THE EFFECTS OF LEARNING EXPERIENCE ON THE ABILITY OF ELEMENTARY SCHOOL STUDENTS TO DEAL WITH MATH PROBLEMS REQUIRING MULTIPLE SOLUTIONS

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Elementary school students tend to produce single solutions for math problems that require two or more solutions. To generate multiple solutions, students must be aware of the existence of multiple solutions to math problems that require them, and then think of multiple solutions. This study examined whether learning experiences that enabled fourth- and fifth-grade students to think of multiple solutions subsequently increased the spontaneous production of multiple solutions to math problems that required them. The results of Investigation 1 indicated that students could spontaneously arrive at multiple solutions immediately after the learning experience, but 6 days later most returned to producing a single solution. The results of Investigation 2 indicated that students could reproduce multiple solutions 3 days after the learning experience, if they were alerted to the existence of multiple solutions. These results suggest that while learning experience increases the likelihood of producing multiple solutions, students tend to stop considering the possibility of multiple solutions existing.

Key words: elementary school students, math problems, multiple solutions, mathematics learning

Japanese junior high school mathematics textbooks (Keirinkan, 1999a; Ministry of Education, Science, Sports and Culture, 1998) include various problems that have multiple, rather than single, values as their correct solutions. For example, the problem “What is the number for which the absolute value is 1?” appears in the first year junior high school textbook in the chapter titled “Numbers and Formulas.” The correct solutions are +1 and –1 (In the Japanese language, singular and plural forms are not differentiated, so the question does not convey how many solutions there might be.) In order to solve this problem, it is first necessary for students to consider two cases: the case of a positive number and that of a negative number. They are then required to think of a solution for each case. There exist several types of problems that require multiple solutions not only in junior high school and high school mathematics textbooks but also in the real world.

All word problems in Japanese elementary school mathematics textbooks (Keirinkan, 1999b; Ministry of Education, Science, Sports and Culture, 1999) have single solutions. I am grateful to Masuo Koyasu for his instructive comments on previous versions of this article. I am also grateful to Aiko Morita, Hiromi Yamagata, Hanae Ando, and Michiko Miyahara for their helpful comments on an earlier version of this article.

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values as their correct solutions. Elementary school students encounter such single-solution word problems in their everyday learning of school mathematics. These problems might encourage students to believe that there is only one correct solution and to stop thinking further once that single solution has been found. Although mathematics education does not aim to promote such a tendency, school textbooks in mathematics generally promote it (e.g., Keirinkan, 1999b; Ministry of Education, Science, Sports and Culture, 1999).

In the fields of educational and developmental psychology, previous studies have primarily used single-solution word problems as cognitive tasks (for reviews, see Baroody & Dowker, 2003; Dunlan, 1998; Ginsburg, Klein, & Starkey, 1998; Nunes & Bryant, 1996; Reed, 1999). Several empirical studies (Kinda, 2003; Reusser & Stebler, 1997; Verschaffel, De Corte, & Lasure, 1994; Wyndhamn & Säljö, 1997; Yoshida, Verschaffel, & De Corte, 1997) have used multiple-solution word problems. The results of these studies provide information regarding such a single-solution tendency in elementary school students.

Verschaffel et al. (1994), Reusser and Stebler (1997), and Yoshida et al. (1997) used countless-solution word problems (see Table 1) to examine whether 4th and 5th grade elementary school students could apply their knowledge of multiple solutions from the real world in the context of solving mathematics problems at school. The results showed that approximately 95% of the students produced only one solution to the problem (Reusser & Stebler, 1997; Verschaffel et al., 1994) even when they were provided with an extra hint, such as “the test contains several problems that are difficult or impossible to solve because of certain unclarities or complexities in the problem statement (Yoshida et al., 1997).” However, in group learning situations wherein students could interact with one another (Wyndhamn & Säljö, 1997), as a result of group discussions, approximately 88% of the 10- to 11-year-old students were able to understand that the problem had countless possible solutions.

