FORMATION OF SUBJECTIVE VALUE: A THEORETICAL AND EMPIRICAL ANALYSIS: I

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Subjective value toward an object is ordinarily formed on the basis of a composite of relevant informations concerning with the object. Three mathematical models were described about the functional relationship between the overall value and the values of individual informations: additive, negative exponential, and positive exponential. All the empirical data presented (preferences towards electric washer, sewing machine, dishwasher, and evaluation of intelligence and of capability of housewives) were accounted for in either of the three functions. Of the respective functional forms, mathematical models were presented about the underlying psychological processes and implications of this analysis in marketing research were also discussed.

In everyday life one is constantly making, consciously or unconsciously, evaluative judgments about such matters as the intelligence of persons, the desirability of commodities, the credibility of arguments, and so forth. One of the characteristics of these psychological processes lies in the fact that the subjective value is rarely formed on a single basis. Ordinarily a person takes into consideration a number of circumstances in the process of forming evaluative judgments about an object, though he may not be conscious of all the circumstances. Perhaps it might be more appropriate to conceive that in many instances the circumstances act as functional, not necessarily conscious, determinants of the subjective value. The

purpose of this study is to present a model for the process through which a subjective value is formed on the basis of a composite of relevant circumstances.

The formation of subjective value from composite basis merits thorough investigation not only from a psychological point of view but also from the standpoint of practical problems. For example, advertisements sometimes enumerate the various characteristics of a commodity, and it might be of importance to the advertising agency to have some insight into how the overall effect is formed from the composite information. The same might be true for business executives who have to make a decision on the advisability, in terms of profit, of manufacturing watches with a particular new characteristic. As a matter of course, adding a new characteristic to a product may be more or less effective in influencing the preference of consumers towards that product on the one hand, while on the other it is almost necessarily accompanied by an increase in the price. In order to arrive at a rational decision in such a situation, it might be helpful to have some knowledge of the laws relating the subjective value of a commodity to a set of

¹ The author wishes to acknowledge with gratitude the collaboration of S. Aoki, H. Kobayashi, and H. Hirasawa. The empirical data have been obtained from their graduation theses, in 1958, 1959, and 1960 respectively. Thanks are also due to H. Mizuta for her help both in analyzing the data and in preparing the first draft (1961). The final preparation of the manuscript took place in 1963 while the author was at the Laboratory of Psychophysics, Harvard University. The author is indebted to Geraldine Stone for her valuable suggestions as to the style.

characteristics borne by that commodity.

The subjective value of a particular object may be characterized as being a function of the relative condition of that object. Suppose that each object is represented by a point c on a continuum C according to its relative condition. The continuum represents all the various conditions of all the objects belonging to the same class, and the condition represents a hypothetical variable in terms of which the subjective value is uniquely determined. For instance, a watch with a particular characteristic i is located at a point c_i somewhere on C , and another watch with a different characteristic j is located at a point c_j . Then with subjective value denoted by v , it is assumed that

$$v_i = f(c_i), \quad v_j = f(c_j) \quad (1)$$

and

$$v_{ij} = f(c_i + c_j) \quad (2)$$

where $(c_i + c_j)$ indicates the condition of an object possessing both characteristics i and j , and v_{ij} represents the subjective value of that object. Hereinafter the function $v = f(c)$ will be called f for the sake of simplicity. It is a matter of definition that additivity is assumed with respect to c . It presupposes that correlated conditions should not be included. Examples of what are here called correlated conditions are pairs, in the following sense. With a typewriter, "easy to carry" and "light in weight" are positively correlated, because the former implies the latter. With a baseball player, "being clever at stealing a base" and "being a slow runner" are negatively correlated characteristics because they are contradictory. However, if the independence of a set of characteristics is assumed, it may be possible to define, as a counterpart of v , a hypothetical variable c such that it behaves according to the principle of additivity. Additivity is not necessarily assumed, however, between the subjective value on the composite basis and the

subjective values of the respective components. In general, v_{ij} is not equal to $(v_i + v_j)$. Gulliksen has showed that the functional form f between v and c can be obtained by analyzing the relationship between v_{ij} and v_i, v_j . His analysis, however, dealt with only the amount of the commodity (Gulliksen, 1956b), whereas the variable c in the present paper refers to more general conditions of objects, as defined above. It should be noted that c is ordinarily not a magnitude to be given in physical terms and cannot be defined independently from v . Hence f cannot be obtained by plotting v directly against c . The relation between v and c is analogous to a unidimensional psychophysical function between sensory magnitude and stimulus, such as a scale of loudness. The difference lies in the fact that a sensory stimulus can be experimentally manipulated and is an independent variable, whereas the condition c , though also manipulable in some way, cannot act as an independent variable. The analysis described below does not require the measurement of c , and f is based solely on a set of scaled values v .

