Some Remarks on Bopp's Field Theory.*

By Eizo Kanai and Shuji Takagi

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§ 1. Introduction and Summary.

The convergence difficulty of the self-force of the electron has been inherent in the classical theory, and it is essentially due to the assumption that the source of the field is a geometrical point. Recently, Bopp has proposed a new method to remove this difficulty by taking the Lagrange-function which involves the higher space-time derivatives of the field quantities.\(^{(1)}\) For example, we would obtain the finite field energy, provided that the scalar potential \(\varphi\) satisfies the following equation in the presence of a point singularity resting at the origin,

\[
\Delta (\Delta - x^2) \varphi = -4\pi e \delta (\vec{x}) \tag{1}
\]

Because \(\varphi\) is given by the suitable linear combination of the Coulomb potential and Yukawa potential having the common source at the origin so as to be finite up to the singularity.

When Lagrange-function contains the second time-derivatives of the field quantities as is in Bopp's case, the field equations are 4-th order with respect to the time differentiation. So we cannot use the ordinary method of field quantization. Nevertheless, we can show that the quantization of such a field is formally possible by the analogous method to the well-known Heisenberg-Pauli's one—what is an aim of this paper.

As a result of this quantization, it becomes clear that the field is made up of two kinds of Bose particles, one of which has the positive energy but another the negative energy. Therefore the total field energy is never positive definite. Moreover, the system cannot form any stable state, when the interaction with the singularity is introduced. This is the greatest difficulty of Bopp's theory. But, the self-energy of the total system still

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remains finite in the quantized form of the theory as in the classical one. Of course, the zero-point energy — which is a quantum effect — is present and its absolute value is infinite.

Here we take the unitaristic standpoint of view as in original Bopp's theory. So, for example, we consider the coordinate of the singular point only as a simple parameter, and describe its motion by the classical equation derived from Born's principle, which is not of the canonical form. Therefore the Hamilton-function of the field does not coincide with the total energy of the system. Finally we pick up such a part of energy that is only dependent upon the coordinates of the singularity and show that it takes the form of the energy expression of a particle with a negative mass, when making a uniform motion with a small velocity.

\section*{§ 2. Classical treatment.}

As the convergence of the self-energy in Bopp's theory is essentially due to the form of the equation like (1), we can discuss the general properties of this theory by taking the scalar theory for simplicity. By using for convenience, the following units and notations:

\begin{align}
  c &= 1, \quad h = 1, \quad (x, y, z, it) \equiv (x_a) \\
\end{align}

and letting \( \varphi(x_a) \) be the scalar potential and

\begin{align}
  f_a &= \frac{\partial \varphi}{\partial x_a} \\
\end{align}

the strength of the field, we define the Lagrange-function as follows:

\begin{align}
  L &= -\frac{1}{2} \left( f_a^2 + \frac{1}{x^2} \left( \frac{\partial f_a}{\partial x_a} \right)^2 \right) + \mu \varphi. \\
\end{align}

Then we can derive the field equations from the following variation principle:

\begin{align}
  \delta \int L \, dx \, dy \, dz \, dt = 0 \\
\end{align}

where \( \varphi(x_a) \) variates independently, and get

\begin{align}
  \frac{\partial F_a}{\partial x_a} &= -\rho, \quad F_a = -\frac{\delta L}{\delta f_a} = f_a - \frac{1}{x^2} \nabla \cdot f_a. \\
\end{align}
or
\[ \Box \left( 1 - \frac{1}{x^2} \Box \right) \phi = -\rho. \] (6')

We can express this equation in the form:
\[ \frac{\partial T_{\alpha\beta}}{\partial x^\gamma} = -\rho f_\alpha. \] (6'')

where \( T_{\alpha\beta} (= T_{\alpha\mu}) \) is the energy-momentum tensor of the field, and its components are given by
\[
T_{\alpha\beta} = \left( F_\alpha F_\beta - \frac{1}{2} \delta_{\alpha\beta} F^2 \right) - \left( u_\alpha u_\beta - \frac{1}{2} \delta_{\alpha\beta} (u^2 + v^2) \right), \quad \Phi = \frac{1}{x} \left[ \frac{\partial \Phi}{\partial x^\alpha} \right] = \frac{1}{x^2} \Box \phi. \] (7)

