Numerical Analysis of Welding Deformation for Large-Scale Structure*

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In recent years, it has been possible to predict the welding deformation by only performing elastic analysis using inherent strain. In elastic analysis with inherent strain, Static implicit FEM is generally used. In Static implicit FEM, huge computing resources are necessary to predict the welding deformation of the large scale structure and it is very difficult to analyze. Therefore, in order to achieve shorter computing time and lower memory consumption, the authors developed a new analysis method using inherent strain based on Idealized explicit FEM.

The developed method is applied to the welding deformation analysis of the fundamental welding structure to verify the validity of the proposed method. As a result, it is found that the developed method has almost the same accuracy as Static implicit FEM. In addition, the developed method can analyze the problem of 571,176 degrees of freedom 4 times smaller in computing time and 10 times less in memory consumption than Static implicit FEM. Thus, it can be concluded that the developed method is very effective method in the welding deformation analysis of large scale structure.

Key Words: Welding deformation, Large scale analysis, Inherent strain, Static implicit FEM, Idealized explicit FEM

1. Introduction

In the construction of large scale steel structures, the dimensional accuracy is strongly influenced by welding deformation and it causes several problems such as reduction of strength and performance. If the welding deformation is unacceptably large, reworking process is necessary and it leads to the increase of working time and labor costs. Therefore, accurate prediction of welding deformation is very important and it leads to improvement of assembly process and cost reduction.

Due to the rapid advances in recent computer hardware and systems, it has been possible to predict the deformation of structures by using simulation such as Finite Element Method (FEM). Welding deformation can be predicted by using thermal elastic plastic FE analysis. However, in thermal elastic plastic analysis, it is necessary to analyze the welding phenomena consecutively and welding have strong non-linearity such as the phenomena of the melting of materials. Therefore, the computing time and memory consumption becomes enormous and it is difficult to predict the welding deformation of large scale structures by using thermal elastic plastic analysis.

On the other hand, it is possible to predict the welding deformation by elastic analysis using inherent strain in case that the inherent strain of welding joint is given. Elastic analyses require much shorter computing times than thermal elastic plastic analyses. However, even in elastic analysis, large simultaneous equations need to be constructed and solved. As a result, the memory consumption becomes large in the analysis of large scale structure. Due to the limitation of computer memory, it is very difficult to analyze the model with more than hundreds of thousands of elements, such as large-scale or complicated structures such as ship hull blocks.

Then, in this study, Idealized explicit FEM (IEFEM) is introduced to welding deformation analysis using inherent strain and shell elements. IEFEM was developed to achieve shorter computing time and lower memory consumption in thermal elastic plastic analysis of welding and it is based on dynamic explicit FEM. In dynamic explicit FEM, it is not necessary to solve large simultaneous equation. Therefore, the computing time and memory consumption becomes small and then, it is possible to analyze large scale problem.

In the present study, IEFEM is applied to elastic analysis using inherent strain and the validity and the usefulness are investigated by applying the developed method to the problem of welding deformation of stiffened thin-plate structures. The results indicate that the accuracy of the IEFEM solution almost equals to that of Static implicit FEM and IEFEM can effectively reduce computing time and memory consumption.

2. Welding deformation analysis using inherent strain

In welding deformation analysis using inherent strain, inherent strain generated in weld joint is given as initial strain. Inherent strain determines the deformation of weld joint and it is shown in the following form.
\[ \varepsilon = \varepsilon' + \varepsilon'' + \varepsilon' + \varepsilon' = \varepsilon' + \varepsilon'' \]

where, \( \varepsilon, \varepsilon', \varepsilon'', \varepsilon', \varepsilon' \) are total strain (geometric strain), elastic strain, plastic strain, creep strain, phase transformation strain and inherent strain, respectively. Then, if creep strain \( \varepsilon' \) and phase transformation strain \( \varepsilon' \) are negligibly small, inherent strain \( \varepsilon' \) becomes equal to plastic strain \( \varepsilon'' \).

In order to determine inherent strain, various methods have been proposed. For example, Ueda et al. proposed that inherent strain is determined by inverse analysis using experimental measurement, and Hata et al. conducted that inherent strain is determined using following empirical formula:

\[ \varepsilon'' = \frac{Q}{S} \]

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where, \( Q \) is heat input, \( S \) is transverse shrinkage, \( Q_{in} \) is net heat input, \( a \) is coefficient of thermal expansion, \( c \) is specific heat, \( h \) is thickness of plate and \( a \) is width to give inherent strain.

\[ \varepsilon'' = \frac{Q}{S} \]

(a) Transverse shrinkage

Transvers shrinkage is calculated through the following Eq. (2)-(4) and inherent strain due to transverse shrinkage \( \varepsilon'' \) is determined as Eq. (5).

\[ [Q'] = 6.27 \left[ \frac{J}{mm^3} \right] \]

\[ S = 1.16 \times 10^{-3} Q' h \ (mm) \]

\[ [Q'] = 20 \left[ \frac{J}{mm^3} \right] \]

\[ S = 1.44 \times 10^{-4} \left[ Q^2 - Q' \right] + 0.0025 h \ (mm) \]

\[ [Q'] = 27.6 \left[ \frac{J}{mm^3} \right] \]

\[ S = a/(\rho c) Q' h \ (mm) \]

\[ \varepsilon'' = S/a \]

\[ \varepsilon'' = \frac{Q}{S} \]

where, \( Q' = Q_{in}/h^3 \), \( S \) is transverse shrinkage, \( Q_{in} \) is net heat input, \( a \) is coefficient of thermal expansion, \( c \) is specific heat, \( \rho \) is density, \( h \) is thickness of plate and \( a \) is width to give inherent strain.

