A NEW OBJECTIVE PROCEDURE FOR THE ORTHOGONAL ROTATION IN FACTOR ANALYSIS (2)

SHIGEO KASHIWAGI

Japanese National Railways

One of the difficulties inherent in the normal varimax method of the orthogonal rotation of factors, i.e., the systematic bias caused by maximization of the sum of the variances of *squared* normalized factor loadings by columns, is pointed out. Then, it is shown that the method we have proposed in the preceding article (1963) provides a satisfactory solution of this difficulty by showing that the result of our solution, when applied on Thurstone's PMA battery, agrees well with his subjective solution.

ON THE SYSTEMATIC BIAS BY THE NORMAL VARIMAX METHOD

The determined rotation angle in the normal varimax method is obtained by maximization of the sum of the variances of *squared* normalized factor loadings by columns. In a certain kind of initial matrices, many negative large factor loadings result by doing this. This difficulty has already been pointed out by the present author (1963). Here, we are going to discuss another difficulty of this method.

Normalization of factor loadings was proposed by Kaiser (1958) in order to correct the systematic bias inherent in the raw varimax method which is caused by maximization of the sum of the variances of *squared raw* factor loadings by columns.

If almost all square roots of communalities of an initial factor matrix are one or nearly one, the normal varimax method is not different from the raw varimax method substantially. And, by normalization in two dimensional cases, we will find all factor loadings on the circumference of a circle with unit diameter and, in three dimensional cases, we will find them on the surface of a sphere with unit diameter. Generally, in n dimensional cases we will find all factor loadings on the surface of a n dimensional sphere

with unit diameter. Then, it becomes apparent that the more the number of factors is, the less the effect of normalization will appear, because as the number of factors increases the greater number of single plane rotations are needed. The normal varimax solution is, in effect, the raw varimax solution of the normalized factor loadings. Therefore, it is apparent that the systematic bias in the raw varimax solution can not be corrected perfectly by the normal varimax method except for two dimensional cases. In the next section we will show how well our method (Kashiwagi, 1963) has solved this point.

RESULTS AND CONCLUSIONS

In Table 1 our solution of Harman and Holzinger's 8 physical variables (Har-

TABLE 1
Harman & Holzinger's 8 physical variable†

Test	Varimax Solution		Subjective		Our Solution	
	A	B	A	B	A	B
1	88	27	88	26	88	26
2	92	21	92	20	92	20
3	89	18	89	17	89	17
4	86	25	86	24	86	23
5	24	90	25	90	25	90
6	18	79	19	79	20	79
7	14	73	14	73	15	73
8	25	68	26	68	26	68
Vp	332	265	335	261	336	260

