Quadrupolar analysis of an interdigitated micro-electrode array for
dielectrophoretic particle transport

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Dielectrophoresis is the motion of the particles due to the interaction between a non-uniform electric field and its induced multipole moments in the particle. Here a numerical solution for the gradient terms in the electric field strength, generated by interdigitated bar micro-electrodes is derived using the finite element method and numerical analysis. These gradients of the electric field produced by such electrodes are mainly effective on the magnitude of the DEP force exerted on micro/nano-particles. In this paper we introduce the relationship between the geometry, the applied voltage and frequency and the height above the interdigitated micro-electrodes with the magnitude of the DEP force for non-uniform electrode widths and gaps by considering the basic dielectrophoresis. In order to explore the effects of higher-order multipoles, a correction factor was added to the basic dipolar DEP force. Based on the simulations, it was found, that the multipole contributions to the induced DEP force acting on particles can be very important specially for the particles of larger radii at lower heights, and also for micro-electrodes having smaller gap/width sizes. The numerical solutions developed in this paper are in an excellent agreement with previous experimental and theoretical reports of particle levitations above the interdigitated micro-electrodes.

(1) Introduction

Electrical-field-induced forces on polarizable particles have been the subject of extensive studies in recent two decades because of their potential applications for particle characterization and manipulation. AC electrokinetics is the study of particle movement arising from the interaction of a non-uniform AC electric field with polarizable particles [1-3]. Particles, including biological cells, become electrically polarized under the influence of applied fields. The induced polarizations can in turn interact with applied fields, resulting in net electrical forces on the particles. Because of the sensitive dependence of these forces on particle dielectric characteristics, these forces can be exploited for particle characterization and separation, trapping and sorting particles such as cells, bacteria, and viruses [4-6]. One common DEP device consists of a thin chamber constructed with a bottom wall that supports a planar interdigitated micro electrode array. The device is filled with a suspending medium into which the sample of particles is introduced. Under AC electrical excitation, the array will produce height-dependent dielectrophoretic forces on particles in the chamber. Current theories for DEP force predictions neglect the effects of multipolar DEP terms and consider that the basic dielectrophoresis. In order to explore the effects of higher-order multipoles, a correction factor was added to the basic dipolar DEP force. Based on the simulations, it was found, that the multipole contributions to the induced DEP force acting on particles can be very important specially for the particles of larger radii at lower heights, and also for micro-electrodes having smaller gap/width sizes. The numerical solutions developed in this paper are in an excellent agreement with previous experimental and theoretical reports of particle levitations above the interdigitated micro-electrodes.

(2) General Equations and Assumptions

By considering the \( \{\tilde{F}_{\text{DEP}}\} \) based on dipole approximation as in Eq. (1), one can construct the desired expression to evaluate the dipolar contribution to the levitation height. Considering the fact that at heights higher than the maximum of the gap and electrode width, the gradient term of the square of the electric field is almost uniform horizontally, it can be defined as a function of the height above the electrodes. So that the levitation height of the particles can be determined as

\[
    h_{\text{lev}} = \nabla E_{\text{rms}}^2 \cdot \left[ \frac{2(\rho_e - \rho_m) k}{3\varepsilon_0 \text{Re}(f_{\text{CM}})} \right].
\]

where \( \nabla E_{\text{rms}}^2 \) is the inverse function of \( \nabla E_{\text{rms}}^2 \). All but one of the terms in this equation are typically easy to obtain. The permittivities of the particles and medium are usually either known or easily measurable, while the polarizability factor is obtained and maximized from proven mathematical formulae. In contrast, the gradient of the square of the applied electric field \( \nabla E_{\text{rms}}^2 \) can only be calculated experimentally or analytically through complicated numerical simulations. However, we are interested in extending this result to include the effects of higher-order multipolar contributions. To consider the contributions of higher-order quadrupolar DEP force to the basic DEP theory, Eq. (1) should be extended into the following form [8]:

\[
    \{\tilde{F}_{\text{DEP}}\} = \frac{2\pi\varepsilon_0 E_{\text{rms}}^2}{\text{Re}(f_{\text{CM}})},
\]

where \( \varepsilon_0 \) is the effective dielectric permittivity of the medium in which the particle is suspended, \( E_{\text{rms}} \) is the root mean square (RMS) value of the field strength for an applied voltage \( V_{\text{rms}} \), and \( \text{Re}(f_{\text{CM}}) \) is the real component of \( f_{\text{CM}} \), the Clausius-Mossotti factor. \( f_{\text{CM}} \) reflects the magnitude and direction of field-induced polarization in the particle at a certain frequency, and is given by

