THE MEASUREMENT OF THE MAGNITUDE OF THE FIELD-FORCE OF A CIRCLE AND CIRCULAR ARCS¹

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This study is the first step to apply the theoretical formula developed with regard to the field of rectilinear figures to that of curved figures.

Figures drawn on a sheet of paper and luminous figures were employed to examine how the reversal of brightness proportion between the figure and its background influenced the magnitude of the field-force (vector field).

A small test-point was presented at various positions away from the center of a circle and circular arcs.

Results indicate that, (1) the reversal of the brightness has little influence on the vector field, (2) the amount of displacement of the test-point varies according to the distance between the figure and the test-point, and is influenced by the length of the circular arc.

The foregoing seems to allow to conclude that the theoretical formula of the rectilinear figures can be applied to estimate the vector field of the curved figures.

PURPOSE

The purpose of this study is to deduce the theoretical formula of the vector field regarding curved figures from that of rectilinear figures by means of measuring the magnitude of the field-force of a circle and circular arcs.

The theoretical formula of the rectilinear figures, which had been established by Yokose, was made public elsewhere in 1957. Yokose and Uchiyama measured the stimulus threshold of a small light point projected at various positions around the luminous figure. This attempt made it clear that the threshold varied accordingly as the distance between the figure and the small point, the form, and size of the figure varied. They assumed a field involving the influence of the figure upon the small point, and called it a "potential field." The following formula has been established under the assumption that the influence-strength of the figure upon the small test-

point is proportional to the total influence of minute parts of the figure, and inversely proportional to the distance between the figure and the test-point. The value of the potential field is given by the formula:

$$Mp = \frac{\sin \theta_1 + \sin \theta_2}{D}^2$$

And, this formula has been proved to be reasonable through many verification experiments.

The potential field itself is considered to be a sort of potential enhancement. But in a case where there exist two fields located closely together, the difference of the enhancement caused by either one of them will generate a shifting flow from one to the other. This is; in the relation of the luminous figure and the small light point, it is generally known that the small

² Mp : Potential

V : Vector

D : The distance between the figure and the small test-point.

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point shifts toward the figure which has stronger field-force. This phenomenon has been verified by Yokose & Kawamura, Yokose & Ichikawa, and Uchiyama & Tsuji.

It was found that the test-point was displaced toward the figure through the center of the angle. Yokose placed the potential values at the both edges of the angle and transposed them toward the direction of the displacement. Hence, the amount of displacement is figured out as a resultant force of the two potential values, and the following theoretical formula of the vector field has been established :

$$V = 2(Mp) \cos \left(\frac{\theta_1 + \theta_2}{2} \right)^2$$

Now, some of the geometrical optical illusions produce such a very impressive phenomenon as displacement. Yokose attempted to justify the theoretical formula through the verification experiment, using Ebbinghouse's illusion, and reached the firm conclusion that the variation of the amount of illusion caused by the variation of stimulus condition tended to be identical with the anticipated value induced by the theoretical formula. Many other verification experiments relating to the various illusions have been carried out so far. Goto proposed an assumption that the filled-unfilled illusion might be caused by the displacement of individual lines involved, and confirmed that the theoretical formula could be applied to the illusion.

Though the theoretical formula has been applied to many types of figures, most of the applications have been restricted to the rectilinear figures. In order to extend the applicability of this theoretical formula, the case study of the curved figures should be intensified.

This study was designed to take up the following three cases ; (1) a segmental line which was tried often in the past, (2) a circle, considered to be one of the simplest types of curved figures, (3) three different kinds of circular arcs, with equal radius

but different length.

The experiment was performed as follows : 1. The amount of displacement of the test-point was measured at various positions away from the center of the figure.

2. The amount of displacement was measured by the apparent space distance between the small measure-point and another one placed at a given interval.

In a case where the vector formula is constructed based on the potential formula, the stimulus threshold of the small light point should be on an identical level with the perception of the distance between the figure and the test-point involved. If there is little individual difference of visual cells of retina in size and sensitivity, and if the cells act upon the all-or-none law, it can be reasoned that the number of the cells affected will vary as the figure alters. Hence, the apparent space distance seems to be influenced by the number of the cells, and a phenomenon may occur that the figure attracts the test-point inward.

In the experiments verifying the vector formula, the figure applied was usually drawn on a white sheet of drawing paper. The small point was also dealt with in the same fashion. However, in the experiments verifying the potential formula, the luminous figure and the small light point were applied on the dark stimulus presenting screen. Then, it was examined whether these two different stimulus conditions had any effect on the displacement of the small test-point or not.

