Permanent Magnetism of Volcanic Bombs.

BY

S. Nakamura and S. Kikuchi.

[Read May 4, 1912.]

1. It is well known that volcanic bombs are magnetic, so strongly magnetic sometimes that when they are brought near to an ordinary compass, the needle is sensibly deflected. These bombs have a peculiar shape, approaching most nearly to that of a spindle, but their two ends are always curved and sometimes twisted. If a bomb be cut at its thickest part perpendicular to its length, then the section has a form like one represented in Fig. 1, consisting of two convex curves $ABC$ and $ADC$ with different curvatures, and joined by small rims $A$ and $C$ running along the whole length of the bomb. The longitudinal section is shown in Fig. 2. The two ends $E$ and $F$ are curved always toward the flatter side $B$. The magnetic axis of a bomb does not coincide with its length nor with the line $BD$. In the present paper, we shall report our experiments conducted on some bombs from the volcano Mihara(1) in the island Ōshima and a bomb from the volcano Aso in the island Kyūshū to determine their magnetic axes and their moments. Our method was magnetometric.

2. Let us suppose that a bomb is imbedded in a nonmagnetic spherical mass, say of gypsum, then this sphere may be considered to be a model of the earth. If the magnetic survey on the surface of this sphere be conducted, then its magnetic moment and the direction of the

---

magnetic axis can be determined. According to the theory of Gauss\((1)\), it is not necessary for the above purpose to determine the three \(X, Y, Z\) components of the earth's field. When the distribution of the vertical \(Z\) component over the earth's surface be known, the magnetic condition of the earth can be made known perfectly. For, according to his theory, if

\[
P^n = g^{m_1}P^n_0 + (g^{m_1} \cos \lambda + h^{m_1} \sin \lambda)P^{n_1} + (g^{m_2} \cos 2\lambda + h^{m_2} \sin 2\lambda)P^{n_2} + \ldots,
\]

where \(g\)'s and \(h\)'s are constants, and

\[
P^{n+m} = \sin^m u \left[ \cos^{n-m}_u - \frac{(n-m)(n-m-1)}{2(2n-1)} \cos^{n-m-2}_u 
+ \frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{24(2n-1)(2n-3)} \cos^{n-m-4}_u \ldots \right]
\]

then the potential \(V\) and the vertical component \(Z\) on the earth's surface at a point, whose longitude is \(\lambda\) and colatitude is \(\theta\), are given by

\[
V = R (P' + P'' + P''' + \ldots),
\]
\[
Z = 2 P' + 3 P'' + 4 P''' + \ldots,
\]

\(R\) being the radius of the earth. Further the component magnetic moments of the earth are given by

\[
a = R^3 g^{m_0}
\]
\[
\beta = R^3 g^{m_1}
\]
\[
\gamma = R^3 h^{m_1}
\]

where \(a\) is the moment parallel to the earth's axis of rotation and \(\beta\) is that moment parallel to an equatorial radius with \(\lambda = 0\), and lastly \(\gamma\) is parallel to an equatorial radius with \(\lambda = 90^\circ\). From \(a\), \(\beta\), \(\gamma\) we can easily deduce the magnitude of the resultant moment and its direction.

Such is the theory of Gauss. Hence it is only necessary for us to calculate \(g^{m_0}\) \(g^{m_1}\) and \(h^{m_1}\) from the observed values of vertical component \(Z\) on several points on the earth's surface. How the calculation is to be carried out is given in full in Gauss's essay, and we need not repeat it here.\((2)\)

3. The preceding idea was put to practice in our experiment as follows. The bomb to be tested is fixed in a frame movable about a horizontal and a vertical axes. The orientation of the bomb can be read off by a horizontal and a vertical circles, just in the same way as

\((1)\) Gauss, Werke V, p. 119.