Above all, for \( u=4 \) equation (6'') gives the following continuity equation
\[
\text{div} \rightarrow S + \frac{\partial W}{\partial t} = \rho \frac{\partial \phi}{\partial t}, \quad W = -T_{\alpha\beta}, \quad S = -iT_{\alpha\beta}. \] (8)

So \( W \) is the energy density of the field. Using this expression, we can compute the total energy of the field produced by a singular charge resting at the point \( x_0 \) and obtain the finite result
\[ E = \int W \, dx \, dy \, dz = 2\pi xe^2. \] (9)

In order to complete Bopp's theory, it is necessary to derive the equations of motion of the point singularities from an unitaristic standpoint of view. For this purpose, substituting the solution of field equations (6) into
\[ L = -\frac{1}{2} \left[ f_\alpha F_\alpha + \frac{1}{x^2} \frac{\partial}{\partial x^\alpha} \left( f_\alpha \frac{\partial \Phi}{\partial x^\alpha} \right) \right] + \rho \phi \]
and subtracting the complete time-differential terms, we get
\[ M = \bar{L} - \frac{d\bar{L}}{dt} = \int \left( \frac{1}{2} \phi \frac{\partial F_\alpha}{\partial x^\alpha} + \rho \phi \right) dx \, dy \, dz = \frac{1}{2} \int (\rho \phi) \, dx \, dy \, dz. \] (10)

When there are many singularities, \( \rho \) is given by the following expression:
We obtain, variating $x_{\nu}$'s independently,

$$\delta \int M \, dt = \sum \int (\overrightarrow{K}_\nu \cdot \delta x_{\nu}) \, dt = 0.$$  \hspace{1cm} (12)

It is just Born's principle to assume

$$\overrightarrow{K}_\nu = 0.$$  \hspace{1cm} (13)

for the equation of motion of the $\nu$-th singularity. In fact, according to Born or Bopp, it is easily shown that (11) gives the ordinary Lagrangian function for many particles each with finite mass and moving in the field produced by other particles, when their accelerations are all small.

Hereafter we take the simplest case of one singularity and denote its coordinate by $x_0$. The eq. (6') becomes

$$\Box (\Box - x^2) \phi = 4\pi c x^2 \sqrt{1 - V^2} \delta(x - x_0), \quad \overrightarrow{V} = \frac{dx_0}{dt}.$$  

Expanding $\phi$ within a unit cube with periodic condition,

$$\phi = \sum_k a(k, t) e^{i k \cdot x}$$

we get

$$\left( \frac{d^2}{dt^2} + k^2 \right) \left( \frac{d^2}{dt^2} + \omega_k^2 \right) a(k, t) = 4\pi c x^2 \sqrt{1 - V^2} e^{-ik \cdot x_0}, \quad \omega_k = \sqrt{k^2 + x^2}.$$  

Then, we solve these equations in following forms

$$a(k, t) = \int A(k, \nu) e^{-i \nu t} \, d\nu,$$

$$A(k, \nu) = 2c x^2 \int \sqrt{1 - V^2} e^{i \nu t - ik \cdot x_0} \, dt' / (\nu^2 - k^2) (\nu^2 - \omega_k^2);$$

$$x_0' = x_0(t'), \quad \overrightarrow{V} = \overrightarrow{V}(t')$$

$$\therefore \quad a(k, t) = 2c x^2 \int_{-\infty}^{\infty} \sqrt{1 - V^2} e^{-ik \cdot x_0} \, dt' a(t, t')$$
where \[ a(t, t') = \int_{-\infty}^{\infty} dv e^{iv(t - t')} / (v^2 - k^2)(v^2 - \omega_k^2). \]

In this place there are two possible paths for integral, namely (1) the retarded case (Fig. 1), where

\[ a^{(1)}(t, t') = \begin{cases} \frac{2\pi}{x^2} \left( \frac{\sin k(t - t')}{k} - \frac{\sin \omega_k(t - t')}{\omega_k} \right) & t \geq t' \\ 0 & t < t' \end{cases} \]

and (2) the advanced case (Fig. 2), where

\[ a^{(2)}(t, t') = \begin{cases} 0 & t > t' \\ \frac{2\pi}{x^2} \left( \frac{\sin k(t - t')}{k} - \frac{\sin \omega_k(t' - t)}{\omega_k} \right) & t \leq t' \end{cases} \]

and

\[ a^{(2)}(k, t) = 4\pi e \int_{-\infty}^{t} dt' \sqrt{1 - \frac{1}{\pi^2} e^{-2\omega_k t'}} \left( \frac{\sin k(t' - t)}{k} - \frac{\sin \omega_k(t' - t)}{\omega_k} \right) \]

