STRATEGIES IN CONCEPT IDENTIFICATION: STOCHASTIC MODEL AND COMPUTER SIMULATION¹ I

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A total of 455 cases, 5 tasks by 91 subjects, of the simplest form of concept identification with stimuli, each consisting of 5 letters, were stratified into 8 groups according to strategies used by the subjects. Two representative strategies were identified through verbal reports of the subjects and also differences in performance: whole-letter strategy to pursue all of an appropriate set of letters simultaneously and one-letter strategy to test one letter at a time and without replacement. Both strategies were simulated by a computer. All the results, the whole cases, 8 separate groups, computer simulations, were also fitted by the original form of Bower-Trabasso stochastic model (1964). Two main findings were as follows: there are substantial individual differences in regard to strategy and the stochastic model gives apparent good fits to each of the heterogeneous set of data which were generated through qualitatively different strategies, provided the value of a parameter, the probability of hitting the correct hypothesis, is appropriately defined.

I. INTRODUCTION

It has been abundantly shown that simple stochastic models are able to numerically account for a set of empirical data of concept identification tasks. For example, the model by Bower and Trabasso (1964) reproduced very systematically over a dozen statistics once two parameters were estimated from the data (Atkinson, Bower, and Crothers, 1965). This model is mainly concerned with concept identi-

fication of the simplest form; stimuli varying over well-defined dimensions and a concept having a single relevant dimension in deterministic manner. It seems to the authors, however, that even the psychological process involved in this simplest situation is in fact not that simple as assumed in this model. Hence, the aim of this study is two-fold: to see what is really implied by the quantitative fit of the stochastic models to the data and also to have some insight into the actual psychological process underlying the simplest possible form of concept identification. For the sake of simplicity, among other stochastic models (Martin, 1965), only the original form of Bower and Trabasso (1964) will be referred to in the present article. The model has already been somewhat modified by the original authors (Trabasso & Bower 1966, 1968). Nevertheless, we shall limit our discussion to the original model and will try to fit it to the data to be described later, because this is the model that has been most extensively

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fitted to the data and is still being used as a frame of reference for theoretical discussion in recent articles (e.g., Gregg & Simon, 1967; Falmagne, 1970; Nahinsky, 1970; Cotton, 1971). The model will be abbreviated hereafter as the B and T model.

The B and T Model

In essence, the B and T model regards the concept identification process as an absorbing Markov chain of extremely simple structure. On each trial n the subject is one of two discrete states: $\bar{S}$ representing the presolution state and S representing the solution state. In so far as the subject is in state $\bar{S}$, the sequence of responses constitutes a Bernoulli series with a constant probability q for the incorrect response for random sequences of stimuli. If the subject is in the absorbing state S , he will always make the correct response. The transition probability from $\bar{S}$ to S is given by a constant qc because the transition is assumed to take place as a result of the following two independent events. Only when the subject finds that the prediction based upon the current hypothesis H_n is wrong, he changes H_n and that probability is by assumption q . In changing H_n , the subject is assumed to take up the correct hypothesis H_0 by chance with a probability c . The notation to be used in connection with the B and T model is summarized below:

- S : Solution state
- $\bar{S}$: Presolution state
- X_n : $\begin{cases} =1, & \text{if the response is incorrect on} \\ & \text{trial } n \text{ (error)} \\ =0, & \text{if the response is correct on} \\ & \text{trial } n \text{ (success)} \end{cases}$
- q : The probability of error in $\bar{S}$
- c : The probability that the relevant cue is selected when the hypothesis is changed
- q_n : The probability of error on trial n for the whole cases; $q_n = E(X_n)$
- T : The total number of errors for a case; $T = \sum_n X_n$

L : The trial number in which the last error occurs for a case

u_j : The number of j -tuples of errors for a case

r_j : The number of runs of length j for a case

R : The number of runs of any length;
 $R = \sum_j r_j$ for a case

c_k : Autocorrelations of lag k ; $c_k = \sum_n X_n X_{n+k}$ for a case

Of each variable v , the mathematical expectation will be denoted by $E(v)$ whereas the average in the data by $\bar{v}$. The two parameters are estimated in the following ways:

$$c = \frac{1}{\bar{T}} \quad (1)$$

$$q = \frac{\bar{T}}{\bar{L}} \quad (2)$$

Solely on the basis of q and c , the followings are predictable: the learning curve q_n , the distributions of T , L , and the sequential statistics concerning with $\bar{u}_j$, $\bar{r}_j$, $\bar{c}_k$, $\bar{R}$, etc. Ordinarily, and as will be shown in this article too, the predictions are in close agreement with the experimental data obtained with a group of subjects as a whole. For the discussions to follow, it will be in order to enumerate the following remarks about the B and T model.

