The orthogonally and the obliquely transformed component analysis in sociometric data¹

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This article presents a new matrix method of analysis of sociometric data, which is more reliable and more efficient than the conventional methods in obtaining full informations relevant to the subgroup structure underlying the sociometric data. The method extracts the orthogonally and the obliquely transformed components, using the procrustean factor rotation in multiple factor analysis. First, a hypothetical target matrix is constructed by principal component analysis of the mutual choice data matrix, and then to approximate the target matrix the orthogonally and obliquely transformed components are obtained based on the choice and received choice data matrices. A tentative application to the results of a sociometric test given to junior high school students suggests that the method is adequate for extracting full informations from both types of sociometric data matrices, and for revealing the salient subgroups hidden in the matrices.

Key words: principal component analysis, procrustes rotation, orthogonal rotation, oblique rotation, group structure, sociometric techniques, human relations.

The following three approaches have, so far, been discussed in the sociometric data analysis. The first approach is concerned with matrix approach, the second one with graphical approach, and the third one with stochastic process approach (Chino, 1984). It may be said, however, that the first approach is the most usable and practical among the three approaches in that it is very easy to manipulate sociometric data.

The conventional matrix approach, nevertheless, may be a time-consuming and unreliable in attaining the final evaluation

or confirmation of the assumed subgroups through the hand-manipulation of data and the subjective judgment of them. As explained so far, it should then be of great importance to seek or develop an objective and improved method to find formal structure of subgroups underlying in a given sociometric data.

Kashiwagi (1972, 1974), as an improved method, applied the principal component decomposition of the mutual choice data matrix ${}_mX$ which was derived from an original choice data matrix X , each element of which was composed of either 1 or 0, where 1 means the choice by a member and 0 otherwise. Then, the subgroups can be identified in terms of the magnitudes of the elements of the corresponding normalized eigenvector matrix. Some em-

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¹ In the data processing, we used the MATRIX procedure in the SAS (Statistics Analysis System) software at Computer Center, University of Tokyo and Data Processing Center, Kyoto University.

irical data have indicated the usefulness of his proposal. This kind of approach, nevertheless, have some problems to be improved in further studies concerning sociometric data analysis in that the approach does not always take care of the information through the members who make or receive their one-way choices.

The main objective of the present study is to establish an objective method with some mathematical models for the description of the group structure by using the information brought about through the sociometric data as fully as possible. For this purpose, we should not neglect all the results based on the one-way choice data together with the mutual choice data in an actual sociomatrix data unlike the proposal explained above by Kashiwagi (1972, 1974). In order to attain the purpose, the senior author has suggested to expand further the proposal of Kashiwagi by making use of the notion of the orthogonally and the obliquely transformed components.

The notion of the orthogonally and the obliquely transformed components is established from the notion of the orthogonal and the oblique procrustean factor rotation in multiple factor analysis. A target matrix, therefore, must be constructed in this analysis as well as in the procrustean factor rotation. The target matrix can be obtained on the basis of the normalized eigenvector matrix by the principal component analysis of mutual choice data, because Kashiwagi suggested that an eigenvector matrix by the principal component analysis of mutual choice matrix is used for the identification of some subgroups. Then, the obtained normalized eigenvector matrix by the principal component analysis of the original data, involving both the one-way choice data and the mutual choice data, is most nearly transformed into the target matrix according to the least-squares criterion of fitness.

Theory and Procedure

First of all, a given sociometric choice matrix is to be transformed into the corresponding 0-1 or binary type matrix through the principle of being 1 as a choice by a member. Rejection or no-choice is, of course, to be shown as 0, and then the matrix is to be denoted by X the size of which is $n \times n$.

The principal component analysis of the mutual choice data based on the binary type of sociomatrix can be done in the following manner.

Before decomposing the corresponding mutual choice matrix ${}_mX$, this matrix should be prepared for the further analysis. The matrix ${}_mX$ can be derived from the sum of the two kinds of matrices, the matrix X and its transposed one X' . If the value of an element is equal to 2, the value can be changed into 1 and otherwise 0. Then, the mutual choice matrix ${}_mX$ is to be a sort of binary type square matrix, the size of which is assumed to be $n_1 \times n_1$ where n_1 is not greater than n .

The mutual choice data matrix, therefore, is decomposed into the main subgroups under the number of components which is enough to explain the group structure, but which is far less than the number of subjects.

