Towards Modeling Sediment Yield and Transport in Tropical Arid & Semi-Arid Lands (ASAL)

By

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Abstract

Scarcity of data in sediment budget for arid and semi-arid lands (ASAL) is a recognized fact despite their environmental fragility. There is therefore urgent need for exchange of current knowledge in this field for effective conservation. We present a comparison among published and proposed models for computation of sediment yield and transport due to rainfall runoff for a single storm by using data from 8 storm events of 1986 in Katorin experimental basin which is located in a semi-arid area of Kenya. We categorize the models into black, grey and white box on the basis of their consideration for physical principles.

Keywords: sediment yield, pick-up rate by rainfall-runoff, stream power.

1. INTRODUCTION

Sediment yield by rainfall runoff is caused by rainfall as the primary and the resulting overland flow as the secondary erosive agent. The simple cause-effect phenomenon would therefore give a black box model whose input is rainfall or flow and sediment transport as the output. We classify such like models, in Fig. 1, as black box: RBB and FBB which signify rainfall and flow as the representative parameter respectively. We next classify models which partially explain the erosion phenomenon by grouping two or more parameters of rainfall, slope, slope length and bed material characteristics developed from statistical considerations with or without consideration of dimensional homogeneity as grey models: RGB and FGB. When homogeneity is considered in the empirical developments, the models lie on the borderline between grey and white box models. Finally we consider models derived from physical principles by mathematical analysis of the raindrop impact and flow dynamics as white box models: RWB and FWB.

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Fig. 1 Classification of Sediment Yield and Transport Models

2. BRIEF DESCRIPTION OF REPRESENTATIVE MODELS

2.1 Black Box Models for Katorin Experimental Basin

Bryan et al. (1988) developed two empirical models one based on rainfall volume (which we code **RBB**) and the other in the form of a rating curve at the outlet of Katorin basin (**FBB**) expressed respectively as follows;

$$S_t = 0.049V_p - 157 ; C_s = 42.7Q^{0.281} \quad (1a, b)$$

in which S_t = suspended sediment load in tons per storm, V_p = rainfall volume in m^3 , C_s = suspended sediment concentration in g/l and Q = water discharge in m^3/s .

2.2 Rain Based Grey Box Models

Ellison (1947) pioneered the development of an empirical model (**RGB1**) which related rainfall parameters to total sediment yield as follows without consideration of dimensional homogeneity;

$$E_e = K_e V_e^{4.33} D^{1.07} I^{0.65} \quad (2)$$

in which E_e = the amount of soil in tons splashed during a 30-minute period, K_e ($= 3.8 \times 10^{-5}$) is constant of the soil, V_e = drop velocity in ft/s , D = drop diameter in mm and I = rainfall intensity in in/hr .

Based on dimensional analysis of a raindrop impact on the water surface, Makhani et al. (1995) proposed the following model (**RGB2**) for computation of bed load transport and expressed in non-dimensional form as;

$$qB / \left[\sqrt{(\sigma/\rho - 1)gd^3} \right] = f_L \cdot D_r^s \cdot [1 - (D_{rc}/D_r)]^n ; D_r = \frac{w_f^2}{(\sigma/\rho - 1)gd} \cdot \frac{r \sin \theta}{\sqrt{gd}} ; f_L = K \left[\left(\frac{L}{d} \right)^c \cdot \tan \theta \right]^e \quad (3a,b,c)$$

in which s ($= 1.75$), c ($= 2.0$), e ($= 1.3$) and n ($= 0.6$) are experimental constants and terminal raindrop velocity in m/s is given by Best(1950) as: $w_f = 9.32 \left\{ 1 - \exp \left[- (D/0.177)^{1.147} \right] \right\}$ for D in cm . The critical rainfall impact parameter D_{rc} is computed by Eq. (3b) for $r \equiv r_c$ from Eq. (4b), w_{fc} = the raindrop final impact velocity for r_c and K ($= 2.2 \times 10^{-13}$) is a scale parameter to be determined for the particular basin. The rainfall intensity at critical conditions r_c is determined from the value of bed shear stress τ_{*s} necessary to sustain a detached particle in motion. By considering the forces on the particle at stopping instant the value of τ_{*s} ($= 0.013$) is determined. Other parameters are expressed as follows;

$$\tau_{*s} = \frac{\sin \theta}{\sigma/\rho - 1} \cdot \frac{h(L)}{d} ; r_{ce} = r_c - r_i = \alpha \cdot \frac{h(L)^m}{L} ; \alpha = \sqrt{\frac{8}{f} g \sin \theta} ; \sqrt{\frac{8}{f}} = B_r + \frac{1}{\kappa} \ln \frac{h(L)}{d} - \frac{1}{\kappa} \quad (4a,b,c,d)$$

For a given value of bed material d and slope gradient, the water depth at the end of the slope length $h(L)$ is obtained from Eq. (4a) and hence the net rainfall intensity corresponding to this condition r_c is evaluated by Eq. (4b) in which r_i = infiltration rate, r_{ce} = effective rainfall at critical conditions, $m = 3/2$, B_r ($\equiv 8.5$) is constant of integration in the log-law velocity profile, f = friction factor and κ ($\equiv 0.41$) is the von Karman constant.