On the basis of the above studies, Kinda (2003) used two-solution (see Table 1) and countless-solution word problems in order to examine the cognitive process of solving multiple-solution word problems among 5th and 6th grade elementary school students. As

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<tr>
<th>Types</th>
<th>Examples</th>
<th>Number of solutions</th>
<th>Number of illustrations</th>
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<tbody>
<tr>
<td>Two-solution word problems</td>
<td>Yamada has a pencil that is 12 cm in length.</td>
<td>2</td>
<td>2</td>
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<tr>
<td></td>
<td>The length of Tanaka’s pencil differs from Yamada’s pencil by 5 cm. How long is Tanaka’s pencil?</td>
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<tr>
<td>Countless-solution word problems</td>
<td>Bruce lives at a distance of 500 meters from the school and Alice lives at a distance of 300 meters from the school. How far do Bruce and Alice live from each other?</td>
<td>Countless</td>
<td>Countless</td>
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the name indicates, the two-solution word problems had only two possible solutions, whereas the countless-solution word problems had countless possible solutions. Kinda (2003) compared the correct response rates of the elementary school students to the problems under three conditions. The correct response rate was the ratio of students that produced multiple solutions under each condition. Under Condition 1, the problems were presented without providing any information on the existence of multiple solutions. In order for the students produce the correct answer under this condition, they would have had to go through the following cognitive processes: (i) notice the existence of multiple solutions; (ii) identify the cases possible; and (iii) think of solutions for each of the possible cases identified. Under Condition 2, the problems were presented along with information suggesting the existence of multiple solutions; hence, students had to go through only (ii) and (iii) of the abovementioned cognitive processes. Under Condition 3, the problems were presented along with multiple identified cases; hence, students had to go through only the third cognitive process. The results indicated that the correct response rates under Conditions 1, 2, and 3 were 0%, 4%, and 42% respectively in the countless-solution word problem, and 11%, 41%, and 88% respectively in the two-solution word problem.

Verschaffel et al. (1994), Reusser and Stebler (1997), and Yoshida et al. (1997) provided students with no prior information regarding multiple solutions; therefore, the conditions employed in their studies were equivalent to Condition 1 in Kinda (2003). In Wyndhamn & Säljö (1997), group discussions enabled students to identify multiple cases; therefore, this situation was almost equivalent to Condition 3 in Kinda (2003).

The difference between the correct response rates under Conditions 1 and 2 implies that students need to spontaneously notice the existence of multiple solutions under Condition 1. Likewise, the difference between correct response rates under Conditions 2 and 3 implies that students need to spontaneously identify the cases possible under Condition 2.

Previous studies have revealed that elementary school students have a single-solution tendency (Reusser & Stebler, 1997; Verschaffel et al., 1994; Yoshida et al., 1997) and that they need to spontaneously notice the existence of multiple solutions and identify the cases possible in order to produce multiple solutions when solving word problems requiring them (Kinda, 2003). However, these studies do not clarify the way in which students are able to spontaneously notice the existence of multiple solutions and identify the cases possible. Therefore, the way in which they will be able to overcome their single-solution tendency is unclear.

This study aims to examine whether learning experiences that alert students to the existence of two solutions subsequently increase their likelihood of producing two solutions when solving two-solution word problems. Elementary school students have never had the experience of solving two-solution word problems in their everyday learning of school mathematics. Students might be able to spontaneously produce two solutions once they acquire the learning experience necessary to produce these. Kinda (2003) demonstrated that under Condition 2, 41% of the students could produce two solutions to the two-solution word problems, and under Condition 3, 88% of the students
could produce them. These students are assumed to have acquired the learning experience of producing two solutions. After these experiences, they might have the ability to spontaneously notice the existence of two solutions and identify the cases possible to the two-solution word problems.