1. The subject is assumed to be in the state $\bar{S}$ on the first trial and presolution responses are assumed independent and stationary with a constant probability of making error, q , over all trials in the state $\bar{S}$. Both the assumptions can be tested directly and are apparently in good agreement with the data (Bower and Trabasso, 1964; Atkinson, et al., 1965).

2. When a current hypothesis H_n is replaced, the probability of the correct hypothesis H_0 being taken up is by assumption a constant c at any trial n in the state $\bar{S}$.

3. Replacing H_n is assumed to take place when and only when an error occurs

and $N_n = 1$.

4. All the fittings are made to the data for a group of subjects. In fact, otherwise the fitting will be impossible.

The assumptions, 1 and 2, that both q and c are constant implies that the subject fails to use memory of information presented on preceding trial sequences to lower the probability q of making error in prediction and also to increase the probability c by narrowing the scope of his search for H_0 . Obviously, this no memory assumption does not seem very realistic and some discussion and modification were made by Trabasso and Bower (1964, 1966, and 1968) in regard to this assumption. The assumption 3 is equivalent to assume that no information is processed during correct response. The situation mentioned in 4 is common to most stochastic models and it tacitly presupposes that all the subjects are identical regarding the values of q and c . Despite these somewhat unrealistic assumptions, it is true that the B and T model is capable of making satisfactory quantitative predictions, and hence it seems worth while to inquire closely into what is really implied by the quantitative fit.

The task of concept identification is of such a kind that can be used as an item of intelligence test. Hence, as mentioned by Trabasso and Bower (1968, p. 218), it is undoubtedly a truism to say that there are individual differences. If individual difference in skill of manipulating information is incorporated into the B and T model, evidently it should be reflected into the value of c rather than that of q . Once Indow tried to see to what extent the predicted distribution of T is sensitive to the variation of c and showed that it would be rather difficult to detect the presence of quantitative individual differences in value of c through the process of fitting (Gregg & Simon, 1967). In the present article, individual differences in strategy of attaining concept will be pursued in detail.

II. EXPERIMENT

Experimentation

Each stimulus consisted of 5 alphabetical letters selected from a population of 10 consonants. The subject was instructed that each card was classified into one of two categories, positive (+) or negative (−), according to whether a particular letter is present or absent in it. Namely, it was clear to the subject from the outset that a concept having a single relevant cue was at issue. Position of each letter was randomized from a card to another so that the relevant letter appeared as r -th letter in a positive card and r varied from 1 to 5. The stimulus cards were presented one by one in a random order which varied from subject to subject. The probability that each of the 10 letters appears in a card is 0.5 and hence the probability of the positive card being presented is also 0.5 with the restriction that neither (+) nor (−) cards are not to be presented more than twice in immediate succession. In total, 91 subjects were employed; 17 male and 12 female regular Keio University students (R. Sts), 28 male and 34 female summer school students of the correspondence course in the university (SS. Sts). The experiment was run individually. Whenever a card was presented, the subject was asked to predict whether it was (+) or (−), and also to describe the reasons for the response. Then, the correct classification of the card was told by E . The sequence was continued under the pace of the subject's verbal reports, until the subject gained complete confidence in the relevant cue H_0 , and the subject was asked to describe in detail how he had come to H_0 and got convinced. Suzuki and Kasahara observed the whole process with great care to discern a pattern of the information processing of the subject. Each subject worked with five different concepts (a different population of 10 consonants and a different H_0 for each) with randomized order from subject to subject. It took between 20 to 30 minutes for a subject to complete all the experiments. In this way, the data of 91×5 cases were obtained in total.

It will be in order to add comments on some characteristics of the present method.

1. The task was a single unit learning with no redundancy, no shift and no reversal of cue and the subject was well informed of it from the beginning. The subject was not told in advance what the 10 letters were. It can be said that each stimulus consisted of 10 dimensions (letters) of binary values (presence and absence), and there was no apriori difference in salience among those dimensions because all were letters. Besides, the subject was aware that the cue letter was randomly chosen. The task was a binary classification (+) and (−) and easy enough so that there was no non-solver but difficult enough to lengthen the dimensional selection phase in learning so that strategies being used by the subject were made explicit as will be stated below.

2. The most frequently employed method to make the strategy explicit was to insert the so-called blank trials (Levin, Leitenberg, & Richter, 1964; Levin, 1966). Relatively few studies used different method (e.g., Byers, 1965; Suppes & Schlag-Rey, 1965; Falmagne, 1969; Troyer & Millward, 1970). In the present experiment, the subject was asked to verbally describe the reasons for making his prediction on each trial. Examples are as follows: "I will say +, because the card has TMSK, and I know the cue must be one of them," or "I think the presence of G is essential and the card does not have it, so I will say −." To ask leading questions was carefully avoided and only when necessary the experimenter asked questions to clarify the stated reasons. This is essentially the procedure of thinking aloud and it is not clear to what extent the procedure was distracting and might tend to suggest certain strategies. The policy of this study was simply to obtain the data under this circumstance, to make full use of the verbal statements in analysis and to try to fit the B and T model to the performance as in ordinary experiments of concept identification.