FORMES OF THE FUNCTION f

With the purpose of determining the form of the function f , experiments were run on various subjective values, preferences towards washers, sewing machines, watches of various types, and evaluative judgments about intelligence. The subject was asked various questions, e.g., "Which do you prefer, a washer having a characteristic i or a washer having characteristics j and k ?" or "Whom do you think superior in intelligence, a person able to perform act i or a person able to perform both acts j and k ?" The scaling method of obtaining v through responses to these questions and the procedure for analyzing the relationship between v_{ij} and (v_i, v_j) will be described later. As shown by Gulliksen (1956b), if experimental errors are

disregarded, f can be uniquely determined by analysis of v (the unit for c is to be given by definition), and all the data the author has so far obtained seem to be accounted for in one or another of the following three forms:

$$\text{Type I: } v = a(c - c_0) \quad (3)$$

$$\text{Type II: } v = V[1 - e^{-a(c - c_0)}] \quad (4)$$

$$\text{Type III: } v = v_0[e^{a(c - c_0)}] \quad (5)$$

The complications that would be introduced by undesirable conditions, e.g., a c_i of negative value, will be discussed later, and, unless otherwise specified, consideration is limited to conditions which are desirable and of positive value. In Type I, v is a linear function of c and the forms of the functions of Types II and III are shown in Fig. 1.

FIG. 1. Subjective value v as functions of condition c in Type II, and in Type III.

Once f , the relationship between v and c is determined in a given situation, it becomes possible to predict how v increases when the object moves up on the continuum C . In that sense, acquiring information about the form of f is important in practical problems because a rational decision will differ according to the form of f . Enumerating characteristics $i, j, k, \dots$ of a commodity in advertisement, or manufacturing objects with various characteristics $i, j, k, \dots$ is equivalent to placing the object in a higher position ($c_i + c_j + c_k \dots$). As a result, the subjective value of that object $v_{ijk\dots}$ will remain the same or increase. It is not always true, however, that the increment in c will be large enough to offset the effects of certain negative factors, such as the distracting effects of the advertisement because too much space or time is involved in the enumeration, or the fact that an increase in the price of the commodity is the natural consequence of the added characteristics. When Type I holds, a displacement from c to $(c + dc)$ will increase v by a constant amount dv , irrespective of the position of c and also of the value of v . When Type II holds, on the other hand, the ratio of dv to dc will be given by (4) as follows:

$$\frac{dv}{dc} = a(V - v) \quad (6)$$

That is to say, the larger the value of v , the smaller will be the increment dv which accompanies the displacement from c to $(c + dc)$. As for the objects which are already in a higher status and are governed by Type II, all the efforts of raising their status further may come to naught. As for the objects which accord with Type III, the situation will be entirely different. Differentiating (5) with respect to c , we obtain

$$\frac{dv}{dc} = av \quad (7)$$

which implies that, the higher the status,

the more it will pay to try to raise the status further. In this case, if a higher position in C is attained by some means or other, there will be the possibility of having a boom, so to speak.

DETERMINATION OF THE FORM OF f

As stated above, the form of the functional relationship between v and c may be one of the crucial factors for decision-making in practical problems, and it may be useful to develop the procedure of making the form of f explicit. Determining f may be of interest as well from the psychological point of view. It is impossible to obtain f by plotting v directly against c for the reason that, whereas a set of v can be given as empirical data in psychological investigations, c is not susceptible to direct observation. By the use of the additivity principle inherent in the definition of c , however, the form of f will be made explicit through analysis of v which presupposes no information about the metric properties of C . The procedures for determining f in (3) were given by Thurstone and Jones (1960) and those in (4) were given by Gulliksen (1956b).

Type I. If it is assumed that Type I holds in a given situation, the following equations are easily given by applying (3) to the subjective values on the in single basis c_i or c_j , etc., and also to the subjective values on the composite basis $(c_i + c_j)$, etc.

$$\left. \begin{aligned} v_{ij} &= (v_i + v_j) + (ac_o) \\ v_{ijk} &= (v_i + v_{jk}) + (ac_o) \\ &= (v_j + v_{ik}) + (ac_o) \\ &= (v_k + v_{ij}) + (ac_o) \\ v_{ijkl} &= (v_i + v_{jkl}) + (ac_o) \\ &= \dots\dots \\ &= (v_{ij} + v_{kl}) + (ac_o) \\ &= \dots\dots \end{aligned} \right\} \quad (8)$$

According to these relationships, the plot

of the left-hand member against the first term of the right-hand member should be linear with unit slope and intercept ac_o . In other words, if a linear relationship with unit slope is obtained by plotting v_{ij} against $(v_i + v_j)$, etc., f will be admitted to be of the form (3).