Classifying all quantities corresponding to above two cases by the suffices (1) and (2), we compute the expressions \( \int M^{(1)}dt \) and \( \int M^{(2)}dt \), which are proved to be equal.

\[ \int M^{(1)}dt = \int M^{(2)}dt. \]

After some calculations we find.
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So according to Born’s principle, the equation of motion for the point singularity becomes

\[
\overrightarrow{K}_o = 2\pi \left\{ \sqrt{1 - \overrightarrow{V}^2} \ \text{grad}_x_0 \ (\phi^{(1)}(x_0, t) + \phi^{(2)}(x_0, t)) + \frac{d}{dt} \left( \frac{\overrightarrow{V}}{\sqrt{1 - \overrightarrow{V}^2}} (\phi^{(1)}(x_0, t) + \phi^{(2)}(x_0, t)) \right) \right\}.
\]

Bopp has shown that Born’s equation of motion does not contain the effect of radiation reaction. Exchange of energy and momentum between field and singularity is always reversible. Hence, to take this effect into consideration, we should replace eq. (13) by the following equation

\[
\frac{d}{dt} \left( \frac{\overrightarrow{V}}{\sqrt{1 - \overrightarrow{V}^2}} (-2\pi\epsilon) (\phi^{(1)}(x_0, t) + \phi^{(2)}(x_0, t)) \right) = -\sqrt{1 - \overrightarrow{V}^2} \ \text{grad}_x_0 (-2\pi\epsilon) (\phi^{(1)}(x_0, t) + \phi^{(2)}(x_0, t)).
\]

(13)

(13) or (13') is clearly relativistic invariant. For, we can get the energy equation by taking the scalar product with \(\overrightarrow{V}\):

\[
\frac{d}{dt} \left( \frac{1}{\sqrt{1 - \overrightarrow{V}^2}} \overrightarrow{W}(x_0, t) \right) = \sqrt{1 - \overrightarrow{V}^2} \frac{\partial}{\partial t} \overrightarrow{W}(x_0, t).
\]

(14)

where

\[
\overrightarrow{W} = \begin{cases} 
-2\pi\epsilon (\phi^{(1)} + \phi^{(2)}) & \text{in Born's case,} \\
-4\pi\epsilon \ \phi^{(1)} & \text{in Bopp's case,}
\end{cases}
\]

and sum up above four equations in the following tensor form.

\[
\frac{d}{d\tau} \left( \frac{dx}{d\tau} \overrightarrow{W}(x_0, t) \right) = -\frac{\partial}{\partial x_0} \overrightarrow{W}(x_0, t).
\]

(15)

where \(\tau\) is the proper time of this singularity. From eq. (14) and eq. (8), we get

\[
\text{div} \overrightarrow{S} + \frac{\partial W}{\partial t} = -\delta(x, x_0) \frac{d}{dt} \left( \frac{1}{\sqrt{1 - \overrightarrow{V}^2}} \overrightarrow{W}(x_0, t) \right).
\]
Therefore, we can interpret \( \frac{1}{\sqrt{1-V^2}} \Psi(x_0, t) \) as the energy concentrated on the singularity. That Born's equation of motion does not contain the radiation reaction can be inferred from the fact that we get the following relation by integrating (14) with respect to time:

\[
\left[ \frac{1}{\sqrt{1-V^2}} \Psi(x_0, t) \right] = \int_{-\infty}^{\infty} dt \sqrt{1-V^2} \frac{\partial \Psi}{\partial t}
\]

\[
= \left\{ \int_{-\infty}^{\infty} dt \sqrt{1-V^2} \frac{\partial \Psi}{\partial t} \left\{ (-2\pi c)(\phi^{(\text{in})} - \phi^{(\text{out})}) \right\} \right. \\
\text{in Born's case}
\]

\[
= \left\{ \int_{-\infty}^{\infty} dt \sqrt{1-V^2} \frac{\partial \Psi}{\partial t} \left\{ (-2\pi c)(\phi^{(\text{in})} - \phi^{(\text{out})}) \right\} \right. \}
\]

\[\text{in Bopp's case.}\]