3. The whole sequence was under the pace of the subject's information processing. On each trial, when the subject made the prediction and completed the verbal description, the

experimenter informed subject of the correct classification. Then when the subject nodded, the next stimulus card was presented and the preceding one was taken away. Though there was practically no time limit in making the prediction and also in observing the card after the correct classification was told, the subject did not take time in both kinds of information processing and the sequence proceeded very rapidly as a routine. This is the reason that the whole experiment was completed within 30 minutes.

Grouping of data according to strategies

As originally pointed out by Bruner, Goodnow, and Austin (1956), various strategies were distinguishable by which the subject processes the information and modifies the current hypothesis H_n . Suzuki and Kasahara, both having observed the whole process, came to the conclusion that 455 cases could be classified into 9 groups (G). First, they evaluated the data independently and then discussed each case to which both did not give the same classification. The protocol of each case was suggestive enough of the involved strategy so that independent classification by the two judges was in fairly good agreement from the beginning and it was not difficult to yield the consensual classification shown in Table 1.

G-1 to G-4 fall under the category that is called wholist strategy (Bruner, et al., 1956) because in principle all the letters in the first positive and/or negative cards are taken into account in forming candidates or non-candidates for the correct cue in memory. This will be called *Whole-letter (L) strategy*. From G-5 to G-8 will be called *One-letter (L) strategy* because in principle only one letter is pursued at a time. There were only four protocols which did not suggest any clear-cut strategy (G-9) and these will be discarded in the discussion to follow. The B and T model was fitted to the data of groups G-1 to G-8 separately and also to the pooled data of the whole 451 cases. The numbers of

TABLE 1
Stratification of the cases according to the strategy and computer simulations

	No. of cases	Proportion of cases		Parameter B & T model	
		R. Sts	SS. Sts	q	c
<i>Whole-letter strategy</i>					
G-1: Memorize the first stimulus in its complete form and take the most efficient method to modify H_n .	135	0.43	0.24	0.57	0.77
G-2: About the same as G-1 but sometimes make slight mistakes.	102	0.34	0.17	0.59	0.69
G-3: Take the whole L into account but not very organized in modifying H_n .	65	0.17	0.13	0.54	0.32
G-4: Memorize a part of the first positive stimulus and apply the same method as G-1 or G-2.	29	0.05	0.07	0.52	0.41
<i>One-letter strategy</i>					
G-5: Memorize the whole L of the first positive card and test each L one by one.	15	0.01	0.04	0.78	0.42
G-6: Memorize a part of the first positive card and test each letter one by one.	41	0	0.13	0.65	0.45
G-7: In modifying H_n , take one L from memory and test it one by one. Rarely take the L already discarded.	42	0	0.14	0.57	0.21
G-8: About the same as G-7 but sometimes take the L already discarded.	22	0	0.07	0.49	0.11
G-9: Not consistent in modifying H_n .	(4)	0	0.01		
Whole cases (except 4 in G-9)	451	(145)	(310)	0.56	0.40
<i>Computer simulation</i>					
Whole-letter strategy Response I.	200			0.45	0.90
Whole-letter strategy Response II.	200			0.62	0.76
One letter strategy.	200			0.56	0.36

cases and the estimated values of q and c are shown in Table 1 with brief descriptions of the respective strategies. Description about the goodness of fit and about objective measures of performance of the respective groups will be given later.

III. COMPUTER SIMULATION

Two sets of computer program were constructed to simulate two most differ-

ent types of strategy: the purest form of Whole-L strategy and the purest form of One-L strategy. The purpose was to create groups of hypothesized subjects who adopt the respective strategies and carry out Monte Carlo experiments comparable with the actual experiment with human subjects which was reported in the preceding section. In Fig. 1 and 2 are shown the essential structures of the programs and the notation used in these

figures is summarized below:

L	Letter or letters
$H_n(+)$	Hypothesis on trial n consisting of candidate L for the relevant cue for the positive concept
$H_n(-)$	Hypothesis on trial n consisting of irrelevant L for the positive concept
$H_n(0)$	No hypothesis has been formed yet
$S_n(+)$	Positive stimulus card on trial n
$S_n(-)$	Negative stimulus card on trial n
$R_n(+)$	Positive response on trial n
$R_n(-)$	Negative response on trial n
$X_n(+)$	Correct response on trial n ($R_n = S_n$)
$X_n(-)$	Error on trial n ($R_n \neq S_n$)
$LM_n(+)$	Candidate L for the relevant

	cue in memory on trial n
$LM_n(-)$	Irrelevant L in memory on trial n
$LM1_n$	Memory at the outset of trial n
$LM2_n$	Memory at the end of trial n
$L_n(S:H)$	L common in S_n and H_n
$L_n(S:\bar{H})$	L in S_n but not in H_n
$N[v]$	Number of v
$L_n(S:LM_n(+))$	L common in S_n and $LM_n(+)$
$L_n(\bar{S}:LM_n(+))$	L in $LM_n(+)$ but not in S_n
$L_n(S:\overline{LM2_{n-1}})$	L in S_n but not in $LM2_{n-1}$

