On Periodic Fluctuations of Convection Currents—
with a Hint on the Origin of
Sun-spots Cycle.

By

Torahiko TERADA.

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Several years ago, while testing the constancy of the zero point of a sensitive balance, Messrs. T. Matoba and Z. Kobayasi, then Students of Physics, observed a peculiar periodic fluctuation of the zero point which was apparently caused by the radiation of an incandescent lamp placed near the glass wall of the balance case. When the lamp was brought near the right side wall, the zero shifted to the right and underwent a slow periodic fluctuation with a period of several minutes. Later, the experiment was repeated by Messrs. K. Nanba and Y. Mimmura, Students of Physics. Thermometers were brought into the case near the right and left side wall to measure the temperature rise due to the radiation. On bringing a 24 C. P. lamp close up to the right side wall, temperatures rose at first rapidly and then slowly, apparently tending to the respective ultimate values. The zero of the balance rapidly shifted towards right and oscillated with a rather irregular period of about ten minutes, with amplitudes of several scale divisions, about a mean position which was about four divisions to the right of the zero with no lamp. In one of the experiments a thin mica plate with an area decidedly larger than that of the scale pan was fastened horizontally to the suspending rod of the pan at midway between the pan and the beam. By this addition, the mean shift of zero was nearly doubled. It was beyond doubt that the convection current set up within the glass case was responsible to the said phenomena. On replacing 50 C. P. lamp, the mean shift was sensibly increased, but the change of the period could not be ascertained. The average shift seems to decrease gradually with time, with the equalizing of the temperature.

From these observations, I was led to consider if a similar periodic fluctuation of convection current could not be found in nature, for example in our atmosphere. To test this conjecture, I carried out some simple calculations as given later. Recently, while engaged in a series of investigations
on the relations between the solar activity and the terrestrial weather, I was frequently led to speculate on the theory of formation of sunspots and naturally hit upon an idea that the periodicity of solar activities may be connected somehow with the fluctuation of some convection system in the solar atmosphere. In an interesting paper on vortex motion read before the recent meeting of the Society, Prof. Fujiwhara alluded to his idea regarding a possible origin of the sun spots cycle. It seems, therefore, not quite out of place to briefly expound here my opinion on the same subject which seems to be closely related with his.

Let ABCD be a system of connected pipe containing a fluid. The vertical parts AC and BD are surrounded by some devices to keep the temperatures at $T_1$ and $T_2$, respectively, where $T_2 > T_1$. On account of the convection current, the average temperatures of the fluid within the vertical pipes are different from those of the mantles. Let the mean temperatures of the fluid in AC and BD be $\theta_1$ and $\theta_2$, respectively. Assume at first that the horizontal parts of the pipe, AB and CD are impermeable to heat. Then the warmer fluid leaving C enters D without changing the temperature and then subjected to cooling during its passage through the vertical portion DB. For simplicity's sake, we regard the temperature within the vertical pipe as uniform in subsequent calculations. Denote the cross section and the velocity for the vertical part by $S$ and $v$, those for the horizontal by $S'$ and $v'$, the density of the fluid by $\rho$, and the mass within each of the vertical column by $M$. Then the rate of exchange of fluid is

$$\frac{dM}{dt} = vS\rho = v'S\rho. \quad (1)$$

The temperature change will be determined by

$$\theta_1 (M-dM) + \theta_2 dM = M (\theta_1 + d\theta_1)$$
$$\theta_2 (M-dM) + \theta_1 dM = M (\theta_2 + d\theta_2), \quad (2)$$

whence

$$d\theta_1 = -d\theta_2 = \frac{dM}{M} (\theta_1 - \theta_2). \quad (3)$$

If we put

$$\tau = \theta_2 - \theta_1, \quad d\tau = -2 \frac{dM}{M} \tau. \quad (4)$$

or

$$\frac{d\tau}{dt} = -2 \frac{M}{M} \frac{dM}{M} = -2 \frac{\tau}{M} \frac{vS\rho}{M} = -2 \frac{v}{h} \frac{\tau}{\rho}$$