2.3 Flow Based Grey Box Models

We propose a bed load model based on stream power which is expressed as follows (**FGB**);

$$q_* I_* = \frac{rL}{\sqrt{(\sigma/\rho - 1)gd^3}} \cdot \frac{i_b}{\sigma/\rho - 1} \quad (5a)$$

$$q_{Be*} = \frac{q_{Be}}{\sqrt{(\sigma/\rho - 1)gd^3}} = A [q_* I_* - (q_* I_*)_c]^\gamma \quad (5b)$$

in which $q (= rL)$ is unit discharge, $A (= 0.0014)$, i_b = bed slope, $\gamma (= 5/3)$ and $(q_* I_*)_c (= 0.981)$ are calibration constants (Fig. 2). Though $\gamma (= 5/3)$ is the same case of the fluvial channel, A and $(q_* I_*)_c$ are different

from the values for fluvial channel because of the rainfall impact.

Fig. 2 Empirical Curve for Sediment Transport Rate Based on Stream Power

2.4 White Box Models Based on Rainfall and Flow

In order to model overland flow on a plane, we define the typical slope length L in white box models based on a kinematic cascade. Thus, the basin is sub-divided into an appropriate number of elements each with an approximate length L . For Katorin basin, Bryan (1988) subdivided the basin into 18 elements each with a third order stream being the main channel. The typical length of slope was 50 to 80 m. We thus adopt $L = 50$ m for the 18 elements in this investigation on white box models. We replace the water depth by the mean overland flow depth in bed load transport models such as that of Ashida et al. (1978) (**FWB1**) as follows;

$$q_{Be*} = \frac{12 - 24\sqrt{i_b}}{\cos \theta} \cdot \tau_*^{(1.5 - \sqrt{i_b})} \left(1 - \alpha_o \cdot \frac{\tau_{*c}}{\tau_*}\right) \left(1 - \alpha_o \cdot \sqrt{\frac{\tau_{*c}}{\tau_*}}\right); \tau_* = \frac{\sin \theta}{\sigma/\rho - 1} \frac{h_m}{d}; \tau_{*c} = 0.04 \times 10^{1.72 \tan \theta} \quad (6a,b,c)$$

in which τ_* = non-dimensional bed shear stress, τ_{*c} = critical non-dimensional bed shear stress and $\alpha_o (= 0.92)$ is an empirical constant. The depth at the end of the slope $h(L)$ and the mean flow depth are approximated from the kinematic flow hydrograph as follows for $m = 3/2$;

$$h(L) = (r_e L / \alpha)^{1/m}; h_m = 0.6h(L) \quad (7a,b)$$

Based on the hydraulics of overland flow, Komura (1976) proposed the following model (**FWB2**);

$$E_v = (K_k C_A C_E / d_m) (f' r)^{15/8} L^{3/8} S_o^{3/2} \quad (8)$$

in which E_v = slope erosion rate in $m^3/hr/m^2$, $K_k (= 1.8 \times 10^{-11})$ is a calibration constant, d_m = mean sediment size of the slope material in mm, $C_A (= 0.3)$ is the bare soil area ratio, $C_E (= 10)$ is the erodibility coefficient, f' = the runoff coefficient, L = slope length in m, S_o = bed gradient and r = gross rainfall intensity in mm/hr.

We propose the following mathematical model based on detachment rate by rainfall and by flow as separate sub-systems with a common deposition process through the same step length Λ ($\approx 50 \sim 100d$), (**FWB3**);

$$q_B(x) = q_{Br}(x) + q_{Bf}(x); \frac{dq_{Br}(x)}{dx} = \frac{q_{Bre}(x) - q_{Br}(x)}{\Lambda}; \frac{dq_{Bf}(x)}{dx} = \frac{q_{Bfe}(x) - q_{Bf}(x)}{\Lambda} \quad (9a,b,c)$$

in which the equilibrium sediment transport rates are uniquely related to the bed shear stress as follows;

$$q_{Bfe}(x) = \frac{A_3}{A_2} p_{sf}(x) \Lambda d; q_{Bre}(x) = \frac{A_3}{A_2} p_{sr}(x) \Lambda d \quad (10a,b)$$