In the following two investigations, the ways in which elementary school students solved the two-solution word problem were examined before and after the learning experience that alerted the students to the existence of two solutions. The results of these investigations will indicate whether elementary school students were able to spontaneously produce two solutions to the problems due to their learning experience. In turn, it is hoped that this will lead to the clarification of an essential condition for elementary school students to be able to overcome their single-solution tendency.

**INVESTIGATION 1**

**Purpose**

The purpose of Investigation 1 was to examine whether learning experiences that alert students to the existence of two solutions consequently increase their likelihood of producing two solutions when solving two-solution word problems.

**Method**

**Participants:** The participants comprised 65 4th grade elementary school students (28 males and 37 females) from two classes of a public elementary school in Kyoto, Japan. The average age of the groups of participants was 9.89 years (range: 9–10). Two of these students (one male and one female) were excluded from the analysis because they were each absent on one of the two investigation days.

**Materials:** A two-solution word problem (see Table 1) was used. In order to avoid repetition of this problem with the same contents as under the conditions in Investigation 1, several versions of this problem were prepared. These versions had two solutions and the same structure and differed only in terms of the nouns and numbers used in their contents.

**Procedure:** Investigation 1 was conducted in two days with an interval of six days in between, with the cooperation of the teachers responsible for each class. The teachers confirmed that the students in the class had no prior experience of solving two-solution word problems.

Three conditions were used in this investigation. In Condition 1, the students were given the problems and simply instructed to “write (the) formula and (the) solution” to the problem, and to “draw (the) appropriate illustration” of the problem. The instructions were given in Japanese — a language in which there is usually no distinction between singular and plural cases. Hence, the instructions did not provide the students with any indications regarding the number of formulas, solutions, and illustrations that were required.

In Condition 2, the students were given the problems and instructed as follows: “write (the) formula and (the) solution. This problem has two solutions. If you know the two solutions, write the two formulas and the two solutions below. Draw (the) appropriate representation of this problem. This problem can be presented in two appropriate illustrations. If you know the two illustrations, draw them below.”

In Condition 3, the students were given the problems in the following two patterns. First, one appropriate solution was presented by “Hanako,” a character that appeared on the investigation booklet, and the students were asked, “Hanako read this problem and thought of the following solution. What kind of formula did Hanako think of? Write the formula that Hanako thought of. What type of illustration did Hanako draw when she was thinking about this problem? Draw the illustration that Hanako thought of.” Next, the other appropriate solution was presented by “Sachiko” and the above questions were repeated. Second, one appropriate illustration was presented by “Taro” and the students were informed, “Taro read this
problem and drew the following illustration. What type of formula did Taro think of? Write the formula that Taro thought of. What type of solution did Taro think of? Write down the solution that Taro thought of.” Next, the other appropriate illustration was presented by “Jiro” and the above questions were repeated.

Under each condition, the students were asked to draw (the) appropriate illustration of the problem as well as to write (the) formula and (the) solution to it. This was because previous studies noted that elementary school students tended to write (the) formula and (the) solution to the problem without comprehending them (for reviews, see De Corte, Greer, & Verschaffel, 1996; Reed, 1999). Therefore, in this study, the students’ illustrations were used as an additional indicator of comprehension.

On the first day of the investigation, the two-solution word problems were presented to the students in the following order: Condition 1 (first presentation), Condition 2 (first presentation), Condition 3 (first presentation), and Condition 1 (second presentation). On the second day of the investigation, six days later, the word problem was presented under Condition 1 (third presentation). Table 2 shows the schedule that was used.

The students were cautioned, “Do not turn the pages before you are told to do so. Wait for the teacher’s instructions. You are not allowed to ask questions about the contents of the problems, so concentrate on the problems on your own.” The teachers were asked not to explain anything regarding multiple solutions before the completion of all the investigations.

