mu_\nu \phi^\nu_{jk} + \phi^\nu_\mu \pi^\nu_{jk}). \]

Let us denote by $\Gamma^i_\mu$ the tensor $-B^i_\mu B^\mu_\nu \phi^\nu_\mu$, then we have

\begin{equation}
\Gamma^i_\mu = \pi^\mu_{jk} - \phi^\mu_\nu \phi^\nu_{jk} + \phi^\nu_\mu \pi^\nu_{jk}. \tag{2.20}
\end{equation}

If we take another non holonomic space defined by

\begin{equation}
\frac{dx^0 + \phi_0 dx^1 + \phi_1 dx^2}{\phi_0 dx^4 + \phi_1 dx^4} = 0, \tag{2.21}
\end{equation}

we obtain the following corresponding fonctions

\[ \Gamma^a_k = \Pi^a_k - \phi_\mu \phi_{\mu k} - \phi_{\mu k}, \]
\[ \Gamma^a_k = \Pi^a_k - \delta^a_i \phi_{\mu k} - \delta^a_i \phi_{\mu}. \]

that is to say,
\[ (2.22) \quad \Gamma^a_k = \Pi^a_k - \phi_\mu \phi_{\mu k} - \phi_{\mu k}, \]
\[ (2.23) \quad \Gamma^a_k = \Pi^a_k - \delta^a_i \phi_{\mu k} - \delta^a_i \phi_{\mu}. \]

The equations (2.23) represent a projective change of the affine connection \( \Gamma^a_k \). Then the common properties for all non holonomic spaces defined by the equation of the form \( dx^0 + \phi_{kl} dx^k = 0 \) may be projective properties of the affine connection derived from \( \Pi^a_k \).

3. The equations of the paths.

Let us consider a path of \( A_{n+1} \)
\[ (3.1) \quad x^a = a^a(t), \]
defined by
\[ (3.2) \quad \frac{d^2 a^a}{d^2 t} + \Pi^a_{\mu} \frac{da^\mu}{dt} \frac{da^a}{dt} = 0, \]
\( t \) being affine parameter.

If we draw the ray passing through each point of the path, then we shall obtain a totally geodesic surface.

The intersection of this totally geodesic surface and our non holonomic space may be determined by
\[ (3.3) \quad \begin{cases} x^a = a^a + \rho & \text{and} \quad (3.4) \quad \frac{dx^0}{dt} + \phi_{\mu} \frac{dx^\mu}{dt} = 0. \end{cases} \]

Substituting (3.3) in (3.2) we obtain
\[ (3.5) \quad \frac{d^2 x^\mu}{dt^2} - \frac{d^2 \rho}{dt^2} + \Pi^\mu_{\nu} \frac{dx^\nu}{dt} \frac{dx^\mu}{dt} + \left( \frac{dx^\rho}{dt} - \frac{d\rho}{dt} \right)^2 = 0, \]
\[ (3.6) \quad \frac{d^2 x^\mu}{dt^2} + \Pi^\mu_{\nu} \frac{dx^\nu}{dt} \frac{dx^\mu}{dt} + 2 \left( \frac{dx^\rho}{dt} - \frac{d\rho}{dt} \right) \frac{dx^\mu}{dt} = 0. \]

If we substitute the value of \( \frac{dx^\rho}{dt} \) obtained from (3.4) in (3.6), we get
\[ (3.7) \quad \frac{d^2 x^\mu}{dt^2} + \Pi^\mu_{\nu} \frac{dx^\nu}{dt} \frac{dx^\mu}{dt} - 2 \phi_{\mu} \frac{dx^\nu}{dt} \frac{dx^\mu}{dt} - 2 \frac{d\rho}{dt} \frac{dx^\mu}{dt} = 0, \]
\[ \frac{d^2 x^\mu}{dt^2} + (\Pi^\mu_{\nu} - \delta^\mu_{\nu} \phi_{\mu}) \frac{dx^\nu}{dt} \frac{dx^\mu}{dt} - 2 \frac{d\rho}{dt} \frac{dx^\mu}{dt} = 0, \]
\[ (3.7) \quad \frac{d^2 x^\mu}{dt^2} + \Pi^\mu_{\nu} \frac{dx^\nu}{dt} \frac{dx^\mu}{dt} - 2 \frac{d\rho}{dt} \frac{dx^\mu}{dt} = 0. \]
We introduce here a parameter $s$ defined by
\begin{equation}
\frac{dp}{dt} = -\frac{1}{2} \frac{dt}{ds^3}.
\end{equation}

Then the equations (3.7) take the form
\begin{equation}
\frac{dx^i}{ds} + \Gamma^i_j \frac{dx^j}{ds} \frac{dx^k}{ds} = 0.
\end{equation}