Although, as will be described at the next stage, the decomposition of the choice matrix X uses the product-sum matrix of $X'X$ or XX' because of the asymmetrical form of the matrix X , the following decomposition will be possible rather than the use of the product-sum of ${}_mX'{}_mX$ or ${}_mX{}_mX'$ because of the symmetrical form of the matrix ${}_mX$. That is to say, the decomposition

$$\begin{aligned} {}_mXH_s &= H_sL_s^{1/2} \quad \text{or} \\ {}_mX &= H_sL_s^{1/2}H_s' \end{aligned} \quad (1)$$

will be possible, where H_s is the normalized eigenvector matrix and $L_s^{1/2}$ is the square root of the corresponding eigenvalue placed in the diagonals and zeros in

the off-diagonals. The latter form of the expression of (1) has been called as the Eckart and Young's decomposition. From the relations above, it will be shown that the same eigenvector matrix H_s can also be given in performing the principal component analysis of the product-sum of ${}_mX'{}_mX$ or of ${}_mX{}_mX'$. In other words, by using the latter form in (1), the relation become:

$$\begin{aligned} {}_mX'{}_mX &= H_s L_s H_s' \text{ or} \\ {}_mX'{}_mX H_s &= H_s L_s. \end{aligned} \quad (2)$$

The expressions (1) and (2) show that they produces the same eigenvector matrix H_s as mentioned above although the value of each element of L_s is the squared one of $L_s^{1/2}$.

Secondly, the two kinds of principal component analyses can be applied to the product-sum matrix XX' and $X'X$, where the former matrix contains the information based on the direct choice by members and the latter one the information based on the received choices by other members. The number of components is equal to the number of components in the decomposition of the mutual choice matrix. The mathematical relations can be expressed in the following manner. As for the former case,

$$XX'H_c = H_c L_c \quad (3)$$

where H_c is the normalized eigenvector matrix and L_c is the diagonal matrix with the eigenvalues as its diagonal elements. The values of the eigenvalues are assumed to be arranged in their descending order. In the same manner, the latter case can be shown as:

$$X'X H_d = H_d L_d \quad (4)$$

where H_d is the eigenvector matrix and L_d is the eigenvalue matrix. It should be noted, here, that positive eigenvalues L_c and L_d are equivalent.

Finally, after preparing the matrices of H_c , H_d , and H_s , the final analyses for the

transformed components will be attained in the following manners. Here note, however, that for the analyses the target matrix $H_{\bar{s}}$ derived from the matrix H_s obtained in (1) or (2) should be constructed by appending the rows, the elements of which are all zeros, at the bottom of the main body of the matrix H_s . In other words, the $H_{\bar{s}}$ may be written as

$$H_{\bar{s}} = \begin{bmatrix} H_s \\ \mathbf{0} \end{bmatrix} \quad (5)$$

where $\mathbf{0}$ shows the appended zero matrix. It should be noted that both the sizes ($n \times c$: c is the number of components) of $H_{\bar{s}}$ and H_c are assumed to be same and so are the sizes of $H_{\bar{s}}$ and H_d .

In regardless of either the orthogonally or the obliquely transformed components, the problems here may be to attain the final solutions of the formulae below; since the solutions of received choice is obtained by changing subscript " c " into " d " in the following formulae, we represent the solutions of direct choice.

$$H_{\bar{s}} = H_c T_c + E_c \quad \text{tr}(E_c' E_c) \rightarrow \min. \quad (6)$$

For these solutions, the established notions of the orthogonal and the oblique procrustean factor rotation in multiple factor analysis may be helpful. Here, it should be noted that we use the reference structure for convenience of explanation because of not finding an appropriate term in this study, though the transformed components are not just equal to the reference structure in the procrustean factor rotation. Then, by the help of the former notion (cf. Green, 1952; Schönemann, 1966), the least square estimates $\hat{T}_{rc}$ for the orthogonal transformation matrix T_c in (6) is:

$$\begin{aligned} \hat{T}_{rc} &= H_c' H_{\bar{s}} (H_{\bar{s}}' H_c H_c' H_{\bar{s}})^{-1/2} \\ &= H_c' H_{\bar{s}} P_c G_c^{-1/2} P_c' \end{aligned} \quad (7)$$

where $P_c G_c P_c'$ is the eigen-solution for $H_{\bar{s}} H_c H_c' H_{\bar{s}}$. By using the relations (6), the matrix to be solved in the orthogonal