the telescope in a theodolite is determined by an azimuth and an
altitude circles. When the frame with the bomb was set at some
distance from a sensitive magnetometer and the resulting deflection of
the needle observed, and then the bomb be made to assume a new
orientation and the observation repeated, then the operation is just the
same as if the magnetometer is carried about the bomb to various points
lying on a sphere, whose centre is the point of intersection of the two
axes of rotation, and whose radius $R$ is equal to the distance between
the point just mentioned and the magnetometer. Thus the operation is
equivalent to a magnetic survey carried out on a miniature earth with
radius $R$. The geographical position of the point, where the magnetic
observation is made, is determined by the two circles, for the vertical
circle gives its longitude $\lambda$, and the horizontal circle its co-latitude $\omega$.
Suppose the frame with the bomb fixed in the position $(\lambda, \omega)$ is set
due west of the magnetometer and in the same level with it, then the
$Z$-component in question is the east west component of the magnetic
field due to the bomb at the point where the magnetometer is placed.
If the component parallel to the meridian of the field due to the bomb be
called $F'$, and the earth's horizontal component be $H$, then the deflec-
tion $\varphi_1$ of the magnetometer needle will be given by
\[ \tan \varphi_1 = \frac{Z}{H + F'} . \]
In order to eliminate $F'$, we place the bomb in the east side of the
magnetometer with its orientation exactly reversed, then the new de-
flexion $\varphi_2$ will be to the other side of the zero and is given by
\[ \tan \varphi_2 = \frac{Z}{H - F'} , \]
whence
\[ Z = \frac{2H}{\cot \varphi_1 + \cot \varphi_2} . \]
In our experiment, we placed the frame with the bomb on the west
side of the magnetometer and then by means of a circular rail with its
centre of curvature at the magnetometer, the frame was brought to its
east side at the same distance $R$ from it, the orientation of the bomb
being naturally reversed by the process. Then the bomb was turned
about the horizontal axis through $180^\circ$, so that $\lambda$ was changed to
$\lambda + 180^\circ$, and then about the vertical axis until $\omega$ was changed to
$180^\circ - \omega$. The resulting deflection was $\varphi_1'$ on the other side of the
zero point. The frame was then brought back to the first position on
the west side of the magnetometer and the latter was deflected \( \varphi_1' \) to
the other side. The deflection of the magnetometer was determined by
lamp and scale method. If the successive scale readings be \( r_1, r_2, r_2', r_1' \)
and the distance between the scale and the magnetometer mirror be \( d \),
then we may take

\[
\tan \varphi_1 = \frac{r_1 - r_1'}{4d} = \frac{\alpha_1}{d}, \text{ say},
\]

\[
\tan \varphi_2 = \frac{r_2' - r_2}{4d} = \frac{\alpha_2}{d}.
\]

Hence

\[
Z = \frac{2H\alpha_1\alpha_2}{d(\alpha_1 + \alpha_2)},
\]

but as the ratio \( \alpha_1 : \alpha_2 \) was always nearly equal to unity, we took

\[
Z = \frac{Ha}{d},
\]

where \( \alpha_1 + \alpha_2 = 2\alpha \). In order to determine \( H \), a magnet with
moment \( M_0 = 355 \) (C. G. S) was placed due west of the magnetometer
at a distance \( R_0 = 137.3 \) cm. in the end-on position, and the resulting
deflection \( \varphi_0 \) was determined. If the scale readings be \( r_0, r_0' \) on rever-
sing the magnet, then

\[
\tan \varphi_0 = \frac{r_0 - r_0'}{4d} = \frac{\alpha_0}{d},
\]

and

\[
H = \frac{2dM_0}{R_0^2\alpha_0},
\]

so that

\[
Z = \frac{2M_0}{R_0^3} \frac{a}{\alpha_0}.
\]

Our experiment consisted of 7 sets for each bomb, for \( \varphi = 0^\circ, 30^\circ, \ldots \)
up to \( \varphi = 180^\circ \), and each set consisted of 12 observations for \( \lambda = 0^\circ, 30^\circ, \ldots \) up to \( \lambda = 330^\circ \), and each observation consisted of four scale
readings as before mentioned. Between each set, the determination of
\( H \) was repeated, in order to eliminate its variation.

4. We examined six different bombs in all.

(1.) Magnetic moment. The moments and the masses of the
bombs are given in the following table.
Thus all bombs are nearly equally magnetized, except V which is brick red in its color. This bomb must have certainly been heated, after it was ejected from the crater and lost its magnetism very much.