$$= -2 \frac{v}{M} \frac{v'S\rho}{h} \quad (5)$$
where \( h \) is the height. When we are to compare this ideal case with the actual case of the balance box, we may roughly put
\[
\frac{S'}{S} = \frac{h}{l},
\]
where \( h \) is the height and \( l \) the length of the box. Thus
\[
\frac{d\tau}{dt} = - \frac{2\tau}{h} v \quad \text{or} \quad - \frac{2\tau}{l} v'.
\]
(6)

If, however, the deviations of the actual temperatures \( \theta_2, \theta_1 \) from the given temperature of the surroundings \( T_2, T_1 \) are sensible there must be a rise of the temperature in absence of \( v \). Hence we assume roughly
\[
\frac{d\tau}{dt} = C - \frac{2\tau}{h} v \quad \text{or} \quad C - \frac{2\tau}{l} v'.
\]
(7)

On the other hand, since the motion of the system is caused by the difference of the densities in \( AC \) and \( BD \), the equation of motion may approximately be put in the form
\[
m \frac{dV}{dt} = g (\rho_1 - \rho_2) h - k V,
\]
(8)

where \( m \) is the total mass of the circulating system and \( V \) is here a kind of average velocity when the cross sections are not uniform. When \( h \) and \( l \) are nearly equal as in the case of balance, \( V \) stand for the actual velocity \( v = v' \); but when \( l \) is very large in comparison with \( h \) as in the case of the most convection current in atmosphere we may neglect the vertical part and put \( v' \) for \( V \). \( k \) is a constant depending on the frictional resistance. Now,
\[
\rho_1 - \rho_2 = \rho \alpha \tau,
\]
(9)

where \( \rho \) is the density at \( \theta = 0 \) and \( \alpha \) the coefficient of expansion. In any case we may put
\[
m = \varepsilon h l \rho_0,
\]
(10)

where \( \varepsilon \) is a constant of the order of magnitude 1. Hence
\[
\varepsilon h l \rho_0 \frac{dV}{dt} = \alpha h l \rho_0 \tau - k V.
\]
(11)

Differentiating with respect to time and replacing the value of \( \frac{d\tau}{dt} \) from (7), we have
\[
\varepsilon h l \rho_0 \frac{d^2V}{dt^2} = \alpha h l \rho_0 (v' - R V) - k \frac{dV}{dt},
\]
(12)

where \( R = \frac{2\tau}{l} \) in either of the two cases above mentioned. Though the coefficient of \( V \) on the right hand member depends on \( \tau \), we may consider
it as constant for the first approximation. Neglecting the friction for simplicity's sake, the period of oscillation becomes

\[ T = 2\pi \sqrt{\frac{z}{2\gamma \alpha \tau}} = 2\pi \sqrt{\frac{z}{2\gamma \alpha \tau}}, \]  

(13)

where we may put the mean value of \( \tau \).

The average velocity will be given by

\[ v_0 = \frac{C}{R}, \]

(14)

where \( C \) is the rate of increase of \( \tau \) in absence of the velocity.

Applying the above to the case of balance, we obtain reasonable values at least as regards the order of magnitude. For example, putting \( l = 20 \) which is the half length of the box and assuming \( \tau = 1 \), \( \gamma = 1000 \) and \( \alpha = 0.004 \), we have

\[ T = \frac{40}{\sqrt{\tau}}. \]

Actual temperature gradient could not be ascertained, but assuming \( \tau = 0.1 \) as a probable value, we obtain \( T \approx 2 \) minutes which is of the right order of magnitude. Again from the mean shift of the zero and the known sensibility of the balance, we can calculate the force exerted by the current upon the scale pan, whence \( v_0 \) may be estimated. Comparing thus obtained value with \( C/R \), we obtain \( C = 0.03 \) in this case which is also of a right order of magnitude.