Type II. If it is assumed that a given situation is of Type II, the following equations are obtained from (4):

$$\left. \begin{aligned} v_{ij} &= A_i v_j + B_i \\ v_{ijk} &= A_i v_{jk} + B_i \\ &= A_j v_{ik} + B_j \\ &= A_k v_{ij} + B_k \\ v_{ijkl} &= A_i v_{jkl} + B_i \\ &= A_j v_{ikl} + B_j \\ &= \dots\dots \end{aligned} \right\} \quad (9)$$

where

$$A_i = \left\{ \left(\frac{1}{V} \right) e^{-ac_o(V-v_i)} \right\} \quad (10)$$

$$B_i = V(1 - A_i) \quad (11)$$

That is to say, if v_{ij} is plotted against v_j , ($i, j=1, 2, \dots$) a family of straight lines will be obtained, a straight line with a slope A_i and intercept B_i for each of the values of V_i . In addition, the family of straight lines will be such that all converge on a point (V, V) . That will be clear if (9) is written in the following form:

$$\begin{aligned} v_{ij} &= A_i v_j + V(1 - A_i) \\ &= V - \left(\frac{1}{V} \right) e^{-ac_o(V-v_i)}(V - v_j) \end{aligned} \quad (12)$$

If either v_i or v_j is equal to V , then v_{ij} is also equal to V . As stated before, V is the maximum possible value for v . Hence, if the indicated plots constitute a sheaf of straight lines intersecting at a point on the line passing the origin with unit slope, then Type II will be admitted to apply and the value of V will be estimated.

Type III. If it is assumed that a given situation is described in terms of Type III, the following equations are given from (5):

$$\left. \begin{aligned}
 v_{ij} &= Gv_i v_j \\
 v_{ijk} &= Gv_i v_{jk} \\
 &= Gv_j v_{ik} \\
 v_{ijk} &= Gv_k v_{ij} \\
 v_{ijkl} &= Gv_i v_{jkl} \\
 &= \dots\dots \\
 &= Gv_{ij} v_{kl} \\
 &= \dots\dots
 \end{aligned} \right\} \quad (13)$$

where

$$G = \left(\frac{e^{a c_0}}{v_0} \right) \quad (14)$$

The equations imply that the left-hand term should be proportional to the product of v 's in the right-hand term. If a straight line passing the origin turns out when v_{ij} , etc., are plotted against $v_i v_j$, etc., the situation is to be interpreted as being of Type III.

The form of the functional relationship between v and c , in so far as the three types are concerned, can be inferred by the procedures which do not require the measurement of c but require only a set of scaled values v . The method of scaling v employed in the present study will be described below.

EMPIRICAL DATA: INDIVIDUAL ANALYSIS

The functional form between v and c may differ not only from object to object, and from one kind of subjective value to another, but also from individual to individual, even when the same kind of subjective value of the same object is at issue. Hence, when these experiments were begun, the analysis was made individually with each of the subjects. The subjective values taken up in the individual analyses were as follows: the desirability of various kinds of washers, sewing machines, and dishwashers, the intelligence of persons, and the capability of housewives. The sets of particular characteristics employed in the various cases are

listed in the appendix. Let i, j, k, l , be alternative subscripts for characteristics in a set, all varying from one to n .

It became apparent that paired comparison provided the easiest method for subjects to use to express their preferences. Hence, comparisons were required between pairs of objects, such as one in condition c_s and the other in a different condition c_t , where c_s or c_t includes not only single conditions c_i but also composite conditions $(c_i + c_j)$, $(c_i + c_j + c_k)$, etc., and $s = 1, 2, \dots, p$; $t = 1, 2, \dots, q$. If composite conditions are considered up to those consisting of m elements, and all the possible pairs are presented, then

$$p = q = \sum_{r=1}^m \frac{n!}{r!(n-r)!} \quad (15)$$

The presentation was visual and/or oral, i.e., the statements describing the conditions of the objects were written on cards and then either shown and/or read to the subject. Comparisons were made on the understanding that, of the two objects presented, conditions not specified were the same. The subject was first asked to indicate which he preferred and then he was required to indicate explicitly the ratio between the subjective values of the two by making a mark along a line. If he preferred c_s to c_t , the right end of the line would represent v_s , the subjective value of the preferred object, and his task was to make a mark somewhere along the line to represent v_t , the subjective value of the less preferred object. With the ratio between the distance from the left end to the mark and the length of the line denoted as r_{st} , it was assumed that the subject could mark the line in such a way that

$$r_{st} = \frac{v_t}{v_s} \quad (16)$$

The length of the line was 8 cm. When c_t was preferred, the mark gave the ratio v_s/v_t , but by taking the reciprocal, the ratio

