SEDIMENT ENTRAINMENT IN RIPPLED BED

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1. INTRODUCTION: Studies carried out in the past on the sediment transport by suspension have been mostly for an idealized boundary condition of flat bed in unidirectional flow. Recently however, there have been the suggestion that the sediment suspension is closely related to the bed form. The sediment was found to be actively entrained in the outer region of flow from side slope of longitudinal trailing ridge of the ripple, due to the action of horse-shoe vortex and burst. The concentration of sediment in suspension was reported to increase significantly, while the mean shear stress remained essentially unchanged during the development of the three dimensionality of the ripples. The present study has been carried out in the same direction to investigate the inter-relationship between the sediment suspension and the vortices formed behind the lee-side of the ripples.

2. EXPERIMENTAL ANALYSIS FOR SUSPENSION DUE TO A LINE VORTEX: Experiments were carried out by rotating a horizontal rod in its own axis over sand bed in a container. With the starting of the rotation of the rod in the initially still water, the bed getting scoured, the sand starts to get suspended. The approximate expression for the resulting total concentration over the area is given as,

\[ C = A_c \exp(-B_o t) \]

where, \( A_c = A_G \left( 1 - \frac{B_o R_o^2}{12 D_r} - \frac{B_o w R_o^3}{36 \pi D_r^2} \right) \) (1)

where \( A_G \) and \( B_o \) are constants and \( D_r \), the radial diffusivity. By using the experimentally obtained total concentrations \( C \) over the area in ppm unit at different times \( t \), the constant \( A_c \) could be found out. Also knowing the representative radius \( R_o \), from the experimentally obtained bed profiles, \( A_G \) and hence the generation (\( = A_G \exp(-B_o t) \)) is obtained readily. This procedure is repeated for four cases of constant rotating speed, with three different initial bed conditions each. The flow field was measured by a double component X-type hot film probe. For this purpose, the tank was cleaned of sand and an artificial bed was made to simulate the sand bed, by

![Fig.1 Circumferential Vel. Profile](image-url)
using 'Aburanendo'. The bed was made circular, with the radius as the representative radius \( R_0 \) obtained in the experiments in sand bed. The circumferential velocity plotted against \( \frac{\ln r}{r} \) as shown in Fig. 1 indicates that the flow field is indeed that of a line vortex as expected. The circulation corresponding to each angular speed was obtained by fitting
\[
\Gamma = \frac{1}{2\pi r} \left( 1 - \exp \left( - \frac{r^2}{C' \Gamma t} \right) \right); \quad C' \Gamma t = \frac{a}{2.24}^2
\]
where \( \Gamma \): circulation; \( a \): radius of the rod

3. GENERATION OF SUSPENDED SEDIMENT DUE TO A LINE VORTEX: For cylindrical co-ordinate system as shown in Fig. 2, the equation for the slow motion of a spherical particle, in a fluid moving with variable velocity, is written as follows, in \( r \)-dirn:
\[
\begin{align*}
\frac{\pi d^3}{6} \left( \frac{dv_r}{dt} - \nu \frac{d\theta}{dt} \right) &= 3\pi ud(W_r - v_r) - \frac{\pi d^3}{6} \rho \left( \frac{dW_r}{dt} - W_r \frac{d\theta}{dt} \right) + \frac{\pi d^3}{6} \left( \frac{dW_r}{dt} - \frac{dv_r}{dt} \right) \\
&- \frac{W_r}{\theta dt} \left( \frac{dv_r}{dt} - \nu \frac{d\theta}{dt} \right) + \frac{\pi d^3}{6} (\sigma - \rho) g \cos \theta - 20.3d^2 \sqrt{\nu u} |\frac{dW_r}{dt}| (W_\theta - v_\theta)
\end{align*}
\]
in \( \theta \)-dirn:
\[
\begin{align*}
\frac{\pi d^3}{6} \left( \frac{dv_\theta}{dt} + \nu \frac{d\theta}{dt} \right) &= 3\pi ud(W_\theta - v_\theta) - \frac{\pi d^3}{6} \rho \left( W_\theta \frac{d\theta}{dt} + \frac{dW_r}{dt} \right) + \\
&\frac{\pi d^3}{6} \left( \frac{dW_r}{dt} + \frac{dv_r}{dt} \right) - \left( W_r \frac{d\theta}{dt} + \frac{dv_r}{dt} \right) + \frac{\pi d^3}{6} (\sigma - \rho) g \sin \theta
\end{align*}
\]
where \( W_\theta, \nu_\theta, W_r, \) and \( v_\theta \) stand for the fluid and particle velocities in circumferential and radial directions respectively. L.H.S. of the equations represent the force required to accelerate the particle and the various terms in the R.H.S. represent viscous resistance force according to Stoke's law, force due to pressure gradient in the fluid surrounding the particle, caused by the acceleration of the fluid, force to accelerate the added mass of the particle relative to the ambient fluid, submerged weight of the particle, and lift force due to velocity gradient (slip shear effect) respectively. Following Ashida et al.\(^3\) the second terms on the R.H.S. of above equations is replaced by \( C_k^2 \rho u_\ast^2 d^2 \) where \( C_k^2 \) is a constant which in the present case also is taken as 2.0. The rearrangement leads to
\[
\begin{align*}
\frac{dv_r}{dt} - \nu \frac{d\theta}{dt} &= A(W_r - v_r) - B u_\ast^2 + C \cos \theta - D |\frac{dW_r}{dt}| (W_\theta - v_\theta) \\
\frac{dv_\theta}{dt} + \nu \frac{d\theta}{dt} &= A(W_\theta - v_\theta) - B u_\ast^2 + C \sin \theta
\end{align*}
\]
where, 
\[ A = \frac{18u}{d^2 (\sigma + \rho)^2} \quad B = \frac{3pC}{d (\sigma + \rho)^6} \quad C = \frac{(a-p)g}{(\sigma + \rho/2)} \quad D = \frac{120\sqrt{p}u}{\pi d (\sigma + \rho)^2} \]  
(7)