A completely identical procedure was applied to the experiment 1 in which the figure drawn on a sheet of paper was used as well as the experiment 2 in which the luminous figure was employed. And the results of the two experiments were evaluated in comparison.

PROCEDURE

Stimulus figures. Figures were a segmental line 30 mm long, four kinds of circular materials of $1/2 \pi$, π , $3/2 \pi$, and 2π all with 15 mm in-

terior radius. They were all 3 mm wide as shown in Fig. 1.

The test-point was presented near around the figure. The scale-point was presented 80 mm off the test-point at the right-hand side. The two measure-points were presented 70 mm underneath and 90 mm off the test-point at the right-hand side. In Exp. 1, all these points presented were solid square of a side 1 mm each. In Exp. 2, the test-point and the scale-point were solid circle 1 mm in diameter, and the two measure-points were solid square of a side 1 mm each. The positional layout of these four points is shown in Fig. 2.

Measuring positions. The test-point was presented at the distance of 2, 5, 10, 15, and 20 mm off the center of the figure, respectively.

General procedure. This experiment was conducted by the method of adjustment in which the experimenter made adjustments. The subject was requested to compare the distance between the test-point and the scale-point (d') with the distance between the two measure-points (d''), and to observe them binocularly and with eye movements.

Subjects. Four students majoring in psychology served as subjects, who has normal visual acuity and enough experience in this type of experiment.

FIG. 1.

$d = 2, 5, 10, 15, 20$ mm S_{po} : Scale-point
 F_g : Figure M_{po} : Measure-point
 T_{po} : Test-point FIG. 2.

Experiment 1

This experiment was designed to measure the amount of displacement of the test-point which was presented at various positions away from the center of the figure drawn on a sheet of paper.

Apparatus. The apparatus used is schematized in Fig. 3. The stimulus presenting screen (B) was set at the distance of 100 cm from the observing window (A). The luminous intensity on the surface of the screen was constantly maintained at 20 m.L. The figure (6% in its reflection rate) was observed through the frame-screen (C). The figure was drawn in black ink on a sheet of drawing paper (86% in its reflection rate). The test-point, the scale-point, and the measure-point at the left-hand side were all drawn in black ink on a piece of transparent glass (a). The measure-point at the right-hand side was drawn on another piece of transparent glass (b). The drawing paper and the two pieces of glass were put one over another, and the distance between the two measure-points could be changed at will by moving horizontally the uppermost laying glass (b) to the right and the left.

Results. Average results of 3 Ss are

A: Observing window
 B: Stimulus presenting screen
 C: Frame-screen
 D: Illuminating lamps
 E: Voltage transformer

FIG. 3.

shown in Figs. 4 and 5.

Fig. 4 indicates that the amount of displacement is plotted against the distance between the figure and the test-point. Fig. 5 indicates that the amount of displacement is plotted against the figure. The experimental value of the amount of displacement may be obtained by assessing the remainder between the apparent distance of the test-point and the scale-point when the figure is present and that distance with the figure absent. So, the positive experimental value means that the test-point should be displaced toward the left (toward the figure), and the negative one means that the test-point should be displaced toward the right (away from the figure).

(1) The amount of displacement toward the circle and the circular arcs decreases as the test-point shifts away from the figure. The trend of the above phenomenon can be verified as in Fig. 4; the plotted curves involving the figure run down as they move toward the right, and the plotted curves involving the distance between the figure and the test-point run almost parallel. As for the segmental line, the trend is in the same as the foregoing at the positions of 5 mm and beyond, while the test-point is perceived to be shifting away from the figure at the position of 2 mm. (2) It can be reasoned that the difference of the figure should correspond to the difference of the length of the circular arc. At the position of 2 mm, the amount of displace-

FIG. 4.

TABLE 1
Analysis of the variance of the results in Exp. 1

Source	SS	df	Variance	F
Distance (D)	287.99	4	72.00	30.77**
Figure (F)	54.78	4	13.70	5.85**
Subject (S)	801.92	2	400.96	171.35**
D×F	218.13	16	13.63	5.82**
D×S	27.79	8	3.47	1.48
F×S	34.61	8	4.33	1.85
D×F×S	17.42	32	0.54	—
Error	703.37	300	2.34	

Total 2146.01 374
** Significant beyond 1% level

ment toward the figure is on the rising trend as the length of the circular arc increases. The diagram of Fig. 4 verifies this tendency; the plotted curves stay in layers, and Fig. 5 confirms that the plotted curve runs upward as it moves toward the right. However, the above mentioned trend is found to take place merely within the distance ranging from 2 mm to 5 mm. Further, the amount of displacement increases remarkably at any distance between the figure and the test-point as the figure $3/2\pi$ is replaced by the 2π .