was transformed into the form (16), to give r_{st} larger than one. Then,

$$\log r_{st} = \log v_t - \log v_s \quad (17)$$

$$\sum_{s=1}^p \log r_{st} = p \log v_t - \sum_{s=1}^p \log v_s \quad (18)$$

Let us define the unit of the scale for v as follows for the sake of computational convenience,

$$\prod_{s=1}^p v_s = 1 \quad (19)$$

The second term of the right-hand member (18) then vanishes to give

$$v_t = \text{antilog} \frac{1}{p} \sum_{s=1}^p \log r_{st} \quad (20)$$

If either set, c_s or c_t ($s, t=1, 2, \dots, p$) covers all the conditions to be scaled, the scaling procedure will be completed at this stage. If all the conditions, $(p+q)$ in number, are divided into two non-overlapping sets, c_s and c_t ($s=1, 2, \dots, p, t=1, 2, \dots, q$), then scaling v_s becomes necessary, and from (17),

$$\sum_{t=1}^q \log r_{st} = \sum_{t=1}^q \log v_t - q \log v_s. \quad (21)$$

Using (20) and (21),

$$v_s = \text{antilog} \frac{1}{q} \times \left(\frac{1}{p} \sum_{s=1}^p \sum_{t=1}^q \log r_{st} - \sum_{t=1}^q \log r_{st} \right). \quad (22)$$

The unit of scaled values v_s, v_t obtainable from (20) and (22) is such that the set of v_s satisfies condition (19). Once v_s, v_t are given, it is a matter of definition to transform the unit into a more appropriate form if necessary. It is of theoretical importance, however, that the origin of the scale is defined absolutely in this method of scaling. To reproduce on the line the ratio r_{st} according to (16) presupposes an absolute zero point on the

FIG. 2. An example showing internal consistency among ratio judgements made by S . Theoretical ratio $\dot{r}$ versus empirical ratio $\dot{r}$.

scale of v . In theory, a ratio would not be defined otherwise.

It might be appropriate to refer briefly to the test of internal consistency of the data in terms of the scaling method stated above. Let us express the ratio given by the subject as $\dot{r}$ and the theoretical ratio as $\dot{r}$. The latter is defined as the ratio between scaled values calculated from (20) and (22). An example is shown in Fig. 2 where $\log r$ is plotted against $\log \dot{r}$. This is not an exceptional case and a coincidence of this order was always obtained between $\log r$ and $\log \dot{r}$. It may be said, therefore, that the subjects could grasp a ratio of the subjective value consistently and also that they could transform it faithfully to the ratio of length along a line. The author participated as a subject in some of the experiments, but the majority of the subjects were university students who were not specially experienced in judgments of this type. None of the subjects complained that the task was too difficult. Making responses as required seems to have offered no difficulty, though it was also true that the subjects were not fully confident in making their marks.

Of the notations given above, n, p, q ,

and m varied from experiment to experiment. Comparisons involving a duplicate characteristic such as c_i versus $(c_i + c_j)$ or $(c_i + c_j)$ versus $(c_i + c_k)$ were avoided as far as possible, but still a number of pairs of that type were presented. If the matrix consisting of r_{st} is incomplete in the sense that it contains a number of vacant elements, it is possible to reduce considerably the number of comparisons of duplicate characteristics. However, the scaling procedure from the incomplete matrix is so laborious to compute (Gulliksen, 1956a) that it is not appropriate in the analysis of individual data. Furthermore it became apparent that the comparisons involving duplicate characteristics offered no special difficulty to the subjects, nor did these comparisons disturb the internal consistency of the responses.

As stated before, all the data the author had obtained so far could be accounted for within the framework of Types I, II, and III. In a few cases, though, Type I and II could be fitted to the same data equally well and distinction between the two was difficult. In fact, when the range of v

treated is not wide enough and V is located far away as compared with the range of v , Types I and II become indistinguishable. Otherwise each result was differentiated clearly into one or another of the types.

FIG. 4. An example of the relationship corresponding to Type II among scaled subjective values.

FIG. 3. An example of the relationship corresponding to Type I among scaled subjective values.

FIG. 5. An example of the relationship corresponding to Type III among scaled subjective values.

Examples of the three types are shown in Figs. 3, 4, and 5, wherein plots were made according to (8), (9), and (13), respectively. With the same kind of subjective value, the type differed from subject to subject. Whether or not the same subject reveals the same type, irrespective of the kind of subjective value, is an interesting question, but the data are not sufficient to answer this question. Individual analysis requires considerable time to carry out, and the same subject participated in only up to two experiments.

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(Received Jan., 16. 1971)