Table 1
The principal component analysis for the mutual choice matrix

Member	Principal components: H_s					
	I	II	III	IV	V	VI
A	.503	0	0	0	0	-.195
B	.441	0	0	0	-.417	.321
C	.441	0	0	0	.417	.321
D	.401	0	0	0	.231	-.387
E	.401	0	0	0	-.231	-.387
F	.131	0	0	0	.521	.477
G	.131	0	0	0	-.521	.477
H	0	-.500	0	0	0	0
I	0	-.500	0	0	0	0
J	0	-.500	0	0	0	0
K	0	-.500	0	0	0	0
L	0	0	-.577	0	0	0
M	0	0	-.577	0	0	0
N	0	0	-.577	0	0	0
O	0	0	0	-.707	0	0
P	0	0	0	-.707	0	0
C.B. ^{a)}	3.350	3.000	2.000	1.000	0.801	0.673

^{a)} contribution: the sum of the squared elements of the eigenvector corresponding to each eigenvalue.

case for $H_{\bar{s}}$ becomes:

$$\hat{H}_{rc} = H_c \hat{T}_{rc}. \quad (8)$$

Further, by the help of the notion of the oblique procrustean factor rotation (cf. Mulaik, 1972), the normalized least square estimates with regard to the matrix T_c in (6) is:

$$\hat{T}_{bc} = (H_c' H_c)^{-1} H_c' H_{\bar{s}} D_c \quad (9)$$

$$D_c = [\text{diag}\{H_{\bar{s}}' H_c (H_c' H_c H_c' H_c)^{-1} H_c' H_{\bar{s}}\}]^{-1/2}. \quad (10)$$

Then, the matrices to be solved for the oblique case can be

$$\hat{H}_{bc} = H_c \hat{T}_{bc}. \quad (11)$$

It should be noted that the matrix $\hat{T}_{bc}' \hat{T}_{bc}$ may be an indicator of showing the degree of the correlation of the obtained transformed component matrix H_{bc} .

On the Data to be Used for Illustration

A part of the data on the sociometric

test given for junior high school students, herein were used illustration. The total number of the participants in this test was 19, and those who made the mutual choices with each other was 16.

Results

1. Principal component analysis of the mutual choice matrix ${}_m X$:

Table 1 shows the eigenvector matrix H_s for the mutual choice matrix ${}_m X$ which is obtained according to the Formula (1). The least number of components in H_s which is not singular in order to explain the group structure clearly enough, is assumed to be six. In addition, the eigenvectors are arranged in the descending order of the values of the square roots of the corresponding eigenvalues. Four salient subgroups may be identified through the operation of the exchanges of rows or members of H_s which is presented in Table 1.

Through the inspection of Table 1, it

Table 2
The principal component analysis for the choice structure matrix

Member	Principal component solution: H_c					
	I	II	III	IV	V	VI
A	-.362	-.167	.038	-.139	-.227	0
B	-.357	.041	-.001	-.439	.463	0
C	-.358	-.260	-.203	.092	-.096	0
D	-.383	-.152	-.017	-.228	.036	0
E	-.282	-.220	-.226	.273	.034	0
F	-.154	.200	.486	.033	.489	0
G	-.286	.439	.296	.382	-.024	0
H	-.070	.372	-.191	-.055	-.150	0
I	-.069	.362	-.290	.016	.119	0
J	-.059	.332	-.229	-.151	-.141	0
K	-.053	.340	-.284	-.021	-.185	0
L	0	0	0	0	0	.577
M	0	0	0	0	0	.577
N	0	0	0	0	0	.577
O	-.184	-.017	.050	.638	.025	0
P	-.223	.233	.340	-.207	-.481	0
Q	-.081	.038	.152	-.144	.012	0
R	-.369	-.168	.044	.020	-.219	0
S	-.188	.126	-.427	.081	.331	0
E.V. ^{a)}	27.074	11.998	8.771	5.000	4.015	4.000

^{a)} Eigenvalue

becomes very apparent that the rows or members A to G may belong to the first group, and that the rows or members H to K may belong to second group. Also, Table 1 indicates that the rows L to N and the members O to P may belong to the third and the fourth groups, respectively. The result shows that the Kashiwagi's proposal is of use in the identification of the main subgroups.