This set of equations of motion are solved for the bed region and central region as follows:

a) **Bed region**: In the region near the bed, \( W_\Theta = \frac{u^* R_0 - R}{\kappa} \ln \frac{R_0}{d} + 8.5u^* \)  
(8)

taking \( R = R_0 + \hat{R} ; \ \Theta = \Theta_0 + \hat{\Theta} ; \ W_\hat{r} = 0 ; \ 2 \frac{d^2 \hat{R}}{dt^2} \frac{d\hat{\Theta}}{dt} \ll \frac{d^2 \Theta}{dt^2} \)

Eqs. 5 and 6 with certain approximation and rearrangement transform to

\[
\frac{d^2 \hat{R}}{dt^2} + A \frac{d\hat{R}}{dt} = R_0 \left( \frac{d\hat{\Theta}}{dt} \right)^2 - \frac{Bu^*}{R_0} \frac{d\hat{\Theta}}{dt} - C \left( \cos \Theta_0 - \hat{\Theta} \sin \Theta_0 \right) - \frac{0.218D(\kappa)}{d} \left( \frac{30}{d} \right)^{1/2}
\]  
(9)

\[
\frac{d^2 \hat{\Theta}}{dt^2} - A \frac{d\hat{\Theta}}{dt} - \frac{C}{R_0} \hat{\Theta} \cos \Theta_0 = \frac{Au^*}{R_0} \ln (1000) - \frac{B}{R_0} u^* \frac{d^2 \hat{\Theta}}{dt^2} + \frac{C}{R_0} \sin \Theta_0
\]  
(10)

These equations can be solved for \( \hat{R} \) and \( \hat{\Theta} \) for initial condition

at \( t=0 ; \ \hat{\Theta} = 0 , \ \frac{d\hat{\Theta}}{dt} = 0 , \ \hat{R} = 0 , \ \frac{d\hat{R}}{dt} = 0 \)
(11)

as

\[
\hat{\Theta} = e^{-At/2} \left[ C_1 e^{Et/2} - C_2 e^{-Et/2} \right] + \frac{H}{G}
\]  
(12)

\[
\hat{R} = \frac{C_3 + C_4 e^{-At} + C_5 e^{-Mt/2} + C_6 e^{-Mt/2} + C_7 e^{-Mt/2} + C_8 e^{-Mt/2} + C_9 t}{C_1 + C_2 e^{-At} + C_4 e^{-Mt/2} + C_5 e^{-Mt/2} + C_6 e^{-Mt/2} + C_7 e^{-Mt/2} + C_8 e^{-Mt/2} + C_9 t}
\]  
(13)

where \( C_1 = -\frac{HM}{2Ge^t} ; \ C_2 = -\frac{HM}{2Ge^t} ; \ M = A-E ; \ N = A+E ; \ H = \frac{Au^* \ln (1000)}{R_0} - \frac{B}{R_0} u^* \frac{d^2 \hat{\Theta}}{dt^2} + \frac{C}{R_0} \sin \Theta_0 \)

\( G = -\frac{C}{R_0} \cos \Theta_0 ; \ C_3 = -(C_4 + C_5 + C_6 + C_7) ; \ C_4 = (C_5 - MC_6 - NC_7 + MC_8 + NC_9 - NC_1/2 + NC_2 + NC_3) / A \)

