Modeling of Weld Pool Oscillation using Energy Method*

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Abstract

An analytic model estimating the natural frequency of a weld pool is derived using energy method. The model is applied to the complex weld geometries, penetration conditions and oscillation modes. In addition to partial and full penetration, transition penetration is introduced and analyzed as the intermediate state between partial and full penetration conditions. Results show that the predicted natural frequency of the weld pool is affected by oscillation modes and geometric parameters such as the eccentricity and sidewall slope of the weld pool as well as material properties and pool dimensions.

Key Words: Weld pool oscillation, Energy method, Natural frequency, Pool geometry, Sidewall slope, Eccentricity, Penetration condition, Oscillation mode

1. Introduction

For the past several years, a number of researchers have studied the oscillation of weld pool for possible application to weld penetration and pool size control of the GTAW process. Weld pool oscillation refers to the periodic motion of the free surface of the molten pool, which results from interaction between the surface tension on the free surface and inertia from the mass of the weld pool. The frequency of pool oscillation depends on variables such as the weld pool geometry, penetration condition, oscillation mode and material property. It has been reported that the oscillation frequency, which is almost same as the natural frequency due to small damping effects of molten metal, decreases with increase in weld pool volume, and also decreases in the full penetration weld compared with the partial penetration weld. The details of dependence of pool oscillation on these variables are complex, making interpretation of the oscillation frequency difficult.

Investigators have derived several theoretical models of the weld pool oscillation in attempts to understand the influence of various parameters¹⁻⁶⁾. The main object of modeling is to find the relationship between the natural frequency and weld pool conditions such as the pool geometry, penetration condition and oscillation mode. Approaches can be classified into those using classical fluid mechanics and those using force

equilibrium. Those using classical fluid mechanics assume the molten weld pool is a cylindrical pool filled with inviscid and irrotational fluid (potential flow) for partial and full penetration welds¹⁻³⁾. Recently, effects of complex oscillation modes and elliptic free surfaces on the natural frequency are estimated for the full penetration weld⁴⁾. Those using force equilibrium which are applied to the full penetration weld, conduct a force balance between the weight of the weld pool and restraining force due to the surface tension^{5,6)}. If the base material is thin, the pool is modeled as a stretched membrane and the natural frequency of the membrane is calculated⁶⁾.

Although the natural frequencies can be calculated for partial and full penetration welds using these models, their application is limited to a simple weld pool shape, typically a circular or elliptic pool surface with vertical sidewall for the full and partial penetration welds. Therefore, it is difficult to estimate the effects of weld geometries such as the sidewall slope and ellipsoidal shape of pool shape for full and partial penetration welds, respectively. Also, transition between the full and partial penetration conditions has not been able to be analyzed.

The current work takes a different approach, applying energy method⁷⁾ to derive a theoretical model which can predict the natural frequency of the full, partial and transition penetration welds with more realistic pool geometries. This model

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allows the study of the effects of oscillation modes as well as geometric parameters such as the non-circular weld pool shape and slope of the weld pool sidewall.

2. Application of Energy Method

Energy method is an approximate approach for vibration analysis of the structure to estimate the natural frequency. Compared with previous approaches which applied analytical methods to simple pool geometries, energy method allows an approximate method to be applied to more complex pool geometries.

2.1 Application to weld pool

If the weld pool is assumed to be a conservative system, the maximum potential energy of the system, $U_{\max}$, is same as the maximum kinetic energy, $T_{\max}$:

$$U_{\max} = T_{\max}. \quad (1)$$

The maximum potential energy is determined by the maximum amount of displacement on the free surface, and the maximum kinetic energy occurs at the time of the maximum velocity of the weld pool system. While the potential energy is generated by surface tension and gravity, effects due to gravity on the natural frequency are neglected in this work because effects due to gravity are reported to be less than 5% of those due to surface tension¹⁾.

In order to apply energy method, a time-dependant displacement function of the weld pool system, $F_t(x, y, z, t)$, is assumed, which simulates the expected pool oscillation. Given the displacement function, maximum potential and kinetic energies are:

$$U_{\max} = \frac{1}{2} \int_A \sigma \left(\left(\frac{\partial F_t}{\partial x} \right)^2 + \left(\frac{\partial F_t}{\partial y} \right)^2 \right)_{\max} dA, \quad (2)$$

$$T_{\max} = \frac{1}{2} \int_V \left(\frac{\partial F_t}{\partial t} \right)_{\max}^2 \rho dV \quad (3)$$

where σ and ρ are the surface tension and mass density, respectively, and the integrals are over the free surface area, A , and volume, V , of the weld pool.