The pick-up rate due to flow depth (sub-script f) of Nakagawa and Tsujimoto (1980) is as follows;

$$p_{sf*}(x) = p_{sf}(x) \sqrt{d/[(\sigma/\rho - 1)g]} = F_o \tau_*(x) \left\{1 - \frac{k_2 \tau_{*c}}{\tau_*(x)}\right\}^m; \tau_*(x) = \frac{\sin \theta}{\sigma/\rho - 1} \frac{h(x)}{d}; h(x) = (r_e x / \alpha)^{1/m} \quad (11a,b,c)$$

Fig. 3 Location of Kiatorin Experimental Basin

in which $F_0 = 0.03$, $k_2 = 0.7$ and $m' = 3.0$. The pick-up rate due to rainfall impact (sub-script r) is given as follows based on the probability of a sand particle being detached within the area of raindrop impact;

$$p_{sr}(x) = p_{sr}(x) \sqrt{d/[(\sigma/\rho-1)g]} = \frac{A_2}{A_3} \cdot \frac{r}{\sqrt{(\sigma/\rho-1)gd}} \left(\frac{d}{D}\right)^3 \cdot N_o(x) \quad (12)$$

in which A_2 , A_3 ($= \pi/4$ and $\pi/6$ for a sphere respectively) are two and three-dimensional geometric coefficients respectively and $N_o(x)$ = the number of detached particles at distance x in the direction of flow which is expressed as follows based on mechanics of flow by Fukada et al. (1992);

$$N_o(x) = \left(\frac{D}{d}\right)^3 \frac{A_k}{(\sigma/\rho-1)gH_{\max}} \frac{w_f^2}{gH_{\max}} \left\{ 0.38 - 0.79A_f^{1/3} \left(\frac{h(x)}{H_{\max}}\right)^{2/3} + 0.33A_f \left(\frac{h(x)}{H_{\max}}\right)^2 \right\}; \quad H_{\max} = (2E_o/\rho g \pi)^{1/4} \quad (13a,b)$$

in which $E_o \equiv 1/2 \rho A_3 D^3 w_f^2$ in J is the kinetic energy of rainfall, $A_f \equiv I_o / \sqrt{gH_{\max}^3}$, $A_k = 0.028$ and $I_o = 0.72$.

3. KATORIIN EXPERIMENTAL BASIN

Kiatorin experimental basin at an altitude of 947 m above sea level is located on the west of Lake Baringo in Kenya at latitude $0^\circ 33' N$ and longitude $36^\circ E$ (Fig. 3). It has an area of 0.33 km^2 and oblong with a shape factor of about 0.65. The length of the basin is about 1300 m, with a perimeter of 3000 m and average width of 200 m. Two river gauging stations are located for the upper sub-basin with an area of 0.21 km^2 and the lower sub-basin which is 0.09 km^2 . The upper sub-basin has a general slope of $10^\circ \sim 26^\circ$ with slope lengths of less than 50m. The soils are mainly silty clay with minor sands which generate much runoff when too dry due to surface sealing or when wet. The soils in the lower sub-basin constitute a thin alluvial layer with sodic smectile and rich in mudstone. The main river channel in the lower sub-basin has a slope of $2^\circ \sim 3^\circ$ with general slope lengths of less than 80 m. The basin relief is 0.063 with a median grain diameter of about $d_{50} = 1.4 \text{ mm}$ with a mass density $\sigma = 1500 \text{ kg/m}^3$. The basin had a vegetation cover of about 20 % in 1986 being composed mainly of annual grasses and shrubs. The Manning's resistance coefficient was about $0.018 \sim 0.04$.

Table 1 Details of Rainfall Runoff and Sediment Transport Rates (adapted from Bryan, 1988)

Storm (1986)		r (mm/hr)	t _R (min.)	f'	Q _{max} (m ³ /s)	**			***		
No.	Date					q' _s	q' _B	q' _T	q _s *	q _B *	q _T *
1	11 April	66	18.2	0.56	2.80	553	47	600	3.783	0.322	4.104
2	28 April	42	38.6	0.49	1.40	430	37	467	1.387	0.119	1.506
3	15 May	42	40.0	0.48	0.89	218	31	249	0.678	0.096	0.775
4	28 July	36	33.3	0.35	2.20	490	22	512	1.832	0.082	1.914
5	11 Aug.	48	25.0	0.38	1.80	332	24	356	1.653	0.120	1.773
6	12 Aug.	12	32.0	0.66	1.50	340	28	368	1.323	0.109	1.432
7	13 Aug.	60	28.0	0.66	2.10	810	40	850	3.602	0.178	3.780
8	14 Aug.	30	26.0	0.29	0.30	48	12	60	0.230	0.058	0.287

** = measured sediment discharge in t/km²; *** = non-dimensional sediment discharge.

The annual rainfall regime in Katorin basin is bi-modal with peaks in April/May and July/August during which 27 and 28 % of the mean annual rainfall of 650 mm is received respectively. The annual potential evapotranspiration is 1900 mm. Thus based on the U_{ai}, UNESCO aridity index which classifies arid climates on the basis of rainfall to evapotranspiration ratio of less than 0.5, it is a semi-arid climate with U_{ai} = 0.34.