The time limit for thinking under each condition was set at approximately 3–4 minutes. The total time required was approximately 25 minutes on the first day of the investigation and approximately 15 minutes on the second day. After the completion of the entire investigation, the teachers explained the approaches for producing two solutions to all the participants.

The aim of this procedure was as follows. Students were required to spontaneously notice the existence of two solutions and identify the cases possible under Condition 1 (first presentation) to produce two solutions. They were also required to spontaneously identify the cases possible under Condition 2 (first presentation). However, they were not required to do these under Condition 3 (first presentation). An assumption was made that the students would be able to produce two solutions more easily under the conditions that required them to go through fewer cognitive processes.

After the students had solved the problem under Condition 1 (first presentation), Condition 2 (first presentation), and Condition 3 (first presentation), they solved the problem under Condition 1 (second presentation) and Condition 1 (third presentation) six days later. The difference between the correct response rates under Condition 1 (first presentation) and Condition 1 (second presentation) will reveal the effects of the learning experience under Condition 2 (first presentation) and Condition 3 (first presentation). The difference between the correct response rates under Condition 1 (second presentation) and Condition 1 (third presentation) will indicate the extent to which the effects of the learning experience were retained with the passage of time.

Results and Discussion

The correct response under Conditions 1 and 2 involved producing two appropriate
formulas, two appropriate solutions, and two appropriate illustrations. The correct response under Condition 3 involved producing two appropriate formulas and solutions for both the presented illustrations and the presented solutions.

The correct response rates under Condition 1 (first presentation), Condition 2 (first presentation), and Condition 3 (first presentation) were 10%, 40%, and 70%, respectively. The correct response rates increased under the condition that required students to go through fewer cognitive processes. These results provide additional confirmation of the results obtained by Kinda (2003).

In order to better understand the ability of each student to respond correctly in the first presentation, they were categorized into four groups (see Table 3). Group 1 comprised the students who could respond correctly under Condition 1 (first presentation), Condition 2 (first presentation), and Condition 3 (first presentation). Ten percent of the students were categorized into this group. It was assumed that they had the ability to spontaneously notice the existence of two solutions and identify the cases possible because they responded correctly under Condition 1 (first presentation).

Group 2 comprised the students who were unable to respond correctly under Condition 1 (first presentation), but they could respond correctly under Condition 2 (first presentation) and Condition 3 (first presentation). Thirty percent of the students were categorized into this group. It was assumed that they lacked the ability to spontaneously notice the existence of two solutions, but had the ability to spontaneously identify the cases possible. This was because they did not respond correctly under Condition 1 (first presentation) but responded correctly under Condition 2 (first presentation).

Group 3 comprised the students who were unable to respond correctly under Condition 1 (first presentation) and Condition 2 (first presentation), but who responded correctly under Condition 3 (first presentation). Twenty nine percent of the students were categorized into this group. It was assumed that they lacked both the ability to spontaneously notice the existence of two solutions and identify the cases possible because they did not respond correctly under Condition 1 (first presentation) and Condition 2 (first presentation).

Group 4 comprised the students who were unable to respond correctly under

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<th>Condition 2</th>
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<tr>
<td></td>
<td>first presentation</td>
<td>first presentation</td>
<td>first presentation</td>
<td>Investigation 1</td>
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<tr>
<td>Group 1</td>
<td>+</td>
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<td>Group 2</td>
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<td>Group 3</td>
<td>+</td>
<td>29 (18)</td>
<td>26 (13)</td>
<td>30 (19)</td>
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<tr>
<td>Group 4</td>
<td>30 (19)</td>
<td>12 (6)</td>
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Note: “+” means that the students solved the problem correctly under the condition.
Condition 1 (first presentation), Condition 2 (first presentation), and Condition 3 (first presentation). Thirty percent of the students were categorized into this group. It was assumed that they lacked both the ability to spontaneously notice the existence of two solutions and identify the cases possible because they did not respond correctly under Condition 1 (first presentation) and under Condition 2 (first presentation). Although the students in Groups 1, 2, and 3 appear to have benefited from the experience of producing two solutions under one or more conditions in the first presentation, the students in Group 4 did not seem to benefit from such experience.

