SHORT NOTES.

Note on the Lifetime of the Neutral Meson.

By Yasutaka Tanikawa.

According to Sakata and the present author (1), a neutral meson can disintegrate spontaneously into two or more photons: Namely, a meson is absorbed by a proton which is in the negative energy state and produces a transitory pair of a proton and an antiproton, which disappear subsequently with the emission of more than two photons. Thus the protons in the negative energy states play the part of catalytic agent. The proper lifetime of a neutral vector-meson due to this process was estimated by Sakata and the author and was found to be of the order of magnitude $10^{-16}$ sec. A more detailed calculation, which was made recently by Nakamura (2), seems to indicate a little longer lifetime.

Meanwhile, the existence of the pseudoscalar-meson has been discussed by several authors with regard to the nuclear phenomena and the cosmic rays. The charge independent character of the nuclear forces appears to require the existence of the uncharged pseudoscalar-meson as well as the charged one. In this note, we want therefore to give the lifetime of a neutral meson described by the pseudoscalar field.

In the calculation based on the ordinary perturbation method using plane wave functions as the basis, it is to be expected that the symmetry between a virtual pair of a proton and an antiproton give cancellations. The general theorem which predicts the appearance of such a cancellation was presented by Furry (3). According to this theorem, it is easily shown that the transition probability of the probability of the process in which a pseudoscalar-meson transforms into odd numbers of photons vanishes identically, this is in contrast with the fact that the disintegration of a vector-meson into even numbers of photons is forbidden.

For the sake of simplicity, we consider only the two quanta disintegration of a meson which contributes mainly to the lifetime. If a meson is assumed to be at rest with respect to a certain coordinate system, the typical processes accompanied by the transitions of protons from negative to positive energy states are represented schematically in Fig. 1.

Here 0, 1 and 2 indicate respectively the absorption of a meson at rest, the emission of one of the photons with no

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Fig. 1.

(1) Sakata and Tanikawa, Phys. Rev., 57 (1940), 548.
(2) Nakamura, Nippon Sugaku-Buturigakkaisi, in press.
mentum \( \gamma \gamma' (|l\rangle = \mu \text{ meson mass}) \) and the emission of another photon with momentum \( \gamma' \), the transitions being represented by the arrows. Other possible processes can be obtained by interchanging the order of 1 and 2. There are also the analogous set in which the roles of protons and antiprotons are interchanged. We have after all twelve different intermediate states. It is noticeable that a peculiar sort of processes seems to be possible, i.e. the absorption of the meson and the emission of the photons take place independently with each other in any negative energy state of protons. This sort of processes is represented schematically in Fig. 2.

The corresponding processes can not be brought about for a meson with any magnitude of momentum. This ambiguity is related closely with the well-known infinite charges of the vacuum and can be avoided by taking into account “the Heisenberg prescription” for the order of the products of the \( q \)-number wave functions of the proton.* We omit therefore the above mentioned processes. The calculation is not so complicated as that in the vector-meson. We find the decay formula of the neutral pseudoscalar-meson.

Here \( \gamma \) denotes a constant characterizing the strength of the pseudo-vector type interaction of the meson with the heavy particles\(^1\). \( J_1 \) and \( J_2 \) represent the following integrals:

\[
J_1 = \frac{1}{\pi} \int_0^\infty \frac{x^2 dx}{(1+x^2)^{5/2}}
\]

and

\[
J_2 = \frac{1}{\pi} \int_0^\infty \frac{dx}{(1+x^2)^{5/2}}
\]

where \( x \) is the wave number of a proton divided by the Compton wave number of proton. The formula (1) is obtained by neglecting the quantities of the order of magnitude \( (\mu M)^2 \) compared to unity, \( M \) being the proton mass. Since \( J_1 \) is a diverging integral, if we want to get a finite result, it is necessary to take into account the following procedure:

(A) We remove the diverging integral \( J_1 \) and leave only the non-diverging integral \( J_2 \). This is corresponding to the subtraction procedure in the Dirac-Heisenberg theory of the positron,

(B) We introduce an upper limit \( x_0 \) for \( x \) marking the validity of our theory and cut off the integrals at this value. Then the above integrals reduce to

\[
G_1 = \frac{1}{\pi} \int_0^{x_0} \frac{x^2 dx}{(1+x^2)^{5/2}}
\]

and

\[
G_2 = \frac{1}{\pi} \int_0^{x_0} \frac{dx}{(1+x^2)^{5/2}}
\]

respectively.

Using the usual values \( f_2 = \frac{1}{10} \) and \( \mu = 178 m \) for the constants in (1), we get

\[
\tau = 1.5 \times 10^{-11} \text{ sec.}
\]

in the case (A).

If we choose

\[
\tau = \text{yes}
\]

corresponding to a cutting off wave number of \( 3 \times 10^9 \text{ m} \), we get

\[
\tau \approx 1.8 \times 10^{-11} \text{ sec.}
\]

* Pauli, Huitieme Conseil de Physique Solvay, Chapter II. If any hermite operator \( F \) of the \( q \)-number theory appears, in the calculation of the \( q \)-number theory, we replace \( \psi^* \psi \) by \( \frac{1}{2} (\psi^* \psi - \psi \psi^*) \) where \( \psi \) is the wave function of the proton.

in the case (B). This value of the lifetime is very sensitive to the cutting off value \( x_0 \).

On the other hand, Nakamura obtained for the lifetime of the vector-meson the value
\[
\tau = 2 \times 10^{-4} \text{ sec.}
\]
in the case (A) and
\[
\tau \approx 2 \times 10^{-4} \text{ sec.}
\]
in the case (B), if we adopt the single force hypothesis.

Our result for the pseudoscalar meson seem to be compatible with the fact of the experimental failure to prove the existence of the neutral meson in cosmic rays. But it is not decided whether or not these values of the lifetime are consistent with the whole cosmic ray phenomena. For instance, Taketani\(^5\) pointed out that in order to explain the experimental result obtained by Schein et al.\(^6\), the lifetime of the neutral meson must be of the order of magnitude \( 10^{-2} \sim 10^{-6} \) sec. This value seems to be compatible with the lifetime of the vector-meson obtained by Nakamura, but not with that of the pseudoscalar-meson.

In conclusion, the author wishes heartily to thank Professor H. Yukawa, and Dr. S. Sakata for many helpful discussions on the subject of the present note. The cost of this research has been defrayed from the Scientific Research Expenditure of the Department of Education.

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\(^1\) According to Bethe, only the tensor type interaction is assumed for the interaction of the vector-meson with the heavy particle.

\(^5\) Taketani, Kagaku, 11 (1941), 523, in Japanese.

\(^6\) Schein, Jesse and Wollan, Phys. Rev., 59 (1941), 615.

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Errata.

Sizuo Ueno and Hironobu Kuze: Note on the Magnetic Moment of the Nurleon.


The expression (4) on page 185 should be read
\[
m = -\frac{\hbar}{2\pi} \left( \frac{4}{\hbar^2} \right) \frac{e^2}{\mu c} s_i(\chi_0)
\]
instead of
\[
m = \frac{\hbar}{2\pi} \left( \frac{4}{\hbar^2} \right) \frac{e^2}{\mu c} s_i(\chi_0)
\]