\[
    f_{\text{CM}} = \frac{(\varepsilon_e - \varepsilon_m) (\varepsilon_e + 2\varepsilon_m)}{(\varepsilon_e - \varepsilon_m)/2 + \varepsilon_m}.
\]

where \( \varepsilon_e \) and \( \varepsilon_m \) are the frequency-dependent complex dielectric permittivities of the particle and its suspending medium.
The time-averaged DEP force including both the dipolar and quadrupolar contributions can be estimated by

\[
\langle \mathbf{F}_{\text{DEP}} \rangle = 2\pi \varepsilon \varepsilon_0 \text{Re}(f_{\text{CM}}) \nabla E_{\text{ext}}^2 + 2\pi \varepsilon \varepsilon_0 \text{Re}(f_{\text{CM}}) \nabla (\nabla E_{\text{ext}} : \nabla E_{\text{ext}}) \tag{3}
\]

Here \( f_{\text{CM}} = (\xi_n - \xi_w)/(2\xi_n + 3\xi_w) \) is the quadrupolar species polarizability, which permits the inclusion of higher-order quadrupolar moment, being related to the contribution from the quadrupole expansion. From the conclusions made up to this point, the dipole gradient term can be expressed as

\[
\nabla E_{\text{ext}}^2 = \frac{-128}{3\pi d^4(1 + w/d)} \cos^2 \left( \frac{\pi(w/d)}{2(1 + w/d)} \right) \times \mathcal{P}(j) V_{\text{ext}}^2 \exp \left( \frac{-2\pi y}{1 + w/d} d \right) \tag{4}
\]

To assess the importance of the quadrupolar contribution to the DEP force, we should determine the dependencies of the quadrupolar gradient which is the gradient of the scalar product of the gradient field intensity term, \( \nabla (\nabla E_{\text{ext}} : \nabla E_{\text{ext}}) \), to the dipolar gradient term, \( \nabla E_{\text{ext}}^2 \), gap width and electrode size of the micro-electrode array, and also the height above the micro-electrodes where the particles are levitated. The time-averaged DEP force including both the dipolar and quadrupolar contributions can be estimated by

\[
\langle \mathbf{F}_{\text{DEP}} \rangle = -256 \varepsilon_n \text{Re}(f_{\text{CM}}) + 10.406 \text{Re}(f_{\text{CM}}) y^2 d^{-138} \times \left( 1 + w/d \right)^{-137} y^{-138} \times (r/d)^3 \left( 1 + w/d \right)^6 \times \cos^2 \left( \frac{\pi(w/d)}{2(1 + w/d)} \right) \mathcal{P}(j) V_{\text{ext}}^2 \exp \left( \frac{-2\pi y}{1 + w/d} d \right) \tag{5}
\]

To get better understand the importance of the quadrupolar contribution to the DEP force, we compare the ratio of the time-average quadrupolar DEP force to the DEP force considering both dipolar and quadrupolar effects, \( (\langle \mathbf{F}_{\text{DEP}} \rangle - \langle \mathbf{F}_{\text{DEP}^{\text{dip}}} \rangle) / \langle \mathbf{F}_{\text{DEP}} \rangle \), as a function of the vertical position above the electrode arrays for latex beads of 3 and 6 µm; see Fig. (2). The interdigitated electrode array consists of electrodes ranging from 5-40 µm gap with spacing equal to the width and is energized by a 8 Vp-p, 1.0 MHz AC signal.

### (3) Result and Discussion

A simple formula was derived to express the DEP dipolar and quadrupolar gradient terms above interdigitated micro-electrode arrays. The effects of the micro-electrode dimensions, applied voltage and frequency were also considered. Equation (5) can be effectively used to predict the possible responses of particles in microfluidic devices for micro-electrode design purposes. While most of our comparisons were done for the case of uniform interdigitated micro-electrodes, Eq. (5) can generally be used for micro-electrodes with different gap-to-width ratios. By having the DEP gradient term values above interdigitated micro-electrode arrays, the DEP force which applies to particles can be easily determined from Eq. (18). By having the amount of the force exerted on particles, the particle dielectrophoretic movements can be predicted which can be used for particle controlled manipulation purposes. Comparisons with previously published experimental [9] and theoretical [10] data on DEP levitation of particles show good agreement with the simulation results.

### References