(b) Angular distortion

Angular distortion is determined as Eq. (6) or Eq. (7) and inherent strain due to angular distortion is obtained as Eq. (8).

\[ \theta = 0.1061 Q'/\left[ ( Q' - 6.16 )^2 + 73.6 \right] \ (rad) \]

\[ \kappa' = \theta/a \]

where, \( \theta \) is angular distortion and \( \kappa' \) is inherent strain due to angular distortion.

(c) Longitudinal shrinkage

Inherent strain due to longitudinal shrinkage is determined from tendon force as follows.

\[ \varepsilon'' = \frac{\Delta L_{av}}{l} \]

\[ \Delta L_{av} = 224.1/(E B) Q' \ (mm) \]

where, \( \varepsilon', \Delta L_{av}, E, B, l \) are inherent strain due to longitudinal shrinkage, average longitudinal shrinkage, Young’s modulus, width of welding line and length of welding line, respectively.

3. Application of Idealized explicit FEM to elastic analysis using inherent strain

Idealized explicit FEM (IEFEM), which was developed in our previous study, achieves fast computation and low memory consumption based on dynamic explicit FEM in welding transient analysis. In dynamic explicit FEM, it is necessary to limit the time increment to a very small value. Therefore, an enormous number of time steps is needed to analyze from the beginning of welding to complete cooling and it is difficult to analyze a welding problem in a realistic computing time. In contrast, in Static implicit FEM, which is generally used in structural analyses, it is necessary to solve large simultaneous equations, which cover the entire domain of analytical model. Therefore, to analyze large-scale or complex model, larger memory and computing time are needed in Static implicit FEM. On the other hand, IEFEM, which is very fast with low memory consumption, is considered to be much more suitable for welding distortion and stress problems.

In the present study, IEFEM is extended to welding deformation analysis using inherent strain with shell elements. In the next section, the basic theory of IEFEM is described.

3.1 Idealized explicit FEM (IEFEM)

In IEFEM, the analysis progresses according to the following procedure as shown in Fig. 1.

1. Load increment is given and the load is held in time step calculation.
2. Until static equilibrium state is obtained, displacement is calculated through Eq. (10) based on dynamic explicit FEM.

\[ \int \left[ M \right] \ddot{\varepsilon} + \left[ C \right] \dot{\varepsilon} + \left[ k \right] \varepsilon \ dv = \left[ f \right] \]

3. If static equilibrium state is obtained, the analysis returns to step 1 (the load increment is given and held).

Eq. (11), obtained by taking the centered difference of the velocity and acceleration, is used to calculate the displacement in dynamic explicit FEM.
where, $\Delta t$ is the time increment. Here, assuming that mass matrix $[M]$ and damping matrix $[C]$ are a node-concentrated diagonal matrix, the matrix operation in Eq. (11) is no longer a simultaneous equation, therefore, low memory consumption is achieved. However, to attain static equilibrium state in steps 2 and 3, numerous time steps are necessary if the conventional mass matrix $[M]$ and damping matrix $[C]$ are used. In next chapter, the modified mass and damping matrix is derived to achieve good convergence\textsuperscript{11).

### 3.2 Modified mass and damping matrix

In one-dimensional vibration theory, the following equation of motion is given for the spring-mass-damper system.

$$m \ddot{u} + c \dot{u} + k u = F$$

(12)

where, $m$ is the mass, $c$ is the damping coefficient, and $k$ is the spring constant. Over damping occurs if $c > 2\sqrt{mk}$, critical damping occurs if $c = 2\sqrt{mk}$, and under damping occurs if $c < 2\sqrt{mk}$. The convergence to static equilibrium is most rapid with critical damping. If the relation between the mass and the spring constant are defined as:

$$m = c \alpha k$$

(13)

where, $\alpha$ is constant.

Then, the relation between the damping coefficient and the spring constant are expressed as:

$$c = 2\sqrt{\alpha} k$$

(14)

Thus, the mass and damping coefficients can be regarded as the value that depends on the spring constant.