\( C_5 = \frac{P_1}{N(N-A)} ; \ C_6 = \frac{P_2}{N(N-A)} ; \ C_7 = \frac{4P_3}{N(N-A)} ; \ C_8 = \frac{4P_4}{N(N-A)} ; \ C_9 = \frac{P_5}{A} \)

with \( P_1 = \frac{R C_1^2}{4} ; \ P_2 = \frac{R C_1^2}{4} ; \ P_3 = -CC_1 \sin \Theta_0 ; \ P_4 = -CC_1 \sin \Theta_0 ; \)

\( P_5 = C_1 \cos \Theta_0 - Bu^* - C_1 \sin \Theta_0 \frac{H}{G} - \frac{0.218D(\kappa)}{d} \left( \frac{30}{d} \right)^{1/2} - R_0 C_1 C_2 MN \)

Assuming that the bed region and the central region can be demarcated by \( \hat{R} = \hat{R}_{\text{crit}} = 10d, \frac{d\hat{\Theta}}{dt}, \hat{\Theta}_{\text{crit}}, \) and \( \frac{d\hat{R}}{dt} \) can be found and used as the initial condition for the central region.

b) **Central region**: The corresponding equation of motion in the central region with \( W_r = 0 ; \ W_\Theta = \frac{\ln r}{r} \), is written after simplification as

\[
\frac{d^2 \hat{R}}{dt^2} + A \frac{d\hat{R}}{dt} = R_0 \left( \frac{d\hat{\Theta}}{dt} \right)^2 + C \left( \cos \Theta_0 - \hat{\Theta} \sin \Theta_0 \right)
\]  
(14)

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The solution of which can be obtained in the similar form. The range \( \Theta_1 - \Theta_2 \) from which the suspension occurs can be obtained from above solutions. The particle is considered to go into the suspension when it reaches above the original bed level.

The amount of generation per unit area per unit time occurring from this range \( \Theta_1 - \Theta_2 \), is obtained with the approach of Ashida et al. with some modification as

\[
\frac{d^2 \Theta}{dt^2} - \frac{d \Theta}{dt} - \frac{C}{R_o} \dot{\Theta} \cos \Theta = \frac{A \dot{\Theta}}{R_o} \frac{\ln R_o}{\kappa} + \frac{C}{R_o} \sin \Theta
\]

(15)

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The amount of generation per unit area per unit time occurring from this range \( \Theta_1 - \Theta_2 \), is obtained with the approach of Ashida et al. with some modification as

\[
Q = \frac{2}{3} \rho K \sqrt{\frac{\Theta}{\pi}} (s+1) u^*_w R_o \left[ \Theta \max \left( \frac{C}{k} \delta - \frac{\pi}{8} C D_{o} \sin \Theta \right) \exp \left( -\frac{\delta^2}{2}\right) d\Theta \right]
\]

in \( \text{gms/cm. width/sec} \)

(16)

4. COMPARISON OF GENERATION RATE FOR A LINE VORTEX: Using the experimentally obtained values of mean \( R_o \) from the bed profiles and \( \beta \) and \( u^*_w \) from the velocity measurements, the theoretical values of the generation is obtained. The results compare well with the experimentally obtained values, as can be seen in Fig. 3.

5. ENTRAINMENT IN RIPPLED BED:

An example of flow pattern that is encountered in the channel with rippled bed is shown in Fig. 4. The vortex structure is very complicated with curving of its axis and stretching. However, as an approximation, it is modelled by a line vortex of certain constant intensity to arrive at an expression for the entrainment in the channel which can take into account the effect of bed form and depth of flow. The sediment in suspension is considered to be due almost entirely to entrainment caused by the vortex on the lee side. It is assumed that the profile of the bed from which the entrainment into the flow can take place due to the action of vortex, can be approximated to a circular arc with its center at a point where the perpendicular bisector of the lee face meets the vertical line passing through the point at a distance \( q \delta \) from reattachment point. Denoting the horizontal distance between crest and toe of the ripple by \( p \delta \), and that between toe and center by \( .5 p \delta \),

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Fig. 3 Expt. & Theo. Results Compared