Next, we transform the field equations into the Hamiltonian form before we perform the quantization. Writing \( L \) in a slightly different form from (4),

\[
L = -\frac{1}{2} \left( f_i f_i - \dot{\phi}^2 + \frac{1}{x^2} \frac{\partial f_i}{\partial x_k} \frac{\partial f_i}{\partial x_k} - \frac{1}{x^2} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i} + \frac{1}{x^4} \dot{\phi}^2 \right) + \rho \phi,
\]

we introduce the following variables:

\[
\phi^+ = \frac{\delta L}{\delta \phi}, \quad \frac{d}{dt} \left( \frac{\delta L}{\delta \phi} \right) = \dot{\phi} - \frac{2}{x^2} \Delta \phi + \frac{1}{x^4} \ddot{\phi},
\]

\[
= \frac{\partial L}{\partial \phi} = -\frac{1}{x^4} \ddot{\phi}
\]

(16)

The Hamiltonian of the field is defined to be

\[
H = \phi^+ \chi + \chi^+ \dot{\phi} - L = \phi^+ \chi + \frac{1}{2} \left( f_i f_i - \chi^2 \\
+ \frac{1}{x^2} \frac{\partial f_i}{\partial x_k} \frac{\partial f_i}{\partial x_k} - \frac{2}{x^2} \frac{\partial \chi}{\partial x_i} \frac{\partial \chi}{\partial x_i} - x^2 \chi^2 \right) - \rho \phi.
\]

(17)

If we make the canonical equations by taking \( (\phi, \phi^+), (\chi, \chi^+) \) as canonical variables,
it is easily shown that (18) coincides with (16) and (6'). The relation between $\overline{H}$ and $E = \int W \, dx \, dy \, dz$ becomes then

$$\overline{H} = E - \int \psi^* \phi \, dx \, dy \, dz.$$  

### § 3. Quantization and the Self-energy.

Having found the Hamiltonian form, quantization is simple. We put, according to Heisenberg and Pauli, the following commutation rules between the field quantities:

$$[\phi(x, t), \phi^+(x', t)] = i \delta(x, x') \quad \text{and} \quad [\chi(x, t), \chi^+(x', t)] = i \delta(x, x').$$  

all other pairs of quantities being taken to be commutable. If we postulate the equations of motion for $\phi$, $\phi^+$, $\chi$, $\chi^+$ to be

$$i \phi^* = [\phi^*, \overline{H}] \quad \text{etc.},$$  

it is well known that the classical equations (18) can be derived from (20) and (21). Hence, for a functional $G(\phi, \chi, t)$ containing the time $t$ explicitly we can put, generally,

$$i \dot{G} = i \frac{\partial G}{\partial t} + [G, \overline{H}].$$  

Particularly, for the Hamiltonian $\overline{H}$

$$\frac{d\overline{H}}{dt} = \frac{\partial \overline{H}}{\partial t} = -\int \frac{d\mu}{dt} \phi^* \phi \, dx \, dy \, dz.$$
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so that $\frac{d\overrightarrow{H}}{dt} \neq 0$ in general. We cannot, therefore, consider the Hamiltonian of the field to be the energy of total system, because it is not a constant of motion. Only when $\frac{dH}{dt} = 0$, especially when the singularity is at rest, $\overrightarrow{H}$ represents the energy of the system.

Here we take the standpoint that only the field quantities are to be quantized (q-numbers!), and the coordinates of the singularity are mere c-numbers. Since the field equations describe the feature of the field when the motion of the singularity has been given, we must add, to complete the problem, the equation which determines the motion of the singularity (though it is only a parameter) and is compatible with the field equations. It is not necessary to write this equation in canonical form. One of the simplest way is to borrow the classical equation

\[
\frac{d}{dt} \left( \frac{\overrightarrow{V}}{\sqrt{1-V^2}} \overrightarrow{\Psi}(x_0, t) \right) = -\sqrt{1-V^2} \text{ grad}_x \overrightarrow{\Psi}(x_0, t)
\]

with $\overrightarrow{\Psi}(x_0, t) = -4\pi e \overrightarrow{\phi}(x_0, t)$.