† Decimal points omitted

TABLE 2

Thurstone's primary mental abilities

Test	Varimax Solution										Thurstone's Subjective										Our Solution																						
	S	P	N	V	M	W	J	R	D	10	11	12	13	14	15	16	17	18	19	20	S	P	N	V	M	W	J	R	D	10	11	12											
4 Reading I	07	03	-03	73	23	11	21	10	-06	-13	19	10	-04	14	42	-02	33	17	29	00	48	05	43	-03	12	03	42	00	15	21	38	-04	56	19	29	54	15	28	11	15	10	24	
5 Reading II	14	12	05	80	20	13	15	07	19	-12	03	-01	-05	12	03	06	10	51	03	22	26	23	12	18	15	18	18	11	09	18	25	21	-12	18	13	42	30	14	29	12	15		
6 Verbal Classification	41	51	03	43	14	18	25	13	15	04	02	-03	-16	12	24	13	36	01	18	12	31	04	09	11	13	14	31	14	16	17	18	17	21	26	14	28	31	17	14	21			
7 Word Grouping	49	63	04	-04	-03	16	53	-06	19	20	-10	03	-08	39	20	08	05	06	41	-10	40	12	27	13	09	41	08	02	13	01	-20	40	02	18	44	-11	20	26	11	20	26		
8 Figure Classification	-07	14	04	48	-11	-11	-11	-11	20	-10	-02	03	-08	39	20	08	05	06	41	-10	40	12	27	13	09	41	08	02	13	01	-20	40	02	18	44	-11	20	26	11	20	26		
9 Controlled Association	08	19	13	55	13	48	04	07	05	16	39	06	17	07	19	03	64	23	33	03	31	-05	32	-03	11	-10	35	03	32	29	54	15	28	11	15	10	24	15	10	24			
10 Inverse Opposites	25	31	-08	59	12	50	05	20	-06	-13	19	10	-04	14	42	-02	33	17	29	00	48	05	43	-03	12	03	42	00	15	21	38	-04	56	19	29	54	15	28	11	15	10	24	
11 Completion	22	16	15	20	03	53	07	13	10	-03	-01	30	-13	16	42	-02	33	17	29	00	48	05	43	-03	12	03	42	00	15	21	38	-04	56	19	29	54	15	28	11	15	10	24	
12 Disarranged Words	24	-05	15	22	02	67	11	-17	12	02	01	03	12	03	06	10	51	03	22	26	23	12	18	15	18	18	11	09	18	25	21	-12	18	13	42	30	14	29	12	15			
13 First & Last Letter	24	-05	15	22	02	67	11	-17	12	02	01	03	12	03	06	10	51	03	22	26	23	12	18	15	18	18	11	09	18	25	21	-12	18	13	42	30	14	29	12	15			
14 Disarranged Sentences	22	57	18	18	13	25	09	06	30	33	10	01	03	12	03	06	10	51	03	22	26	23	12	18	15	18	18	11	09	18	25	21	-12	18	13	42	30	14	29	12	15		
15 Diagrams	07	00	17	05	17	66	11	03	00	16	-11	10	-06	-09	33	04	50	15	37	01	19	31	-02	13	11	-11	32	04	15	23	40	09	11	26	36	35	10	15	10	24			
16 Inverse Synonyms	00	27	17	45	12	53	13	-05	23	00	-11	08	-08	-09	33	04	50	15	37	01	19	31	-02	13	11	-11	32	04	15	23	40	09	11	26	36	35	10	15	10	24			
17 Block Counting	14	17	11	-07	-04	41	13	-12	-15	-21	31	-01	14	41	21	22	01	03	-13	02	36	-02	37	02	13	40	05	11	03	11	06	25	60	-09	02	14	19	10	24	15	10	24	
18 Cubes	74	23	08	-04	-06	22	12	18	05	04	06	-04	-15	41	21	22	01	03	-13	02	36	-02	37	02	13	40	05	11	03	11	06	25	60	-09	02	14	19	10	24	15	10	24	
19 Lozenges A	67	02	13	29	-15	-08	06	03	23	07	10	30	-02	45	07	12	08	03	00	02	34	19	-01	28	53	48	19	-07	18	11	-15	10	23	19	08	35	41	11	10	24	15	10	24
20 Flags	83	-04	20	-02	-04	04	-07	-03	01	01	15	-05	-23	64	00	26	-02	-04	01	-11	22	18	06	43	18	47	14	03	04	32	02	09	57	06	26	13	36	10	24	15	10	24	
21 Form Board	70	11	01	26	02	25	23	16	-08	-08	05	37	-21	-06	42	01	03	06	19	16	13	33	10	50	28	40	73	10	13	15	05	29	12	18	-03	18	11	10	24	15	10	24	
22 Lozenges B	75	10	08	00	23	03	01	22	19	10	02	07	-05	-03	01	12	03	06	10	51	03	22	26	23	12	18	15	18	11	09	18	25	21	-12	18	13	42	30	14	29	12	15	
23 Surface Development	71	07	07	16	27	10	02	07	05	-03	01	12	-12	55	00	16	05	28	-06	04	17	15	31	09	28	58	12	07	10	13	15	05	29	12	18	-03	18	11	10	24	15	10	24
24 Punched Holes	67	07	09	36	03	00	27	06	25	-07	07	31	11	-34	02	07	03	15	-07	18	28	19	13	15	08	-04	45	19	22	07	35	09	27	22	15	10	24	15	10	24			