An analysis of variance was carried out as shown in Table 1. The results of the analysis on the distance between the figure and the test-point (D), the difference of the figure (F), and the subject (S) are all significant. The interaction between the distance and the figure (D×F) is also significant. They are all significant beyond the 1% level.

FIG. 5.

Experiment 2

This experiment was designed to measure the amount of displacement of the light test-point which was placed at various positions away from the center of the luminous figure, being projected from behind.

Apparatus. The apparatus used is schematized in Fig. 6. The stimulus presenting screen (B) was set at the distance of 70 cm from the observing window (A). The figure was cut out into the shape of segmental line, $1/2\pi$, π , $3/2\pi$, and 2π , respectively, in the center of five separate sheets of black paper (3% in its reflection rate). And, they were all pasted on five

- A: Observing window
B: Stimulus presenting screen
C: Light source
D: Projector
E: Screen
F: Voltage transformer

FIG. 6.

FIG. 7.

separate pieces of transparent glass. The figure was illuminated from the back by scattering light which came through the clouded-glass facing against one side of the light source (C). Both the test-point and the scale-point were projected as a circle 1 mm in diameter on to the stimulus presenting screen (made of a black sheet of paper) by a projector (D). The two measure-points were presented underneath the test-point at the right-hand side. To provide the measure-point at the left-hand side, a square of a side 1 mm each was cut out on the stimulus presenting screen. And then, in order to provide a movable measure-point which should be positioned at the right-hand side, a horizontal line 1 mm wide and 30 mm long was cut out on the stimulus presenting screen, while a vertical line 1 mm wide and 10 mm long was cut out on another sheet of paper which was fastened on a piece of transparent glass. Then, the stimulus presenting screen with the horizontal line and another piece of glass with the vertical line were put together so that the point where the horizontal line and the vertical one crossed each other produced the right-hand side measure-point of a square of a side 1 mm each. By moving horizontally the uppermost laying piece with the vertical cut-out line in both directions of the right and the left, the distance between the two measure-points could be changed at will. The luminous intensity of both the figure and the points were kept at 1 mL.

Results. Average results of 3 Ss are shown in Figs. 7 and 8.

(1) A similar trend is found in this Exp. 2 as in Exp. 1 concerning to the effect

FIG. 8.

of the distance between the figure and the test-point. The plotted curves run down as they move toward the right as in Fig. 7, and the plotted curves run almost parallel as in Fig. 8. The foregoing can be interpreted to demonstrate that the distance between the figure and the test-point should produce a dominant effect on the amount of displacement of the test-point. (2) It is clearly recognized how the difference of the figure influences the amount of displacement; i.e. as in Fig. 7, the plotted curves run almost parallel. And, as in Fig. 8, the amount of displacement of the test-point increases as the length of the circular arc increases. The decrease of the amount of displacement observed when the figure is changed from the segmental line to the $1/2\pi$ and the increase seen when the figure is switched from the $3/2\pi$ to the 2π are more remarkably noted in Exp. 2 than in Exp. 1.

An analysis of variance was carried out as shown in Table 2. The results of the analysis on the distance between the figure and the test-point (D), the difference of the figure (F), and the subject (S) are all significant. The interaction between the distance and the figure ($D \times F$), and between the figure and the subject ($F \times S$) are also significant. They are all significant beyond the 1% level.

DISCUSSION

Estimate of results. When the vector formula given in the foregoing Purpose is applied to the field around the circle, the vector (V) at point (P) in Fig. 9 is represented as follows:

FIG. 9.

TABLE 2
Analysis of the variance of the results in Exp. 2

Source	SS	df	Variance	F
Distance (D)	288.56	4	72.14	68.70**
Figure (F)	278.94	4	69.74	66.42**
Subject (S)	254.60	2	127.30	121.24**
$D \times F$	183.23	16	11.45	10.90**
$D \times S$	6.46	8	0.81	—
$F \times S$	69.80	8	8.73	8.31**
$D \times F \times S$	11.28	32	0.35	—
Error	314.58	300	1.05	—
Total	1407.45	374		

** Significant beyond 1% level

$$V = 2(Mp) \cos \theta \quad (\text{I})$$

Now, the potential (Mp) at point (P) is

$$V = \frac{2 \sin \theta}{D} \quad (\text{II})$$

Consequently, the vector formula pertaining to the circle is established as follows:

$$V = \frac{4 \sin \theta \cdot \cos \theta}{D} \quad (\text{III})$$

The anticipated value in Fig. 10 can be obtained by applying the above vector formula to the segmental line and the circle used in this experiment. In the diagram of Fig. 10, the distance between the figure and the test-point is indicated on the horizontal axis, and the vector (V), which is regarded as the relative value of the amount of displacement of the test-point, is given on the vertical axis. It results from this formula that the figure's part immediately

FIG. 10.

confronting the test-point seems to cause the maximum influence on the test-point. Consequently, the anticipated value seems to be almost leveled off in all four circular figures.