We have in this case

\[
\frac{d\overrightarrow{H}}{dt} = -\frac{d}{dt} \left( \frac{V^2}{\sqrt{1-V^2}} \overrightarrow{\Psi}(x_0, t) \right).
\]

Hence,

\[
\frac{dH}{dt} = 0, \quad H = \overrightarrow{H} + \frac{V^2}{\sqrt{1-V^2}} \overrightarrow{\Psi}(x_0, t) = E + \frac{1}{\sqrt{1-V^2}} \overrightarrow{\Phi}(x_0, t)
\]

$H$, which may be called the energy of the system, consists of two parts. We may consider $\frac{1}{\sqrt{1-V^2}} \overrightarrow{\Phi}(x_0, t)$ as the energy concentrated on the singularity, while $E$ may be called the energy of the field.

To see the particle character of the field, it is convenient to go to the momentum space. We put, namely,

\[
\overrightarrow{\psi} = \sum_k q(k) e^{ik \cdot x}, \quad \overrightarrow{\psi}^* = \sum_k p(k) e^{-ik \cdot x},
\]

\[
\overrightarrow{\chi} = \sum_k Q(k) e^{ik \cdot x}, \quad \overrightarrow{\chi}^* = \sum_k P(k) e^{-ik \cdot x}.
\]

Then we get the following commutation relations:
\[ [q(k), p(k')] = [Q(k), P(k')] = i\delta_{kk'} \]  \hspace{1cm} (25a)

while all the other pairs are commutable. Further, we perform another transformation of variables, putting,

\[ q(-k) = \frac{1}{\sqrt{2k}} \left( A(-k) + A^*(k) \right) + \frac{1}{\sqrt{2\omega_k}} \left( B(k) + B^*(-k) \right) \]
\[ \rho(-k) = -\frac{i\omega_k}{\sqrt{2k}} \left( A(-k) - A^*(k) \right) - \frac{k}{\sqrt{2\omega_k}} \left( B(k) - B^*(-k) \right) \]
\[ Q(k) = i\frac{k}{2} \left( A(-k) - A^*(k) \right) + i\frac{\omega_k}{2} \left( B(k) - B^*(-k) \right) \]
\[ P(-k) = \frac{1}{\sqrt{2}} \left( \omega_k \left( A(-k) + A^*(k) \right) + \omega_k \left( B(k) + B^*(-k) \right) \right) \]  \hspace{1cm} (26)

In these variables we find the commutation rules to be

\[ [A(k), A^*(k')] = [B(k), B^*(k')] = i\delta_{kk'} \]  \hspace{1cm} (26a)

all the other pairs being commutable. For the energy of the system we obtain, in terms of \( A, A^*, B \) and \( B^* \),

\[ \mathbf{H} = \mathbf{H}_0 + \mathbf{H}' \]  \hspace{1cm} (27)

with \( \mathbf{H}_0 = \sum_k \left\{ k \left( A^*(k) A(k) + \frac{1}{2} \right) - \omega_k \left( B^*(k) B(k) + \frac{1}{2} \right) \right\} \)  \hspace{1cm} (27a)

\[ \mathbf{H}' = -\frac{4\pi e}{\sqrt{1 - V^2}} \sum_k \left\{ e^{i\kappa} \left( A^*(k) \sqrt{2k} + B(k) \sqrt{2\omega_k} \right) \right. \]
\[ + e^{-i\kappa} \left( A(k) \sqrt{2k} + B^*(k) \sqrt{2\omega_k} \right) \]  \hspace{1cm} (27b)

As a consequence of (26a) the operators

\[ \mathcal{N}_A(k) = A^*(k) A(k) \]
\[ \mathcal{N}_B(k) = B^*(k) B(k) \]

has all positive integers and 0 as proper values. Thus, we know that the free field \( \mathbf{H} = \mathbf{H}_0 \) consists of two kinds of particles. This is the result of the fact that our Lagrangian contains the second order derivatives of the field variables with respect to time. Instead of the initial conditions in the classical theory, we may take here the initial values of the complete set of observables such as, say, \( \mathcal{N}_A(k), \mathcal{N}_B(k) \).
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To get the self-energy of the system it is convenient to make the following canonical transformations:

\[
A(k) = A'(k) + \frac{4\pi e}{\sqrt{1-V^2}} \frac{e^{ik\cdot x_0}}{k \sqrt{2k}}
\]

\[
B(k) = B'(k) - \frac{4\pi e}{\sqrt{1-V^2}} \frac{e^{-ik\cdot x_0}}{\omega_k \sqrt{2\omega_k}}
\]

(28)

Then we have

\[
H_0 = \sum_k \left\{ \frac{1}{2} \left( A'^* (k) A'(k) + \frac{1}{2} \right) - \omega_k \left( B'^* (k) B'(k) + \frac{1}{2} \right) \right\} + \Delta E
\]

(29)

where \(\Delta E = -2\pi e^2/(1-V^2)\).

