Analysis of Discharge Parameters for EDM Using the LC Pulse Generator
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Abstract
The LC generator has many advantages in electrical discharge machining (EDM). It can generate high open voltage for EDM gap and high discharge current pulse with short duration with very low voltage input. The shape of current pulse generated by the LC generator with higher rising speed shows superiority in machining with efficiency higher than the conventional RC generator. However, the pulse shapes of discharge current, relative peak value and duration of open voltage need to be analysed. This paper aims to investigate the influential parameters of the LC generator to analyse the discharge current and open voltage for EDM. The results of investigation are proved to agree with the real discharge experiments.

Key words: Electrical discharge machining (EDM), LC pulse generator, Discharge current, Open voltage, Delay time

1. Introduction
In electrical discharge machining (EDM), conventional generators mainly include transistor pulse generator and RC pulse generator. RC pulse generator is simple to be fabricated, however, it needs time to charge the capacitor, which leads to the low frequency for discharging and influences the efficiency in EDM; transistor pulse generator has high ability in controlling gap current and pulse duration by changing the duty of transistor and the resistance, but it is hard to obtain short pulse duration. In EDM, both generators require power supplys offering high voltage to form gap breakdown [1].

The electrostatic induction feeding method has drawn lots of attention in recent years because of the advantages that discharge occurs only once in half cycle, which will make machining more stable and capacitance can prevent leak current flowing through the gap [2]. However, electrostatic induction feeding method requires providing high voltage and current at the same time although the average power of power supply may be very low.

The newly developed LC pulse generator [3] can realize short duration and high rising speed discharge current pulses for EDM using a power supply of 5 V voltage, which is not possible with the conventional generators. The LC pulse generator provides better machining efficiency compared to the RC generator under the same discharge frequency. Furthermore, the LC pulse generator is able to machine high resistivity materials such as single crystal SiC which cannot be machined easily with conventional generators. However, the pulse shapes of discharge current, relative peak value and duration of open voltage need to be analysed by mathematical methods. This paper aims to introduce how the parameters influence on the open voltage and discharge current waveforms.

2. Principles of proposed generator
Fig. 1 shows the LC pulse generator. In the circuit shown, \( u_0 \) is the constant DC power source; \( C_1 \) is the capacitor used to prevent leak current flowing through the gap; \( L \) is the inductor used to store energy and inductively boost the gap voltage; and \( S \) is the switch for controlling the current direction.

![Fig. 1. Proposed LC generator.](image)

When the switch \( S \) is turned on, \( u_0 \) is applied to \( L \), causing current from the power source's positive terminal to flow through \( L \) and charge the inductor. The increase in current causes back electromotive force across \( L \) due to the Faraday's law of induction which opposes the change in current. Thus, the current through \( L \) increases slowly while energy from the power source is stored in the inductor's magnetic field.

When the switch \( S \) is turned off, the inductor prevents the drop in current by developing a very large induced voltage of polarity in the same direction as \( u_0 \). Current therefore flows through the \( C_1 \) and the EDM gap thereby applying large open voltage to the working gap.

3. Influential parameters of open voltage waveforms analysis

When the switch \( S \) is turned off, the inductor prevents the drop in current by developing a very large induced voltage of polarity in the same direction as \( u_0 \). Current therefore flows through the \( C_1 \) and the EDM gap thereby applying large open voltage to the working gap.

![Fig. 2. Open voltage and inductance current waveforms without discharging.](image)

Period 1 is charging inductance period; period 2 is voltage application period; period 3 is recover period; period 4 is oscillation period.

3.1 Charging inductance period
When switch \( S \) is on, the inductance would be charged by the power supply. In this case, the state function can be written as follow:

\[
\frac{di}{dt} = u_{in}
\]  

(1)

Since the rising speed of inductance current is constant by the equation (1), the maximum inductance current \( i(0) \) can be written as follow:

\[
i(0) = \frac{u_{in}}{2fL}
\]  

(2)

In this equation, \( f \) is the switching frequency of switch \( S \) and duty ratio of switch is 50%.

3.2 Switching off period
1) Voltage application period

The state function of circuit when switch is off can be written as follows:

\[
\begin{align*}
\frac{L}{C} \frac{di}{dt} &= u_{in} - u \\
C \frac{du}{dt} &= i
\end{align*}
\]  

(3)

Here, \( u \) is the voltage at the point shown in Fig. 1. \( C \) is the resultant capacitance of \( C_1 \) and the gap capacitance. Laplace Transform of the state function is expressed as:

\[
\frac{U(s)}{s} = U(s) = sL_i(s) - L_i(0)
\]  

(4)

\[
I(s) = sC(U(s) - C_0(0))
\]  

The initial condition of \( u(0) \) is 0. If solving \( u \) in complex frequency domain, the solution can be written as follows:

\[
U(s) = \frac{u_{in}}{s} + \frac{L_i(0)}{s^2LC + 1}
\]  

(5)

The definition of \( \omega \) is given to simplify the solution:

\[
\omega = \frac{1}{\sqrt{LC}}
\]  

(6)

Then \( u \) in complex frequency domain can be written as follows:

\[
U(s) = \frac{u_{in}\omega}{s} \frac{\omega}{\omega^2 + \frac{1}{s^2}} + \frac{L_i(0)}{s^2} \frac{\omega}{\omega^2 + \frac{1}{s^2}}
\]  

(7)

According to convolution equation and inverse Laplace transform, the potential \( u \) in time domain can be written as follows:

\[
u(t) = \frac{\omega}{\sqrt{C_0}} \int_0^t u_{in}\omega \sin[\omega(t - \tau)]d\tau + \frac{L_i(0)}{\sqrt{C_0}} \sin(\omega t)
\]  