4. DETAILS OF MEASURED RAINFALL, DISCHARGE AND SEDIMENT YIELD

The last three columns in Table 1 for non-dimensional sediment discharge per unit width were computed based on total length of basin L' = 1300 m, d₅₀ = 1.4 mm and σ = 1500 kg/m³). The maximum 10 minute rainfall intensity is used as the representative rainfall intensity since the time of concentration t_c for the basin is 10 ~ 15 minutes. However, t_c is also dependent on storm dynamics (Makhanu, 1991). The effective rainfall duration t_R was computed to keep the total amount of rainfall constant as those used by Bryan (1988) in Eq. (1a).

$$q_{B*} = q'_B \cdot F_a; \quad q_{s*} = q'_s \cdot F_a; \quad q_{T*} = q'_T \cdot F_a; \quad F_a = L'/1000\sigma(60t_R)\sqrt{(\sigma/\rho - 1)gd^3} \quad (14a,b,c, d)$$

in which q'_s, q'_B, q'_T = the measured suspended, bed load and total sediment yield from the basin respectively in tons per km² per storm, t_R = equivalent rainfall duration time in minutes which would give the same amount of rainfall volume for t_R as for the mean rainfall intensity and the actual rainfall duration and sand diameter d is used interchangeably with the median sand diameter (d ≡ d₅₀). The ratio of bed load to suspended load for this basin was about 7 % for the 1986 storms. The observed critical rainfall intensity for no sediment discharge was 4 ~ 6 mm/hr whereas by Eq. 4(b) it was found to be about 3 mm/hr.

5. RESULTS AND DISCUSSION

Results of sediment transport rates are shown in Fig. 4. Two parameters are further adopted:

$$Score = 1 \text{ for } 0.5 \leq \frac{q'_{B*i}}{q_{B*i}} \leq 2.0; \quad R = 1 - \frac{\sum_{i=1}^N (q'_{B*i} - \bar{q}_{B*})^2}{\sum_{i=1}^N (q'_{B*i} - q_{B*i})^2}; \quad \bar{q}_{B*} = \frac{1}{N} \sum_{i=1}^N q_{B*i} \quad (15a,b,c)$$

in which q'_{B*i}, q_{B*i} = computed and measured sediment transport rates as either total or bed load for the i-th storm respectively. The Score indicates nearness to actual measured value, N = number of storms and R indicates consistency among the different storms. The Score and R respectively for each model were as follows: RBB(6, 0.366); FBB(8, 0.681); RGB1(5, 0.353); RGB2(7, 0.672); FGB(7, 0.577); FWB1(4, 0.320); FWB2(5, 0.488) and FWB3(7, 0.596). In Fig. 4(a) the one-term dimensional models are simple to calibrate but they have limited transferability to another basin or season beyond the water year of calibration. The dimensional models in Fig. 4(a) are not as flexible as that in Fig. 4(b). Model RGB2 was superior to FGB1. Further, model RGB2 and FGB performed almost equally. The white box models based on average bed shear stress (FWB1), based on bare area ratio (FWB2) and on pick-up rate of rainfall impact and flow (FWB3) are shown in Figs. 4(e), (f) and (g) respectively. Model FWB3 was better in comparison to the other two white-box models (FWB1 & FWB2) in both accuracy and consistency.

[Conversions: $q_{B*} \equiv 0.08q_{T*}$; $q_{S*} \equiv 0.92q_{T*}$]

(Note: Sediment transport rates reported as bed load except for models RBB, FBB & RGB1 where rates are reported as total load)

Fig. 4 Results of Model Computations for the Storms of 1986 in Katorin Experimental Basin, Kenya

6. CONCLUSIONS

Various models for prediction of sediment yield and transport classified according to the consideration of physical factors are discussed. The performance of the models is evaluated by use of field data from the tropical ASAL of Eastern Africa. With correct calibration and determination of principal parameters such as length of slope, soil cover and elements in which morphological factors may be uniform, it is possible to select suitable models from these or others that may predict sediment transport rates fairly accurately. The results of this paper may contribute towards filling the gap of information in the area of sediment budget for tropical arid and semi-arid climates.

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