From (3.4), we have
\[ \frac{d^2 x^i}{dt^2} = -\varphi_{k,l} \frac{dx^j}{dt} \frac{dx^k}{dt} \frac{dx^l}{dt} + \varphi_{l,k} \frac{dx^l}{dt} \frac{dx^j}{dt} \frac{dx^k}{dt} + 2 \left( \frac{dx^k}{dt} - \frac{dp}{dt} \right) \frac{dx^j}{dt}, \]

therefore
\begin{equation}
\frac{d^2 x^i}{dt^2} = -\varphi_{k,l} \frac{dx^j}{dt} \frac{dx^k}{dt} + \varphi_{l,k} \frac{dx^l}{dt} \frac{dx^j}{dt} - 2 \varphi_{l,k} \frac{dx^j}{dt} \frac{dx^l}{dt} - 2 \frac{dp}{dt} \frac{dx^j}{dt}.
\end{equation}

From the equations (3.4), (3.5) and (3.10), it follows that
\begin{align*}
\left( \frac{dp}{dt} \right)^2 \frac{d^2 p}{dt^2} &= -(\Gamma^i_j - \varphi_{k,l} \varphi_{k,j} - \varphi_{l,k} \varphi_{i,j}) \frac{dx^i}{dt} \frac{dx^j}{dt}, \\
\left( \frac{dp}{dt} \right)^2 \frac{d^2 p}{dt^2} &= -\Gamma_{j,k} \frac{dx^j}{dt} \frac{dx^k}{dt}.
\end{align*}

But, on the other hand, the first member of this equation may be calculated as follows,
\begin{align*}
\left( \frac{dp}{dt} \right)^2 \frac{d^2 p}{dt^2} &= \frac{1}{4} \left( \frac{dt}{ds} \right)^4 + \frac{1}{2} \frac{dt}{ds} \left( \frac{dt}{ds} \right)^2 \left( \frac{dt}{ds} \right)^2 \\
&= \frac{1}{2} \left[ \frac{dt}{ds} \frac{dt^2}{ds^3} - 3 \left( \frac{dt}{ds} \right)^2 \right] = \frac{\{t\}_s}{2 \left( \frac{dt}{ds} \right)^3},
\end{align*}

where $\{t\}_s$ is the Schwarzian derivative of the function $t$ with respect to $s$, say...
The equations (3.9) determining the system of paths in the nonholonomic space $A_{n+1}$, the equation (3.12) determines a parameter on each path. This parameter, corresponding to the projective normal parameter, is determined up to a linear fractional transformation. We can interpret this property as follows.

All the paths being on the same totally geodesic surface generated by the rays determine the same path of $A_{n+1}$ up to a displacement defined by

$$x^o \rightarrow x^o + \text{constant}, \quad x^i \rightarrow x^i.$$  

The affine parameters on these paths are obtained by a linear fractional transformation from one another. For, if we suppose that a path defined by (3.1) is on a totally geodesic surface generated by the rays, all the other paths on the same plane must have the form

$$x^o = a^o + \sigma(t), \quad x^i = a^i,$$

that is to say,

$$x^i = a^i + \delta_i^o \sigma(t).$$

Substituting these equations in (3.2), we have

$$\frac{d^2 x^o}{dt^2} + H_{o_\nu} \frac{dx^o_x}{dt} \frac{dx^x}{dt} - 2\sigma' \frac{dx^o_t}{dt} - \delta_i^o \sigma'' + \delta_i^o \sigma' = 0,$$

where

$$\sigma' = \frac{d\sigma}{dt}, \quad \sigma'' = \frac{d^2\sigma}{dt^2}.$$  

Under the above mentioned supposition that (3.14) represents a path, $\delta_i^o (-\sigma'' + \sigma')$ must be proportional to $dx^o/dt$, therefore we have the differential equation

$$-\sigma'' + \sigma' = 0,$$

whose general solution is

$$\sigma = -\log(at + \beta).$$

Substituting this in (3.14), we have

$$\frac{dx^o_t}{dt} = \frac{3}{2} \left( \frac{ds^o_t}{dt} \right)^{\frac{3}{2}},$$

consequently, we obtain

$$\{t\} = -2 \Gamma_\nu \frac{dx^o \cdot dx^x}{ds \cdot ds}.$$  

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$$\sigma = -\log(at + \beta).$$

Substituting this in (3.14), we have
The affine parameter \( i \) for the path (3.15) may be defined by

\[
\frac{\dd x^i}{\dd t^2} = \frac{2\alpha}{\alpha t + \beta}, \quad \text{or} \quad \frac{\dd x^i}{\dd t} = -\frac{2\alpha}{\alpha t + \beta},
\]

from which we have

\[
i = \gamma t + \delta.
\]


We shall define the \( A_{2+n} \)-components of the curvature tensor (1.5) by

\[
W_{jkh} = B_{jkh} B_{k}^{m} B_{h}^{m}_{\mu} \Pi_{\mu}^{n}.
\]