2. The orthogonally transformed component analysis of the matrices H_c and H_d :

Table 2 and Table 3 present the matrices H_c and H_d in the Formulae (3) and (4), respectively, and the orthogonally transformed components $\hat{H}_{rc}$ and $\hat{H}_{rd}$ are presented in Table 4 and Table 5, respectively. The members A to P have their mutual choices and the members Q to S do not have their mutual choices in the tables above except Table 1. Table 4 and

Table 5 present the results of the orthogonally transformed components based on the data matrix of making choices and those based on the data matrix of receiving choices, respectively.

The four subgroups which can be found in Table 1 may be identified in the both tables, although both results do not always give perfectly a simple structure type of solutions unlike the one of Table 1. Here note, however, that the members L to N in both Table 4 and Table 5 are as completely independent of the other members as are in Table 1.

3. Obliquely transformed component analysis of the matrices H_c and H_d :

Table 6 and Table 7 show the obliquely transformed components $\hat{H}_{bc}$ and $\hat{H}_{bd}$ together with the matrices $\hat{T}_{bc}'\hat{T}_{bc}$ and $\hat{T}_{bd}'\hat{T}_{bd}$ as the indicators of the degrees of the interrelations of the transformed components above, respectively. As well as

Table 3
The principal component analysis for the received choice structure matrix

Member	Principal component solution: H_d					
	I	II	III	IV	V	VI
A	-.408	-.188	-.264	.196	-.287	0
B	-.392	-.153	-.023	.179	.248	0
C	-.426	.134	.452	-.322	-.024	0
D	-.299	-.241	-.100	.395	.241	0
E	-.394	-.136	.067	-.402	.261	0
F	-.371	-.182	-.283	-.160	-.270	0
G	-.098	.069	.163	-.181	-.475	0
H	-.071	.335	-.415	-.033	-.061	0
I	-.078	.368	-.123	-.194	.478	0
J	-.128	.473	-.303	.180	-.044	0
K	-.162	.447	-.140	-.110	-.132	0
L	0	0	0	0	0	-.577
M	0	0	0	0	0	-.577
N	0	0	0	0	0	-.577
O	-.127	.251	.379	.093	.008	0
P	-.120	.179	.281	.471	-.244	0
Q	-.029	.057	.164	.015	-.244	0
R	-.133	.189	.231	.363	.239	0
S	0	0	0	0	0	0
E.V. ^{a)}	27.074	12.000	8.770	5.001	4.015	4.000

^{a)} Eigenvalue

the results of the orthogonally transformed components, the four subgroups may be found very clearly even through the slight inspection. And the subgroup, as is indicated in the sixth component of both Table 6 and Table 7, can be identified as the one in the third component of Table 1. The members F to G here, changed to belong to another subgroup so that some changes seem to appear as compared to the orthogonally transformed components, and then there may arise the necessity for more detailed or thorough inspection.

The member G is changed to belong to a member of the subgroup consisting of the members O to P in Table 6, while he belonged to the subgroup consisting of the members A to G presented in Table 1. This kind of change may, however, occur so often because the orthogonally and the obliquely transformed component analysis do not always take care of the magnitudes

of the eigenvalues.

Like the orthogonally transformed component analysis, the obliquely transformed component analysis can provide the information which is not gained by the analysis of mutual choice data matrix. In other words, although it may be impossible to determine which subgroup is to be appropriate for a member who makes no mutual choice with other member in Table 1, both the orthogonally and the obliquely transformed component analysis may do.

Taking the member R, for instance, while he is excluded in the result of the principal components in Table 1, he may belong to the subgroup consisting the members A to F in Table 6. The similar kinds of differences between the results through the choice data matrix and the received choice data matrix may be found. As in the case above, taking the member R for instance, while he may belong to the

Table 4
The orthogonal transformed component for the choice structure matrix

Member	Orthogonal transformed component: $\hat{H}_{rc}$					
	I	II	III	IV	V	VI
A	.347	-.012	0	-.213	.196	-.162
B	.490	-.101	0	.365	.197	.334
C	.387	.050	0	-.093	-.125	-.278
D	.440	-.012	0	.018	.166	-.029
E	.305	.112	0	-.050	-.324	-.202
F	.118	.201	0	.007	.006	.696
G	.032	-.113	0	-.487	-.266	.431
H	-.030	-.441	0	-.078	-.058	.015
I	.028	-.387	0	.121	-.237	.109
J	-.005	-.456	0	-.003	.008	-.024
K	-.038	-.457	0	-.062	-.111	-.078
L	0	0	-.577	0	0	0
M	0	0	-.577	0	0	0
N	0	0	-.577	0	0	0
O	.057	.221	0	-.336	-.525	.052
P	.032	-.236	0	-.513	.410	.070
Q	.071	-.009	0	-.036	.172	.125
R	.326	.043	0	-.279	.064	-.154
S	.251	-.216	0	.298	-.386	.036
Contr. ^{a)}	.994	.993	.999	.994	.995	.994