The abovementioned four groups accounted for 98% of the students. These results indicated that most students who responded correctly under Condition 1 (first presentation) could respond correctly under Condition 2 (first presentation) and Condition 3 (first presentation), and that most students who responded correctly under Condition 2 (first presentation) could respond correctly under Condition 3 (first presentation). These results corresponded to the assumption regarding the cognitive processes required under the three conditions.

The changes in the correct response rates under Condition 1 were examined for each group (Fig. 1). In Group 1, the correct response rates were 100%, 100%, and 100% under Condition 1 (first presentation), Condition 1 (second presentation), and Condition 1 (third presentation), respectively. This result shows that after they had responded correctly under Condition 1 (first presentation), students could consistently notice the existence of two solutions and identify the cases possible.

In Group 2, the correct response rates were 0%, 68%, and 37% under Condition 1 (first presentation), Condition 1 (second presentation), and Condition 1 (third presentation), respectively. This result showed that 68% of the students were able to spontaneously
notice the existence of two solutions and respond correctly under Condition 1 (second presentation) as a result of the learning experience under Condition 2 (first presentation) and Condition 3 (first presentation). However, six days later, only 37% of the students could respond correctly under Condition 1 (third presentation). This shows that the effect of the learning experience had not been well retained with the passage of time.

In Group 3, the correct response rates were 0%, 39%, and 17% under Condition 1 (first presentation), Condition 1 (second presentation), and Condition 1 (third presentation), respectively. This result showed that 39% of the students were able to spontaneously notice the existence of two solutions and identify the cases possible and respond correctly under Condition 1 (second presentation) as a result of the learning experience under Condition 3 (first presentation). However, six days later, only 17% of the students were able to respond correctly under Condition 1 (third presentation). This also showed that the effect of learning experience had not been retained with the passage of time.

In Group 4, the correct response rates were 0%, 0%, and 11% under Condition 1 (first presentation), Condition 1 (second presentation), and Condition 1 (third presentation), respectively. The students in this group had acquired the experience of solving the problem, but they lacked the experience of producing two solutions under each condition in the first presentation. The reason that 11% of the students could respond correctly under Condition 1 (third presentation) was unclear. It is possible that during the interval period, these students had learned to produce two solutions outside the investigation design, such as from their friends.

In summary, the results of Investigation 1 clarify the effects of the learning experience on students’ ability to notice the existence of two solutions. The students in Group 2 were able to spontaneously notice the existence of two solutions as a result of their learning experience. However, the effects of the learning experience had not been well retained with the passage of time.

It needs to be noted, however, that the results of Investigation 1 did not clarify the effects of the learning experience on the students’ ability to spontaneously identify the cases possible. In Group 3, the effects of the learning experience on this ability have not been examined independent of the effects on the ability to spontaneously notice the existence of two solutions. Therefore, the reason that the correct response rate in Group 3 decreased from what it was under Condition 1 (second presentation) to that under Condition 1 (third presentation) is unclear.

**INVESTIGATION 2**

**Purpose**

The purpose of Investigation 2 was to examine the effects of the learning experience on the ability to spontaneously identify the cases possible without the need to spontaneously come up with the existence of two solutions.
Method

Participants: The participants comprised 50 5th grade elementary school students (26 males and 24 females) from two classes of a public elementary school in Kyoto, Japan. The average age of the groups of participants was 10.20 years (range: 10–11). The school in Investigation 2 was different from that in Investigation 1.

Materials: The problems used here were identical to those used in Investigation 1.

Procedure: Investigation 2 was conducted in two days with an interval of three days in between, with the cooperation of the teachers responsible for each class. The teachers confirmed that the students in the class had no prior experience of solving two-solution word problems. The difference in the interval period between Investigation 1 and Investigation 2 was due to the scheduling conditions at the schools subjected to the investigation.