The effect of friction is difficult to estimate. But the oscillations dies out rather slowly so that in most cases ten or more oscillations could be observed without any marked decrease of amplitudes.

It is interesting to apply the above theory to the case of our atmosphere and inquire if there does not exist a similar fluctuation in different cases of convection currents, such as the land and sea breezes, the monsoon, or much larger system of the trade wind and antitrade. In these cases, the assumption above made that the temperature remains constant during the horizontal transport, must be abandoned even if we put aside considerations of adiabatic expansion and condensation, or the elongation of effective length by the deflection due to the rotation of earth. We may assume, for example, that \( \theta_1 \) and \( \theta_2 \) changes during the transport by some amounts proportional to the time required for travelling the distance \( l \), which is nearly equal to \( l/v \), where \( v \) is the mean velocity. Thus we assume that

\[ \theta_1 \text{ changes to } \theta_1 + s_1 \frac{l}{v}, \]
\[ \theta_2 \quad \text{changes to} \quad \theta_2 - s_\gamma \frac{l}{v_\gamma}. \]

Under this assumption, \( R \) must be replaced by

\[ R' = \frac{2}{l} \left( \tau_\gamma - s_\gamma \frac{l}{v_\gamma} \right), \tag{15} \]

where \( s_\gamma = s_\gamma + s_\gamma \).

If \( \tau_\gamma = s_\gamma \frac{l}{v_\gamma} \), \( T \) becomes infinitely large which is evident. In any case, the effect of \( s_\gamma \) is to prolong the period, as is evident from the general consideration. If we put

\[ \tau_\gamma - s_\gamma \frac{l}{v_\gamma} = \gamma \tau \tag{16} \]

\( \gamma \) is a fraction which is smaller than unity and decreases with \( l \). Hence, neglecting \( s_\gamma \), we may obtain the lower limit of \( T \).

Putting \( \varepsilon = 1 \) and \( \varepsilon = 1 \), we obtain for the values of \( l = 1, 10, 100, 1000 \) and \( 10000 \) kms. the corresponding values of \( T = 2.7 \) days, \( 27 \) days, \( 271 \) days, \( 7.4 \) years and \( 74.2 \) years respectively. Hence in any case of the terrestrial convection system, the period is of decidedly higher order than the period of the corresponding heat supply.

It remains, however, to be considered, if it is not possible that the convection system such as the trade wind is divided up into a series of horizontal vortices, with their diameter of the same order of magnitude as the height of the wind layer, and each of these whirls is subjected to the periodic fluctuation as is here considered, in which case the resultant effect will be quite similar to the hitherto considered case. In this case, the corresponding \( \tau \) must be substituted by \( \gamma h/l \), while \( l \) in (13) is replaced by \( h \). Hence in this case

\[ T = \pi \sqrt{\frac{2 \gamma h^3}{g \alpha \tau}}. \tag{17} \]

On applying this to the actual cases, it is found that the above modification is also inadequate for explaining any observed fluctuation of the terrestrial convection system. The observed periodic fluctuation of land and sea breezes, monsoon etc. are therefore not of the character here considered, but probably of purely dynamical origin as is ordinarily believed. (1)

(1) Messrs. U. Doi and T. Ito investigated the fluctuation of weak winds observed during the turning of wind in Tokyo and came to the conclusion that the fluctuation is due to the Helmholtz waves produced at the boundary of wind layers. The result will be published in the Journ. Met. Soc. Japan in a future.
Another possibility still remains that a kind of “combination frequency” may arise by the periodic excitation of these systems by the daily or yearly variation of the solar radiation. This point seems to be worth a further consideration.

Next turning our attention to the solar atmosphere, the matter shows somewhat different aspects.