Taking account of

\[
B_{k}^{j} = \delta_{k}^{j}, \quad B_{j}^{i} = \delta_{j}^{i} - \xi_{j}^{i} \varphi, \quad \Pi_{\mu}^{n} = \Pi_{\mu}^{n}_{a} = \Pi_{\mu}^{n}_{b} = 0,
\]

we obtain

\[
W_{jkh} = \frac{\partial \Pi_{j}^{i}}{\partial x^{k}} - \frac{\partial \Pi_{k}^{i}}{\partial x^{j}} + \Pi_{j}^{i} \Pi_{k}^{m} - \Pi_{j}^{m} \Pi_{k}^{i} - \Pi_{j}^{k} \Pi_{k}^{i},
\]

or

\[
W_{jkh} = \left( \frac{\partial \Pi_{j}^{i}}{\partial x^{k}} - \frac{\partial \Pi_{k}^{i}}{\partial x^{j}} + \Pi_{j}^{i} \Pi_{k}^{m} - \Pi_{j}^{m} \Pi_{k}^{i} - \Pi_{j}^{k} \Pi_{k}^{i} \right) + \Pi_{j}^{i} \Pi_{k}^{m} - \Pi_{j}^{m} \Pi_{k}^{i} + \Pi_{j}^{k} \Pi_{k}^{i}.
\]

From the equations (4.3), it follows that the tensor \( W_{jkh} \) does not depend upon the vector \( \varphi_{a} \) that is to say the tensor \( W_{jkh} \) is independent of the choice of the non holonomic space.

Replacing the equations

\[
\Pi_{j}^{i} = \Gamma_{j}^{i} - \xi_{j}^{i} \varphi, \quad \varphi_{a} = \varphi_{a},
\]

and

\[
\Pi_{j}^{i} = \Gamma_{j}^{i} + \delta_{j}^{i} \varphi, \quad \varphi_{a} = \varphi_{a},
\]

obtained from (2.19) and (2.20) respectively in (4.2), we get after calculation,

\[
W_{jkh} = R_{jkh} + \Gamma_{j}^{i} \delta_{k}^{i} - \Gamma_{k}^{i} \delta_{j}^{i} + \delta_{j}^{i} (\varphi_{a} - \varphi_{a}),
\]

where

\[
R_{jkh} = \frac{\partial \Gamma_{j}^{i}}{\partial x^{k}} - \frac{\partial \Gamma_{k}^{i}}{\partial x^{j}} + \Gamma_{j}^{a} \Gamma_{k}^{i} - \Gamma_{j}^{i} \Gamma_{k}^{a}.
\]

But, on the other hand, it follows from (4.3) that

\[
\varphi_{a} - \varphi_{a} = - (\Gamma_{j}^{i} - \Gamma_{j}^{i}),
\]

therefore, we have the following expression of \( W_{jkh} \).
Let us now consider the case where the affine connection $\Pi_{\mu}^{\nu}$ is normal.

The normal connection is characterized by

\begin{equation}
\Pi_{\mu}^{\nu}=0,
\end{equation}

therefore

\begin{equation}
W_{\mu}^{i\mu}=B_{i}^{\alpha}B_{\beta}^{\gamma}B_{\gamma}^{\delta}\Pi_{\alpha}^{\beta}\Pi_{\mu}^{\nu} = B_{i}^{\alpha}B_{\beta}^{\gamma}(\delta_{\alpha}^{\gamma} - \delta_{\gamma}^{\alpha})\Pi_{\mu}^{\nu}
\end{equation}

\begin{equation}
= B_{i}^{\alpha}B_{\beta}^{\gamma}\Pi_{\mu}^{\nu} = 0.
\end{equation}

Contracting in (4.5) with respect to the indices $i$ and $\mu$, we have the equations

\begin{equation}
0 = R_{ij} + n\Gamma_{ij} - \Gamma_{ij},
\end{equation}

which give

\begin{equation}
\Gamma_{jk} = -\frac{1}{n^2-1}(nR_{jk} + R_{jk}),
\end{equation}

where

\begin{equation}
R_{jk} = R_{jk}^{a}.
\end{equation}

Substituting (4.9) in (4.5), we obtain

\begin{equation}
W_{jk}^{a} = R_{jk}^{a} - \frac{1}{n^2-1}[(nR_{jk} + R_{jk})\delta_{a}^{j} - (nR_{jk} + R_{jk})\delta_{a}^{i}] + \frac{1}{n^2-1}\delta^{j}R_{jk}^{a} - R_{jk}^{a}.
\end{equation}

The curvature tensor obtained here is the Weyl\(^{10}\) projective curvature tensor.

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\(^{10}\) Weyl, loc. cit.