\(H - \Delta E\) has the same spectrum as the Hamiltonian of the free field. \(\Delta E\) represents, therefore, the energy-shift caused by the presence of the singularity, and can be interpreted as the self-energy of the system. Thus we see that the finiteness of the self-energy in classical Bopp's theory is kept also when we go into the quantum theory. Particularly, when the singularity is at rest, we find

\[
\Delta E = -2\pi e^2
\]

(9')

which has the opposite sign to (9). This corresponds to the circumstance that we have calculated in (9) the energy of the field only — in the classical theory the self-energy is always interpreted in this meaning —; while in (9') we have calculated the energy of the system consisting of the field and the singularity. Further, we should remark the fact that \(\Delta E\) are not transformed as the time-component of a four-vector when the velocity of the singularity varies.

\(H_0\) contains the zero-point energy whose absolute value is infinity. But the more serious difficulty in Bopp's theory is the appearance of negative energy particles in \(H_0\). In consequence, \(H\) is not positive definite and it is impossible to save this difficulty by taking account of the hole theory, since the particles of our problem obey Bose statistics. The system, therefore, can not form the stable state even if we neglect the zero-point energy. It seems for us that the success of Bopp's theory is to be confined to the classical region.

In the previous section we have separated $H$ into $H_0$ and $H'$. Another separation is possible which will now be made. $A(k)$, as it satisfies the following equation

$$i\overrightarrow{A}(k) = [A(k), \overrightarrow{H}] = kA(k) - \frac{4\pi e}{\sqrt{1 - \nu^2}} e^{i k z_0} / \sqrt{2k}$$

consists of two parts:

$$A(k) = A_0(k) + u(k)$$

where $A_0(k)$ is the solution of the homogeneous equation made from above one corresponding to the free field, and $u(k)$ is the non-homogeneous solution of this equation, namely:

$$u(k) = \frac{-4\pi i e}{\sqrt{2k}} \int_{-\infty}^{t} dt' e^{i k z_0} e^{i k(t-t')} / \sqrt{1 - \nu^2}.$$

Similarly we can write for $B(k)$

$$B(k) = B_0(k) + v(k)$$

with

$$v(k) = \frac{-4\pi i e}{\sqrt{2\omega_k}} \int_{-\infty}^{t} dt' e^{-i k z_0} e^{-i\omega_k(t-t')} / \sqrt{1 - \nu^2}.$$

Here we can consider $u$, $v$ as c-numbers. Inserting these expressions into (27) we obtain

$$H = \overrightarrow{H}_0 + \overrightarrow{H}_1 + \overrightarrow{H}_2$$

with

$$\overrightarrow{H}_0 = \sum_k \left\{ k \left( A_0^*(k) A_0(k) + \frac{1}{2} \right) - \omega_k \left( B_0^*(k) B_0(k) + \frac{1}{2} \right) \right\} \quad (30a)$$

$$\overrightarrow{H}_1 = \sum_k (k u^*(k) u(k) - \omega_k v^*(k) v(k)).$$

$$-\frac{4\pi e}{\sqrt{1 - \nu^2}} \sum_k \left\{ e^{i k z_0} \left( \frac{u^*(k)}{\sqrt{2k}} + \frac{v(k)}{\sqrt{2\omega_k}} \right) + e^{-i k z_0} \left( \frac{u(k)}{\sqrt{2k}} + \frac{v^*(k)}{\sqrt{2\omega_k}} \right) \right\} \quad (30b)$$
$H^o$ corresponds to the energy of the free field, and $H^i$ is a function of the coordinates of the singularity only, while $H^s$ may be interpreted as the interaction term. Especially when the singularity is moving uniformly with small velocity $\vec{V}$, $H^s$ becomes simply as follows,

$$H^s \approx -2\pi\varepsilon^2/\sqrt{1-V^2} \quad (31)$$

so that, in this case $H^s$ takes the form as if it were the energy of the particle with negative mass.

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E. K. Department of Physics, Kyoto Imperial University.
S. T. Department of Physics, Nagoya Imperial University.

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