^{a)} contribution: the sum of the squared elements of the eigenvector corresponding to each eigenvalue.

two kinds of subgroups consisting of the members A to F and of the members O to P by the result based on the received choice data matrix as shown in Table 6, he may not belong to the subgroup consisting of the members A to F as shown in Table 7.

Thus far, the introduction of the two types of transformed component analyses may provide a great deal of benefits for determining the salient subgroups based on the choice and the received choice data matrix, and it may be said that they are the extensions of the proposal by Kashiwagi (1972, 1974) based only on the mutual choice data matrix.

Discussion

In this paper, the great emphasis is placed on the following two points. One is to pay attention to the information ob-

tained through the choice and the received choice data matrices, another is to estimate the number of the main subgroups in their data. These points come from the notion of the oblique procrustean factor rotation.

On the assumption of the existence of salient subgroups, sociometric data here are intended to analyze so as to utilize the informations as fully as possible through the choice and the received choice data matrix. In terms of this purpose, the orthogonally and the obliquely transformed component analyses are herein discussed by making use of the result obtained through the principal component of the mutual choice data matrix. Then the present proposal may be useful and practical when some salient subgroups are assumed to exist in the mutual choice data matrix discussed in this study.

As we discussed above, the orthogonally

Table 5
The orthogonal transformed component for the received choice structure matrix

Member	Orthogonal transformed component: $\hat{H}_{rd}$					
	I	II	III	IV	V	VI
A	.344	-.001	0	-.021	.516	.081
B	.438	.001	0	-.062	.054	-.267
C	.416	-.016	0	-.189	-.396	.375
D	.350	.079	0	-.106	.220	-.427
E	.495	-.025	0	.258	-.301	.068
F	.340	-.042	0	.228	.334	.258
G	.009	.058	0	-.113	.070	.527
H	-.060	-.485	0	.106	.203	.045
I	.073	-.475	0	.127	-.360	-.212
J	-.051	-.542	0	-.160	.207	-.031
K	.006	-.462	0	-.079	.022	.233
L	0	0	-.577	0	0	0
M	0	0	-.577	0	0	0
N	0	0	-.577	0	0	0
O	.069	-.046	0	-.405	-.230	.082
P	-.004	.042	0	-.611	.173	.031
Q	-.025	.065	0	-.180	.033	.228
R	.102	-.080	0	-.426	-.118	-.287
S	0	0	0	0	0	0
Contr. ^{a)}	.994	.995	.999	.993	.993	.994

^{a)} contribution: the sum of the squared elements of the eigenvector corresponding to each eigenvalue.

and the obliquely transformed component analysis may bring the information concerning the differences between the results based on the choice data and the received choice data. In particular, the obliquely transformed component analysis may be useful to estimate the number of dimensions which is necessary for determining the main subgroups. The suggestion may be done through the consideration of the values of the elements for $\hat{T}_{bd}'\hat{T}_{bc}$ and for $\hat{T}_{bd}'\hat{T}_{ba}$.

It is to be noted here that the six tables except Table 1 can be classified into two kinds of them. One includes Table 2, Table 4, and Table 6, and another, Table 3, Table 5, and, Table 7, respectively. The former tables present the results based on the analysis of the choice data matrix or the product-sum XX' , and the latter, ones the results based on the analysis of the received choice data matrix or the

product-sum $X'X$, respectively. The differences between those two kinds of tables may be apparent though they are not so large. For example, the differences to be noted may be found between the fifth components in the former tables and the same components in the latter tables, and those in Table 5 and Table 7 may not be neglected.