Three conditions identical to those in Investigation 1 were used here. On the first day of the investigation, the two-solution word problems were presented to the students in the following order: Condition 1 (first presentation), Condition 2 (first presentation), Condition 3 (first presentation), and Condition 2 (second presentation). On the second day of the investigation, three days later, the two-solution word problem was presented under Condition 2 (third presentation). The schedule used is shown in Table 2.

Whereas the problem in Investigation 1 was repeatedly presented under Condition 1, in the case of Investigation 2, it was repeatedly presented under Condition 2. Under this condition, students were required to spontaneously identify the cases possible in order to produce the two solutions, but they did not need to spontaneously notice that the problem had two solutions.

The cautionary instructions provided to the students, the time limit for thinking under each condition, and the other procedures were identical to those used in Investigation 1.

The aim of the procedure in Investigation 2 was as follows. After the students had solved the problems under Condition 1 (first presentation), Condition 2 (first presentation), and Condition 3 (first presentation), they solved them under Condition 2 (second presentation) and under Condition 2 (third presentation) three days later. The difference between the correct response rates under Condition 2 (first presentation) and Condition 2 (second presentation) will reveal the effects of the learning experience under Condition 3 (first presentation). The difference between the correct response rates under Condition 2 (second presentation) and Condition 2 (third presentation) will reveal the extent to which the effects of the learning experience were retained with the passage of time.

Results and Discussion

The correct response under each condition was identical to that in Investigation 1. The correct response rates under Condition 1 (first presentation), Condition 2 (first presentation), and Condition 3 (first presentation) were 14%, 58%, and 84%, respectively. The correct response rates increased under the condition that required the students to go through fewer cognitive processes. These results provide additional confirmation of Kinda’s (2003) results and those of Investigation 1.

In order to better understand the ability of each student to respond correctly in the first presentation, they were categorized into four groups (see Table 3). The percentages of the students in each group were 14% in Group 1, 44% in Group 2, 26% in Group 3, and 12% in Group 4. These four groups accounted for 96% of the students. Similar to Investigation 1, these results corresponded to the assumption regarding the cognitive processes required under the three conditions.

The students in Groups 1 and 2 were assumed to have the ability to spontaneously identify the cases possible in the first presentation because they responded correctly under Condition 2 (first presentation). It was also assumed that the students in Groups 3 and 4 did not have the ability to spontaneously identify the cases possible in the first presentation because they did not respond correctly under Condition 2 (first presentation).
Although the students in Groups 1, 2, and 3 had acquired the experience of producing two solutions under one or more conditions in the first presentation, the students in Group 4 did not acquire this experience.

The change in the correct response rates under Condition 2 were examined for each group (Fig. 2). In Group 1, the correct response rates were 100%, 100%, and 100% under Condition 2 (first presentation), Condition 2 (second presentation), and Condition 2 (third presentation), respectively. In Group 2, the correct response rates were 100%, 95%, and 95% under Condition 2 (first presentation), Condition 2 (second presentation), Condition 2 (third presentation), respectively. These results show that the students were able to spontaneously identify the cases possible almost consistently after they had responded correctly under Condition 2 (first presentation).

In Group 3, the correct response rates were 0%, 62%, and 77% under Condition 2 (first presentation), Condition 2 (second presentation), and Condition 2 (third presentation), respectively. This result indicates that 62% of the students were able to spontaneously identify the cases possible and respond correctly under Condition 2 (second presentation) as a result of the learning experience under Condition 3 (first presentation). Three days later, 77% of the students were able to respond correctly under Condition 2 (third presentation). The effect of the learning experience was completely retained despite the passage of time.