(2) Direction of the magnetic axis. The magnetic axis of the bombs were found to coincide neither with longer directions nor with their shortest directions (B D in Fig. 1 and 2), but always inclined to them. Suppose a bomb be held in such a way that the line B D is vertical. The magnetic axis was was found to make an angle of 40°–60° with B D, and its azimuth was any whatever. One thing which we found always to be held in the bombs so far examined, is that the north pole of a bomb lies on its flatter side B. (See Plate I).

Melloni (1) was of the opinion that the permanent magnetism of some volcanic rocks is imparted to them by the inductive action of the earth's magnetism at the time of their cooling. Folgheraiter (2) has examined magnetic conditions of some Italian volcanic rocks and Etruscan vases, and accepting Melloni's opinion tried to find the direction of earth's total intensity in by-gone ages. Brunhes and David (3) are also of similar opinion. The magnetism of volcanic bombs also must be due to the same origin, being magnetized when they were ejected from craters. As the mode of formation of these peculiar shaped bombs is not yet clearly known, we can not say anything definite about the direction of the magnetic axis in them. But we can not omit to describe an experiment made by us on the bomb IV. It was heated in a gas furnace to about 800° C, a temperature high enough to deprive of its magnetism and then cooled in situ. The position of the bomb in the furnace was such that its length coincided as nearly as possible with the magnetic meridian, and the line B D was vertical with its flatter side B turned upward. If the magnetism of bombs be due to

<table>
<thead>
<tr>
<th>Bomb.</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment (C. G. S.)</td>
<td>16.4</td>
<td>4.5</td>
<td>18.7</td>
<td>13.4</td>
<td>4.8</td>
<td>20.7</td>
</tr>
<tr>
<td>Mass (gram).</td>
<td>1021</td>
<td>384</td>
<td>—</td>
<td>554</td>
<td>1673</td>
<td>601</td>
</tr>
<tr>
<td>Moment for unit mass.</td>
<td>0.016</td>
<td>0.012</td>
<td>—</td>
<td>0.024</td>
<td>0.003</td>
<td>0.034</td>
</tr>
</tbody>
</table>

the terrestrial magnetism, then this bomb must have its magnetic axis parallel to the direction of the total intensity during the cooling of the bomb. We found that this was the case as approximately as we can judge, and moreover the north pole was found in the lower side $C$ with greater curvature. This is in contradiction with the sense of the magnetic axis of bombs before examined. This may be explained in two ways; at the time when the bomb was formed on the volcano, either the direction of total intensity was opposite to that at present (1), or the flatter side $B$ was the lower side. Whatever it may be, we think it very probable that the line $B D$ was vertical at the time of cooling, the projecting rims $A$ and $C$ being produced by the flowing down of plastic skin of the bomb. The moment, which was $13.4$ (c. g. s.) before heating, was found to be reduced to $10.4$.

---

A Supplement to
"On the Body Tides of the Earth."

BY

T. SUDA.

In Art. III, Lunar Disturbance of the Earth's Potential etc. of my paper "On the Body Tides of the Earth," given in the Proceedings 2nd Ser., Vol. VI, No. 16, the excess of the observed $M_z$ variation of latitude over the theoretical $M_z$ change of plumb line was directly taken as corresponding to the lunar disturbance of the earth's potential; but this requires a correction for the tidal displacement of the observing station.

The surface displacement of a homogeneous incompressible gravitating elastic sphere due to the action of forces derived from the potential $W_z$, a spherical solid harmonic of the second order, is given by

$$r = 3 \frac{\rho \alpha}{2 g \rho \alpha + 19 \mu} W_z,$$

$$s = 3 \frac{\rho \alpha}{2 \left(2 g \rho \alpha + 19 \mu\right)} \frac{\partial W_z}{\partial \phi}.$$

(1) Föppl's paper, loc. cit.
Bomb II supported on three nails. Oblique line $N S$ shows the magnetic axis.

Bomb IV (before heating) supported on three nails.