Meanwhile, Yokose and Ito conducted a series of experiments to relate the potential formula to the curved figures. In the course of this study, Ito established a "formula covering the whole range of the field-strength at optional points inside as well as outside a circle and circular arcs." It is

$$Mp = 2 \int_0^x \frac{R \cdot d\theta}{R^2 + (R+D)^2 - 2R(R+D) \cos \theta} \quad (IV)$$

Hence, the potential (Mp) at point (P) in Fig. 11 is inversely proportional to the square of the distance between minute parts of the figure and the point, and proportional to the total influence of minute parts of the figure.

When the potential value obtained from the formula (IV) is applied to the formula (I), the anticipated value in Fig. 12 is gained. The plotted curves indicate that the amount of displacement varies according to the difference of the figure. But the scale of the vertical axis does not accord with the one in Fig. 10, and the value in Fig. 12 is larger than the one in Fig. 10. The anticipated value and the experimental value will be compared later on.

General results. First of all, let us note the following point; the two different stimulus conditions such as figures drawn on sheets of paper and luminous ones raise a problem involving the reversal of bright-

FIG. 11.

ness proportion. But the reversal of the brightness has little effect upon the amount of displacement of the test-point. The brightness proportion between the luminous figure and its background is more conspicuous, and the total amount of displacement of three S s grows larger as well. The two S s, who participated in both experiments, demonstrate little difference to be noted. As for what effect the distance between the figure and the test-point produces, it is not recognized that the two different stimulus conditions make any difference on the distance. It should be considered that in a case of using the luminous figure, the figure's part which does not immediately confront the test-point seems to cause more distinguished influence on the test-point. The foregoing can be verified by the fact that the experimental value pertaining to the figure drawn on a sheet of paper follows a similar pattern with the anticipated value shown in Fig. 10, where only the figure's part immediately confronting the test-point is involved. While, the experimental value relating to the luminous figure resembles the anticipated value shown in Fig. 12, where the length of the circular arc is taken into account.

As the test-point is shifted away from the figure, the amount of displacement of the test-point tends to decrease. That the influence which the figure causes on the surrounding field is inversely proportional to the distance between the figure and the

FIG. 12.

test-point has been esteemed as one of the fundamental assumptions constructing the theoretical formula, which is described in Figs. 10 and 12. As far as the segmental line is concerned, the amount of displacement of the test-point makes a remarkable decrease when the test-point is placed nearest to the figure. The anticipated value in Fig. 10 demonstrates the same tendency as the above result.

When the figure is switched from the segmental line to the $1/2\pi$, more conspicuous decrease of the amount of displacement is detected at the measuring positions of 10 mm and beyond. And, this may be proved by the fact that the length of the $1/2\pi$ is actually shorter than that of the segmental line. A remarkable increase of the amount of displacement is recognized in Exp. 1 as well as Exp. 2 when the figure is switched from the $3/2\pi$ to the 2π . The greatest difference observed between the two figures is due to the fact that the part of the circular arc confronting the test-point indirectly and almost perpendicularly is counted in a case of the 2π . Hence, it is found that any part of the figure which is placed face to face to the test-point perpendicularly tends to exert more influence on the amount of displacement than the part facing the test-point from the direction of diagonal angle.

As far as these experiments are concerned, the reversal of brightness proportion has no significant effect on the amount of displacement of the test-point. But it is found that the distance between the figure and the test-point has a great effect on the amount of displacement. Further, the theoretical formula proves to be able to anticipate such a tendency as above. Even where it confronts the test-point indirectly, the part of the figure has a little influence on the test-point. The formula (IV) anticipates the foregoing. Therefore, it is important to find out at what angle the figure and the test-point are confronted each other either directly or indirectly.

These experiments were conducted at only a few restricted measuring positions

around the circle and the circular arcs. More extensive experiments may be necessary for the verification of the above mentioned results.

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