The values of interrelational matrices denoted by $\hat{T}_{bc}'\hat{T}_{bc}$ and by $\hat{T}_{bd}'\hat{T}_{ba}$ in Table 6 and Table 7 may suggest the dimensions for the given sociometric data matrix X . The values for the interrelation between second and fifth components and for the one between fourth and the fifth components in Table 6 are very large, and the value for the interrelation between sixth and the fifth components in the same table is rather large which may not be neglected. From these facts, it may be suggested that fifth and sixth components

Table 6
The oblique transformed component for the choice structure matrix

Member	Oblique transformed component: $\hat{H}_{bc}$					
	I	II	III	IV	V	VI
A	.368	.039	0	-.195	.107	-.156
B	.466	-.042	0	.282	.089	.260
C	.389	.032	0	-.107	.083	-.250
D	.443	.036	0	-.002	.137	-.051
E	.299	.046	0	-.093	.019	-.159
F	.121	.192	0	-.099	-.034	.681
G	.053	-.180	0	-.576	-.487	.519
H	-.037	-.444	0	-.090	-.396	.038
I	.001	-.429	0	.062	-.396	.125
J	-.016	-.442	0	-.006	-.338	-.017
K	-.048	-.471	0	-.068	-.401	-.049
L	0	0	-.577	0	0	0
M	0	0	-.577	0	0	0
N	0	0	-.577	0	0	0
O	.068	.095	0	-.406	-.187	.150
P	.077	-.146	0	-.463	-.168	.091
Q	.079	.028	0	-.037	.032	.108
R	.349	.062	0	-.275	.063	-.126
S	.209	-.287	0	.205	-.229	.044
Reference axis correlations						
Comp. I	1	.055	0	-.168	.139	-.001
Comp. II	.055	1	0	.058	.857	-.054
Comp. III	0	0	1	0	0	0
Comp. IV	-.168	.058	0	1	.408	-.278
Comp. V	.139	.857	0	.408	1	-.399
Comp. VI	-.001	-.054	0	-.278	-.399	1

are to be dependent on the other four components. Although the detailed values are not same, a similar trend can be found in Table 7.

Through these analyses, we can conclude that the main subgroups derived from the choice and the received choice data matrices here consist of the four subgroups; the members A to F as the first subgroup, members H to K as the second subgroup, members L to N as the third subgroup, members O to S as the fourth subgroup, respectively.

Here, it should be noted that the result through the principal component analysis of mutual choice data matrix in Table 1 failed to identify the existence of fourth

subgroup which is possible to include the members Q to S in the choice data matrix of Table 6 and is possible to include the members Q to R in the received choice data matrix each. The differences between the results in Table 6 and the one in Table 7 should be noted though they may be so slight.

In closing this section, the final comment on the computations for the orthogonally and the obliquely transformed component analysis should be made.

In order to avoid the singularities of the matrices in the solutions through (7) to (11), either the preparations for the initializations for the computations or some trials for the determination of the dimen-

Table 7
The oblique transformed component for the received choice structure matrix

Member	Oblique transformed component: $\hat{H}_{bd}$					
	I	II	III	IV	V	VI
A	.415	-.017	0	-.023	.514	.126
B	.430	.011	0	-.056	.345	-.214
C	.385	-.003	0	-.277	.154	.424
D	.360	.084	0	-.068	.302	-.370
E	.438	-.006	0	.200	.350	.066
F	.384	-.053	0	.194	.520	.261
G	.052	.045	0	-.166	.043	.536
H	-.042	-.494	0	.112	.184	.045
I	-.001	-.455	0	.122	.022	-.214
J	-.025	-.550	0	-.143	.130	.012
K	.017	-.467	0	-.106	.121	.258
L	0	0	-.577	0	0	0
M	0	0	-.577	0	0	0
N	0	0	-.577	0	0	0
O	.060	-.040	0	-.428	-.130	.142
P	.050	.032	0	-.599	-.089	.124
Q	.001	.059	0	-.200	-.046	.247
R	.089	-.070	0	-.403	-.083	-.211
S	0	0	0	0	0	0

Reference axis correlations						
Comp. I	1	.026	0	-.107	.861	.147
Comp. II	.026	1	0	.008	-.233	-.052
Comp. III	0	0	1	0	0	0
Comp. IV	-.107	.008	0	1	.214	-.259
Comp. V	.861	-.233	0	.214	1	.141
Comp. VI	.147	-.052	0	-.259	.141	1

sions for the assumed subgroups should be done. If the appropriate dimensions are not given, the orthogonal or the oblique transformations may not be achieved. In this study, the six components are most appropriate through several trials with regard to the determination of the dimensions in that the final solutions can be attained.

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