(8)

In fact, the initial current should be equal to the maximum inductance current shown in Equation (2). Thus, \( u \) can be written as follows if solving the integral part:

\[
u(t) = u_{in}(1 - \cos(\omega t)) + \frac{\omega u_{in}}{2fL}
\]  

(9)

To get the open voltage duration, \( t \) should be solved under \( u(t)=0 \) condition:
Therefore, the solution is expressed as follows:
\[ t = 2n\sqrt{LC}, \]
\[ t = 2m\sqrt{LC} + 2\sqrt{LC}\arctan\left(\frac{\omega}{2L}\right) \]  
(11)

The first time of \( u(t) = 0 \) after switching off should be as follows:
\[ t_u = 2m\sqrt{LC} - 2\sqrt{LC}\arctan\left(\frac{\omega}{2L}\right) \]  
(12)

\( t_u \) means the open voltage duration. Since the \( \omega \) is always very large, \( t \) equals to the equation as follows approximately:
\[ t_u = \pi\sqrt{LC} \]  
(13)

Besides, according to Equation (9), the maximum open voltage can be written as follows:
\[ u_{\text{peak}}(t) = u_{\text{in}} + \sqrt{1 - \frac{1}{4f^2LC}} \]  
(14)

![Fig. 3. Proof of open voltage duration influenced by inductance and capacitance, and peak open voltage influenced by inductance, capacitance and switching frequency. (Voltage limit of Zener diode is 650V in this experiment)](image)

2) Recover period

In this period, the current flows in reversed direction and \( u \) equals to 0V all the time because of the reversed bias of the MOSFET. Since the voltage and current of power source has opposite direction, the inductance energy is lost in the power source.

3) Oscillation period

When the inductance energy is consumed by power source completely, the circuit can be seen as a LC oscillating circuit. The solution is the same as Equation (9) except for different initial condition shown as follows:
\[ u(t) = u_{\text{in}}(1 - \cos(\omega t)) + \frac{\sqrt{C}}{\sqrt{2}}\sin(\omega t) \]

Thus, \( u \) is shown as follows:
\[ u(t) = u_{\text{in}}(1 - \cos(\omega t)) \]  
(15)
The period \( T \) of this LC oscillation is:
\[ T = 2\pi\sqrt{LC} \]  
(17)

And the peak voltage of this LC oscillation is:
\[ u_{\text{peak}}(t) = 2u_{\text{in}} \]  
(18)

2.3 Discharge period taking resistance into consideration

In fact, the pulse generator cannot be an ideal model. The internal resistance \( R \) of fore-circuit including resistance in inductor, power source, cable, switch and others and internal resistance \( r \) of discharge circuit including resistance in the cable, tool and workpiece have influences on the discharge current.

When switch is on, the state function can be seen as follows:
\[ iR + L\frac{di}{dt} = u_{\text{in}} \]  
(19)

Thus, the maximum inductance current is shown as follows:
\[ i(0) = \frac{u_{\text{in}}}{R}(1 - e^{-\frac{R}{\omega L}}) \]  
(20)

When the switch is off, the state function is written as follows:
\[ \begin{cases} \frac{d}{dt}(iR + L) = u_{\text{in}} - u \\ \frac{d}{dt}C(\frac{iu - u}{iR}) = i \end{cases} \]  
(21)

Laplace transform of Equation (21) is expressed as:
\[ \begin{cases} \frac{u_{\text{in}}}{s} - U(s) + sL(u) - \frac{R}{l}(0) + R(s) \\ I(s) = sC(u) - Cu(0) - srC(i(s)) + rC(0) \end{cases} \]  
(22)

Solving the above equations, the current can be obtained as:
\[ i(s) = \frac{Cu_{\text{in}} + sLC(i(0)) + rC(0)}{1 + sRC + s^2LC} \]  
(23)

Usually in LC pulse generator,
\[ \frac{1}{LC} \left( \frac{r + R}{2L} \right)^2 > 0 \]  
(24)

Defining \( \omega' \) as:
\[ \omega' = \sqrt{\frac{1}{LC} \left( \frac{r + R}{2L} \right)^2} \]  
(25)
The inverse Laplace transform can be expressed as:
\[ i(t) = (0)e^{-\frac{rR}{2L}}\cos(\omega't) + \frac{2Cu_{\text{in}} + rC(i(0)) - rC(0)}{2\omega'LC} e^{-\frac{rR}{2L}}\sin(\omega't) \]  
(26)

When discharge occurs, the state function can be seen as follows:
\[ i = \frac{u_{\text{in}} - u}{R} \left( 1 - e^{-\frac{R}{\omega L}} \right) + (i_{\text{di}}) \]  
(28)

Therefore, the discharge current can be written as follows:
\[ t_i = -\frac{L}{R}\ln(1 - \frac{i(i_{\text{di}})}{u_{\text{in}}}) \]  
(29)

![Fig. 4. Proof of discharge current influenced by inductance, switching frequency and discharge delay time, and discharge current duration influenced by inductance](image)

4. Conclusions

By mathematical analysis, the open voltage and discharge current were investigated carefully. With increasing the inductance and capacitance, the open voltage duration increases and open voltage decreases. The open voltage also decreases when the switching frequency increases. In discharging period, discharge current has negative correlation with inductance, capacitance and discharge delay time. Discharge current duration increases with increasing the inductance. Besides, it is found from the calculation results that the discharge current duration is not influenced by the material resistance \( r \) if the peak discharge current is the same. This is one of the advantages of the LC pulse generator.

References