The programs were written in FORTRAN. Each stimulus card $S_n(x)$ consisting of five letters (L) is generated and labelled as positive ($x=+$) or negative ($x=-$) according to whether the relevant L is involved or not. Five letters are

FIG. 1. Structure of Whole-letter strategy simulation.

$S_n(-)$ but not in $H_n(-)$ is (are) added to $H_n(-)$ to construct $H_{n+1}(-)$ as indicated in the block second from the left. How to read the processes indicated in the remaining blocks will be clear. The process is continued until $H_n(+)$ is narrowed down to the one that consists of the relevant L only ($H_n(+)=H_0$).

Let us proceed to One-L strategy. The processes up to the stage where $R_1(x)$ is generated to the first stimulus card $S_1(x)$ and $X_1(x)$ is determined are the same with what was described above (the right side branch in Fig. 2). Then, for $S_1(+)$, a L randomly selected from the five in $S_1(+)$ constitutes $H_2(+)$ whereas, for $S_1(-)$, no hypothesis is constructed so that $H_2(0)$. In this strategy the hypothesized subject has to retain the L in memory, some as candidate(s) for the relevant L, $LM_n(+)$, and some as irrelevant letter(s), $LM_n(-)$, and the memory has to be changed during trial n . Hence, the memory at the outset of trial n is denoted by $LM1_n$ which is to be changed to $LM2_n$. For $S_1(+)$, $LM2_1$ consists of all the five letters in $S_1(+)$ as the candidates. Then, the second card $S_2(x)$ is generated and, if $H_2(0)$, the process comes into the right side branch again. When it gets back to the stage "0", $LM2_1$ is changed to $LM1_2$ by adding to $LM2_1$ letter(s) which is (are) in $S_2(x)$ but not in $LM2_1$. If $H_2(+)$ and if it is not the correct cue, then the process proceeds into the left side branch and $R_2(+)$ is generated only when the letter in $H_2(+)$ is in $S_2(x)$. If $R_2(x)=S_2(x)$ to give $X_n(+)$, the hypothesis is retained so that $H_3(+)=H_2(+)$, whereas, if $R_2(x)\neq S_2(x)$ to give $X_n(-)$, firstly, $LM1_2$ is changed to $LM2_2$ in the manner as indicated in the block in the central part of Fig. 2, and secondly $H_2(+)$ is modified into $H_3(+)$. For example, for $S_2(+)$, $H_3(+)$ consists of a L randomly selected from the letters which are in common between $S_2(+)$ and $LM2_2(+)$. The process is continued until such $H_n(+)$ is obtained that coincides with H_0 . Note that $H_n(+)$ always consists of one L and

$H_n(-)$ does not exist in the present simulation. Note also that all the letters having been encountered up to trial n are all retained in the memory correctly as $LM_n(+)$ or $LM_n(-)$.

In each case, 200 runs were obtained with the aid of a computer, IBM 7041/1401, and if $X_n(+)$ and $X_n(-)$ are defined as $X_n=0$ and $X_n=1$ respectively, each of the three sets of 200 series each consisting of 1 or 0 is treated exactly in the same way as X_n in the actual experiment with 200 human subjects. The B and T model was fitted to each of those results and the values of the parameters, c and q , are given in the bottom of Table 1.

Obviously, the maximum amount of information in stimulus cards $S_n(x)$ is being utilized in Whole-L strategy, whereas One-L strategy is not so efficient. Nevertheless, it should be more efficient than the B and T model because the hypothesized subject has perfect memory LM_n and sampled L without replacement. The programs were not intended to carry out realistic and exact simulations of actual psychological processes. If so intended, it would have been necessary to use some parameters. For example, the program by Williams (1971) has a set of parameters such as size of focus, buffer size, forgetting rate, and initial salience of respective dimensions. Also the forgetting rate and some other parameters are in the simulations by Trabasso and Bower (1966) and by Wickens and Millward (1971). Notice that the present programs do not have any free parameters. The purpose was to see what sequences are like that are generated by the respective idealized strategies extracted from the qualitative data of the actual experiment, to compare, not to fit, those sequences with the series of actual data in stratification and to fit the B and T model to those sequences with the known structure. Hence, the programs are more specific than the CLS by Hunt, Marin, and Stone (1966) and that by Allen (1962).

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