The reason why the sun’s atmosphere shows different angular velocities for different latitudes has been the subject of many conflicting theories. But, it seems to me highly probable that the phenomena are related somehow with a system of convection current, more or less similar to that in the terrestrial atmosphere. The distribution of the angular velocity will be explained at least qualitatively if we assume two systems of meridional circulation, one corresponding to our system of the trade and antitrade and another in the higher latitude circulating in the opposite sense; above these systems of circulation may be superposed a layer corresponding to our stratosphere. If we are looking upon the system from above the surface, the atmosphere near the equator will be seen leading the rotation of the sun as a whole, while in the higher latitude it will be seen lagging behind the former.

Such a convection could be caused by any absorbing layer near the equator. Such a ‘blanketing’ layer is conceivable, especially in the initial stage of the star’s history. Even in the present stage, the zodiacal light or the coronal rays especially at the minimum of the activity may afford an example of such things. Again, according to Prof. Nagaoka, some portion of corpuscular radiation emitted from the sun’s surface may return to the surface and cause local heating by its bombardment. There are some reasons to believe that such heating may be greater in the equatorial zone than near the poles. Moreover, if the convection is once started, it may eventually be maintained as long as the necessary temperature gradient reigns in the solar atmosphere, since in this case, \( \frac{d\tau}{dt} \) may contain a term proportional to \( \frac{dv}{dt} \) which may compensate the frictional dissipation.

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(1) Emden’s theory gives an utterly different system from that here assumed; but the tacit assumptions therein involved that the radiation is uniform and that the atmosphere is not very thin, are not quite indisputable.

(2) Due to the stirring up of the underlying layer which supplies heat to the hotter column. The case is somewhat analogous to the oscillation of organ pipe maintained by an air current.
Since in the case of the sun, there is apparently no alternation of day and night, nor of the seasons, it seems possible that the periodic fluctuation if once started may continue undisturbed, however long the period may be. This point deserves special notice as an essential difference when compared with the case of the earth.

Calculating $T$ in this case, we obtain for $l = 108 \times 10^7$ km., $g = 25000$, $\alpha = 0.04$,

$$T = 1500 \sqrt{\tau} \text{ years.}$$

Hence, if $\tau = 100^v$, $T = 150$ years.

If on the other hand, (17) holds in this case, and the ratio $l/h$ be of the order $10^3$, we will have $T = 47.5/\sqrt{\tau}$. In this latter case, $\tau = 19^C$ will suffice to give $T = 11$ years.

Assuming that the mean velocity of the current is at most of the order of a few degrees per day, we obtain for the value of $C$ a figure of the order of $\tau \cdot 10^{-7}$.

From the above, it will be remarked that the present idea seems rather promising, though a further development is encountered by difficulties mainly due to the want of the necessary data.

The above idea is not quite irreconcilable with Prof. Fujiwhara's which suggests that the sunspots cycle is caused by the periodic processes of growth and decay of vortices. In the stage where the convection current is gradually speeding up, the vortices may grow by encroaching upon the smaller ones, while in the declining stage of the current, it may gradually be splitting up and dissipating.

At least, it is very probable that the duration of the cycle is determined in some way by the periodic fluctuation of some current caused by the similar thermal origin even if the above conjecture as regard the mode of circulation of the atmosphere may turn out inadequate, since the kinematic viscosity of the solar atmosphere seems to be considerable(1) and the oscillation of the inertial type seems rather improbable, in spite of the larger linear dimensions of the atmosphere compared with ours.

It may be remarked that the present theory has something common with Hahn's theory, in attributing the origin of the cycle to a thermal cause. But the above may, I hope, be of some special interest, inasmuch as it attempts to connect the origin of the periodicity directly with the probable cause of the law of rotation of the solar atmosphere.

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(1) For example, see Newall's recent estimation: Monthly Notices of R.A.S. 82 (1921), p. 119.
Lastly, it may be added to avoid misunderstanding that the latter part of the present paper is not intended to establish any definitive theory of the circulation of the solar atmosphere, but merely introduced in order to draw attention to a possibility which seems to have been utterly neglected hitherto, in spite of its promising features. (1)

(1) Examples of the similar fluctuation of convection current may probably be sought in some natural water basins.