In Group 4, the correct response rates were 0%, 0%, and 33% under Condition 2 (first presentation), Condition 2 (second presentation), and Condition 2 (third presentation), respectively. The students in this group had acquired the experience of solving the problem, but they lacked the experience of producing two solutions under each condition in the first presentation. The reason that 33% of the students could respond

Fig. 2. Correct response rates under Condition 2 in Investigation 2.
correctly under Condition 2 (third presentation) was unclear. The students might have learned to produce two solutions outside the investigation design.

In summary, the results of Investigation 2 clarify the effects of the learning experience on students’ ability to identify the cases possible. The students in Group 3 were able to spontaneously identify the cases possible as a result of their learning experience. The effect of the learning experience was retained with the passage of time.

It is possible that the interval periods influenced the results. The intervals were six days in Investigation 1 and three days in Investigation 2. The influence of interval periods has to be examined in future studies.

**General Discussion**

The goal of this study was to examine whether learning experiences that alert students to the existence of two solutions consequently increase their likelihood of producing two solutions when solving two-solution word problems. In addition to the results obtained in previous studies (Kinda, 2003; Reusser & Stebler, 1997; Verschaffel et al., 1994; Wyndhamn & Säljö, 1997; Yoshida et al., 1997), the results of Investigations 1 and 2 reveal the following.

First, the results of Investigations 1 and 2 revealed that elementary school students were able to spontaneously produce two solutions as a result of their learning experience. Students spontaneously noticed the existence of two solutions and identified the cases possible to the two-solution word problem immediately after the experience of producing two solutions. Although they were not provided with any feedback regarding the correct response, the students understood that two solutions were more appropriate to the problem than a single solution.

It is assumed that immediately after the learning experience, elementary school students acquire the ability to produce multiple solutions in solving math problems requiring them; however, they do not encounter such experiences in their everyday learning of school mathematics. With the help of sufficient exposure to such problems that require multiple solutions in the course of their daily school mathematics learning, students would be able to produce multiple solutions and overcome the single-solution tendency.

However, this study does not clarify whether the school grades of students have an influence on the results. Lower grade elementary school students might find it more difficult to produce multiple solutions than 4th and 5th grade students. Further, this study does not clarify whether the mathematical thinking of students at a certain grade is encouraged or discouraged as a result of their learning experiences of producing multiple solutions. Further studies are required to examine these aspects. Such examinations will reveal the difficulties involved and the importance for the students in each grade to gain learning experiences in producing multiple solutions.

Second, the results of Investigation 1 indicate that elementary school students can not reliably retain the ability to spontaneously notice the existence of two solutions with
the passage of time; the results of Investigation 2 indicated that they could retain the ability to spontaneously identify the cases possible. It could be assumed that the students in Group 3 of Investigation 1 could not continue to produce two solutions under Condition 1 after an interval of six days primarily because their ability to notice the existence of two solutions to a problem had decreased.

In the everyday learning of mathematics at school, elementary school students always encounter single-solution word problems and produce a single solution. They are not required to spontaneously notice the existence of multiple solutions in order to respond correctly to the problems they are provided. This encourages them to stop considering the possibility of multiple solutions existing. Students therefore have a tendency to revert to their original strategy and produce only one solution even after the experience of producing two solutions.

However, students are able to retain the ability to spontaneously identify the cases possible for multiple solutions, and can produce simple multiple solutions when they are provided with information suggesting the existence of these. The learning experience was found to be effective for students in at least this aspect.

Further examination is required to clarify the way in which elementary school students might be able to completely overcome their single-solution tendency. If their school mathematics textbooks were designed to contain a combination of multiple-solution word problems and single-solution word problems, students would have access to more opportunities to produce multiple solutions, and they would be able to produce multiple solutions more consistently.

Mathematics education does not aim to encourage students to form a single-solution tendency. As described in previous studies (for a review, see Reed, 1999), students are expected to solve various problems that they encounter in the real world by using their mathematical knowledge. Therefore, further examination of this aspect would be a worthwhile venture.

REFERENCES


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