Fig. 4 Flow pattern over Ripple Bed
\[ R_0 = \Delta (c.5 + \cot^2 \phi); \quad \theta_{\text{max}} = \cos^{-1} \{1-\Delta/R_0\} = 62^\circ \text{ for } \phi = 40^\circ \] (17)

\[ p = \frac{\Delta}{\Lambda} \cot \phi; \quad (q\Delta)^2 + \left[ R_0 + \frac{q+1.5p}{1-p} \right]^2 = R_0^2; \quad \theta_{\text{min}} = -\sin^{-1}(q\Delta/R_0) \] (18)

where \( \phi \): angle of repose, \( \Delta \): height of the ripple, and \( \Lambda \): length of the ripple. Following the same procedure explained before, the limit angles \( \theta_1 \geq \theta_{\text{min}} \) and \( \theta_{\text{max}} = 62^\circ \), from where the entrainment occurs, under the action of \( u^* \), provided the vortex is continuously present, can be obtained. In achieving this, the proportionality constant \( \Gamma \) for the velocity profile, \( v_\theta \propto \frac{\ln r}{r} \) is taken as the experimentally obtained empirical result, \( \Gamma = 0.473T - 104 \) (19)

where \( \Gamma \) the circulation is given by

\[ \Gamma = \frac{1}{2} U_{\text{sep}} T_e \quad \text{with} \quad T_e = \frac{2\Lambda}{SU} \quad \text{and} \quad U_{\text{sep}} = 1.31U \] (20)

where the Strouhal Number \( S \) is taken as 0.15 and \( U \) is the average velocity in the channel, and \( U_{\text{sep}} \), the velocity at separation point. The total energy gradient in the flow over the rippled bed can be considered as the sum of the gradients due to the head loss due to the skin friction along the stoss side and the head loss due to the expansion on the lee side i.e. \( S = S' + S'' \); which leads to,

\[ \frac{U}{u^*} = \frac{1}{\Lambda} \frac{\ln \frac{11h}{d}}{\sqrt{1 - \frac{\Delta}{\Lambda} \left( \cot \phi - \frac{1}{2} \left( \frac{1}{\kappa} \ln \frac{11h}{d} \right)^2 \right)}} \quad \text{and} \quad u^* = \frac{\sqrt{gS''h}}{\sqrt{2h\Delta}} = \frac{U \Delta}{\sqrt{2h\Delta}} \] (21)

The amount of entrainment per unit time per unit width due to assumed line vortex behind the ripple, if always present, is then given by Eq.16 with \( u^* \) replaced by \( u^* \). However, the vortex behind the ripple is not present continuously, but is intermittent in time as well as space. If \( f \): frequency of occurrence of vortex per unit area, \( T \): period of occurrence of boil, \( A_r \): representative area from which one boil occurs, then \( f A_r = 1/T \). Also, \( UT/h = \text{constant} \), where the value of constant is approximately 6. Further if \( T_e \): duration of existence of vortex, and \( n \): number of vortex in certain area \( A_v \) at any time \( n/A_v = f T_e = T_e U/(A_r 6h) \). Representative area can be taken as ripple area which can be approximated as \( A_r = \text{const.} \Delta^2 \), with const. =1.

\[ n/A_v = 2(\Delta/\Lambda)(K'_{u}h\Lambda) \] (22)

where \( K'_{u} = 0.9 \). Non-dimensional volumetric entrainment is then given by

\[ E_s = \frac{Q}{su} \frac{n}{A_v} \times \text{representative length of axis of vortex} \] (23)
As an approximation, representative length of axis of the vortex can be taken as $\hat{A}$, with any uncertainty being absorbed by the constant. Substitution for $Q_{su}$ then leads to

$$E_s = \frac{A1}{A \xi_0 3nK_u} \sqrt{\frac{3}{s+1}} \frac{R_o}{h} \int_0^\theta \int_0^{\theta \max} \frac{C_k n}{\xi_0} - \frac{\pi C_k \xi_0^2 \cos \theta}{8 \xi_0} \exp \left(-\frac{n}{2}\right) \, d\theta$$  

(24)

6. RESULTS AND DISCUSSION: The above analysis is applied to obtain the entrainment for various values of $u_*$ in the case of three different sand diameters, at different non-dimensional water depths $Z$. These analyses are carried out only for those values of the parameters $u_*$ and $Z$ which fall in the ripple region. The values of $u_*$ fall in the reported range of the fluctuation in the vertical velocity near the trailing ridge. The results obtained are shown in Fig. 5, in terms of the reference level concentration ($E_s$) along with other established relations. The entrainment is found to be weakly dependent on the depth of flow, and attains a maximum at certain $u_*$. It can be inferred from the results that the Fig. 5 Reference Level Concentration entrainment rate depends on the stage of development of the ripple.

7. CONCLUSION: An expression for the entrainment rate in the channel in the presence of ripple bed has been arrived at. The results are found to compare reasonably well with the observed data. Indeed the values of various constants used can be improved upon, if the reliable information is available to compare the entrainment with. However, the expression obtained itself can be said to reflect the various phenomena present in the process of entrainment in the channel.

REFERENCES:
6. Tamai N. et al., 1985, Ann. Conf. JSCE.