Based on the above consideration, the diagonal elements of the mass matrix and the damping matrix are determined, respectively, as:

$$c_{ii} = 2\sqrt{\alpha} k_{ii}$$

(15)

$$m_{ii} = c\alpha k_{ii}$$

(16)

where, $c_{ii}$, $m_{ii}$, $k_{ii}$ and $\alpha$ are diagonal elements of the damping matrix, the mass matrix, the stiffness matrix and coefficient more than 1, respectively. For coefficient $\alpha$, 1.0 is used in this research.

The damping matrix $[C]$ and mass matrix $[M]$ are assumed to be a diagonal matrix. The stiffness matrix of an element is calculated from the following equation.

$$[K] = [B]^T [D] [B]$$

(17)

where, the matrices $[B]$ and $[D]$ are the displacement-strain and stress-strain relation, respectively.

In this way, the modified mass matrix and damping matrix are defined as the rapid condition to converge to the static equilibrium state.

### 4. Evaluation of performance of IEFEM

As described in previous chapter, IEFEM is extended to the welding deformation analysis using inherent strain. In this section, the accuracy and performance of the developed method is verified by applying the developed method to the analysis of thin stiffened plate structure.

#### 4.1 Analysis model and conditions

The analysis model and FE mesh divisions are shown in Fig. 2. In this model, rectangular shell element is used. Heat input is assumed to be $Q = 623 \text{ (J/mm)}$. Welding lines are assumed as red line in Fig. 2. Young’s modulus and Poisson’s ratio are assumed to be $E = 210 \text{ (GPa)}$ and $\nu = 0.3$. Plate thicknesses are $t_1 = 9 \text{ (mm)}$, $t_2 = 9 \text{ (mm)}$ and $t_3 = 7 \text{ (mm)}$ which are shown in Fig. 2.

#### 4.2 Verification of accuracy

Figures 3 (a), (b), and (c) show the displacement distributions in $x$, $y$, $z$ direction obtained by Static implicit FEM (upper images) and those by IEFEM (lower images), respectively. As shown in these figures, it is found that the results obtained by IEFEM and Static implicit FEM agree well with each other.
Next, Fig. 4 shows the distributions of displacement in z-direction along line A-B (transverse section). In this figure, the open circles are the results obtained by IEFEM and the filled triangles are those by Static implicit FEM. From Fig. 4, it is found that the displacement in z-direction obtained by IEFEM agree very well with that by Static implicit FEM.

Figure 5 shows the distribution of shrinkage in x-direction between E'-E and F'-F which are defined in Fig. 2. Similarly, Fig. 6 shows the distribution of shrinkage in y-direction between F-E and F'-E'. The open marks are the results obtained by IEFEM and the filled marks are those by Static implicit FEM. From both figures, it is clearly seen that the residual deformation obtained by IEFEM is also in good agreement with that by Static implicit FEM.

Thus, it was shown that IEFEM has almost the same accuracy as Static implicit FEM.

4.2 Computing time and memory consumption

In this section, computing time and memory consumption are compared using the same analysis model which is shown in the previous section to discuss the performance of IEFEM. The analysis conditions are the same as those used in the previous section and the number of DOFs is changed as follows: 36,414, 143,748, 286,488, 476,460, 571,176 and 713,466. In Static implicit FEM, the skyline solver is used.

Figure 7 shows the relation between computing time and DOFs. The open circles show the results of IEFEM, the filled triangles show those of Static implicit FEM. In the figure, computing time of Static implicit FEM increase in proportion to the square of the degrees of freedom. On the other hand, computing time of IEFEM are in proportion to the degrees of freedom. Especially in the analysis of 571,176 DOFs, IEFEM is approximately 4 times
faster than Static implicit FEM. These results indicate that IEFEM is more effective in the larger-scale problem.

Figure 8 shows the comparisons of memory consumption. From the figure, it is found that IEFEM is also effective in memory consumption compared to Static implicit FEM. In the analysis of the model with 571,176 DOFs, the memory consumption of IEFEM is 1.31 GB, which is 15 times smaller than that of Static implicit FEM.

The results, which are described here, indicate that IEFEM has almost the same accuracy as Static implicit FEM and IEFEM is superior in both computing time and memory consumption to Static implicit FEM. These results mean that IEFEM enables ultra large scale analysis with more than 10,000,000 elements on a single-PC level hardware, which is currently impossible with commercial FEM software. Therefore, IEFEM has potential for high utilization as a next-generation analytical tool.

5. Conclusions

In this study, Idealized explicit FEM (IEFEM) was extended to the analysis of welding deformation using inherent strain. And the developed method was applied to the welding deformation problem. The results were compared with those obtained by Static implicit FEM in the analysis of the same models. The conclusions are summarized as follows:

1. IEFEM is found to be far superior to Static implicit FEM in the computing time especially for a large-scale welding deformation analysis with more than tens of thousands of nodal points.
2. The large-scale welding deformation analysis, which requires extremely high memory consumption in Static implicit FEM, can be performed with far less memory consumption using IEFEM.
3. Through the comparison of the accuracy of IEFEM and Static implicit FEM on welding deformation, it is found that IEFEM can provide almost the same accuracy as Static implicit FEM.

Reference