25 Mechanical Movements	42	-07	-06	17	03	00	27	06	25	-07	07	31	11	-34	02	07	03	15	-07	18	28	19	13	15	08	-04	45	19	22	07	35	09	27	22	15	10	24	15	10	24			
26 Identical Forms	37	57	-10	29	13	08	07	-06	-01	12	-20	-06	-20	32	60	01	-02	13	-06	05	15	29	13	-18	07	18	-05	02	02	10	04	11	54	37	32	25	10	24	15	10	24		
27 Pursuit	63	68	20	04	10	19	10	-01	-12	-20	-06	-20	32	60	01	-02	13	-06	05	15	29	13	-18	07	18	-05	02	02	10	04	11	54	37	32	25	10	24	15	10	24			
28 Copying	46	26	10	09	16	12	49	19	-09	01	07	15	08	27	36	18	-05	22	03	37	17	10	34	25	10	32	18	25	07	20	11	15	05	33	15	00	16	11	10	24	15	10	24
29 Areas	42	-08	10	12	24	08	37	36	07	11	13	20	07	22	-02	13	03	28	08	46	30	07	11	28	17	25	04	23	30	38	20	43	23	-09	06	56	20	10	24	15	10	24	
30 Number Code	45	16	65	09	19	13	21	-05	03	-17	30	10	-06	11	21	63	23	19	03	04	20	21	23	45	10	08	-06	54	-02	04	17	32	-15	-10	06	40	22	15	10	24	15	10	24
31 Addition	14	10	78	-04	11	08	10	-07	05	-04	01	-07	00	-06	03	76	14	05	04	-02	04	22	10	13	02	08	-06	54	-02	04	17	32	-15	-10	06	40	22	15	10	24	15	10	24
32 Subtraction	07	07	74	14	-09	18	-05	02	13	23	11	08	12	-02	01	07	32	00	30	04	08	06	01	01	20	04	06	44	24	06	28	11	-22	-08	01	35	42	10	24	15	10	24	
33 Multiplication	14	12	80	-01	08	15	-02	27	-15	05	04	00	-17	-02	01	81	-02	07	28	07	14	-03	14	02	05	13	-03	73	14	04	17	05	-04	-21	09	23	29	10	24	15	10	24	
34 Division	21	07	60	11	09	12	09	43	13	08	-03	16	-17	06	-02	62	09	-02	18	30	35	20	01	-13	15	14	02	64	30	10	08	13	01	20	10	17	22	10	24	15	10	24	
35 Tabular Completion	23	04	04	13	-02	03	03	08	15	04	-07	03	-08	-03	01	39	13	17	07	48	18	28	19	13	15	08	-04	45	19	22	07	35	09	27	22	15	10	24	15	10	24		
36 Estimating	21	09	36	12	25	06	53	12	30	21	13	07	-03	14	03	02	31	10	-06	39	-07	-07	38	05	05	09	04	-08	36	-02	04	16	30	03	26	17	10	24	15	10	24		
37 Number Series	18	14	40	21	08	-03	12	60	14	09	13	07	-03	00	06	43	-07	13	14	36	53	02	-01	03	10	10	12	61	34	09	-07	01	24	21	-01	09	15	10	24	15	10	24	
38 Numerical Judgment	35	06	38	18	26	10	26	14	09	13	07	-03	-08	01	08	43	-07	13	14	36	53	02	-01	03	10	10	12	61	34	09	-07	01	24	21	-01	09	15	10	24	15	10	24	
39 Arithmetic Reasoning	19	-07	13	34	11	26	24	07	65	-00	17	30	-10	-																													

man, 1960) are shown together with the values which are determined by the normal varimax (Harman, 1960) and the subjective methods (Harman, 1960). In this example we can find how well our solution agrees with the subjective result. But, as its normal varimax solution which corrects for the systematic bias also agrees well with it, it is difficult to determine which result is closer to the subjective result. Nevertheless, if we compare our solution for Thurstone's Primary Mental Abilities of 12 centroid factors (1938) with its normal varimax solution of 13 centroid factors (1960)¹ in Table 2, the former is more appropriate than the latter in that it results in the better levelling of factor contributions (e.g., compare factor contributions of factor S and V in our solution with them in the normal varimax solution). And, if we take the subjective result (Thurstone, 1938) as the criterion, our solution agrees with it very well.

From these examples, we believe that our method satisfies most of Thurstone's criteria of the orthogonal rotation in factor analysis (e.g., positive manifold, simple structure and interpretability of factors).

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¹ It should be noted that Thurstone performed his subjective rotation with regard to 12 centroid factors but not with regard to 13 centroid factors.