When the molten pool is assumed to oscillate sinusoidally, the time-dependant displacement

function and its velocity are given by:

$$F_t(x, y, z, t) = F(x, y, z) \sin(\omega t), \quad (4a)$$

$$\frac{\partial F_t}{\partial t} = \omega F \cos(\omega t). \quad (4b)$$

where ω is the frequency in radian/second. The maximum displacement and velocity become:

$$F_t(x, y, z, t)_{\max} = F(x, y, z), \quad (5a)$$

$$\left(\frac{\partial F_t}{\partial t} \right)_{\max} = \omega F(x, y, z). \quad (5b)$$

When the surface tension and density are assumed to be constant, the natural frequency is derived as:

$$\omega^2 = \frac{\sigma \int_A \left(\left(\frac{\partial F}{\partial x} \right)^2 + \left(\frac{\partial F}{\partial y} \right)^2 \right) dA}{\rho \int_V F^2 dV}. \quad (6)$$

Therefore, given the displacement function and pool geometry, the natural frequency is calculated by integration over the free surface and volume of weld pool.

The coordinate systems and pool geometries for various penetration conditions are shown in Fig. 1. $W(z)$ and $L(z)$ describe the width and length of the weld pool, H and D are the plate thickness and pool depth, respectively. In the case of full penetration as shown in Fig. 1a, the pool geometry is simulated as a tapered cylinder with elliptic free surfaces. Fig. 1b illustrates the partial penetration weld geometry which is simulated by the ellipsoidal volume with the elliptic free surface. The transition penetration beginning to melt-through to the bottom surface is described similar to the partial penetration case, except that the volume is truncated at $z = -H$ as shown in Fig. 1c.

2.2 Determination of displacement function

In order to determine the displacement function, a trial function is selected such that it approximates the expected pool oscillation, and satisfies the volume constant condition and no-slip condition on the solid-liquid interface. These conditions are described as:

$$\Delta V = 0 \quad (7)$$

$$F(x, y, z) = 0 \quad \text{on solid-liquid interface.} \quad (8)$$

Fig. 1 Weld geometry and coordinate system. (a) full, (b) partial, (c) transition penetration weld.

The assumed displacement function is not unique, and any function can be a displacement function if it satisfies two conditions. A given function may be better or worse than others in accuracy to estimate the natural frequency.

In this work, a polynomial is used for the displacement function which consists of an amplitude function, $A\left(\frac{z}{D}\right)$, and mode shape function, $B\left(\frac{2x}{L(z)}, \frac{2y}{W(z)}\right)$:

$$F(x, y, z) = KA\left(\frac{z}{D}\right)B\left(\frac{2x}{L(z)}, \frac{2y}{W(z)}\right) \quad (9)$$

where K is a proportional constant. The amplitude function describes how the displacement varies in the z -direction:

$$A\left(\frac{z}{D}\right) = \left(1 + \frac{z}{D}\right)^n \quad (10)$$

where D is a generalized pool depth, and n is a parameter to determine how fast the amplitude of pool oscillation varies along the pool depth for the specific penetration condition. When the value of n is positive and becomes larger, the amplitude of pool oscillation decreases faster along the pool depth, which is typical for the partial penetration weld pool. The amplitude function produces variation from 1 at $z=0$ to 0 at $z=-D$. Therefore, the assumed amplitude function satisfies the boundary condition at the bottom surface in the case of the partial penetration weld. It also gives non-zero values in the cases of full and transition penetration welds because the generalized pool depth, D , is greater than the plate thickness, H (Fig. 1).

The mode shape function describes the amplitude variation on the x - y plane. The simulated

Fig. 2 Oscillation modes. (a) symmetric, full penetration, (b) sloshing, full penetration, (c) symmetric, partial penetration, (d) sloshing, partial penetration, (e) symmetric, transition penetration.

symmetric and sloshing modes of pool oscillation are illustrated in Fig. 2. While the symmetric modes are expected in the stationary and slow traveling conditions, the sloshing modes are expected in the fast traveling condition. Fig. 2a and 2b show the symmetric and sloshing modes for the full penetration weld, Fig. 2c and 2d represent the symmetric and sloshing modes for the partial penetration weld, respectively. Fig. 2e illustrates the symmetric mode for the transition penetration condition which occurs when the bottom pool size is much smaller than the top pool size. It has features of full and partial penetration welds such that the top and bottom surfaces oscillate similar to the partial and full penetration cases, respectively. In addition to $L(z)$ and $W(z)$, another ellipse represented by $L_1(z)$ and $W_1(z)$ is introduced within the weld pool. This ellipse is the node of oscillation so that no displacement occurs along the perimeter of the ellipse. Only the symmetric mode is considered in this work.

The mode shape functions are described by polynomial functions:

$$B\left(\frac{2x}{L(z)}, \frac{2y}{W(z)}\right) = 1 + C_2 r^2 + C_4 r^4, \quad (11a)$$

$$B\left(\frac{2x}{L(z)}, \frac{2y}{W(z)}\right) = r + C_3 r^3, \quad (11b)$$

$$r^2 = \left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2, \quad (11c)$$

where the C 's are arbitrary constants. More complex oscillation mode can be simulated by increasing the order of the polynomial. As there is no displacements along the y -axis for the sloshing mode, Eqn. 11b can be modified further:

$$B\left(\frac{2x}{L(z)}, \frac{2y}{W(z)}\right) = \left(\frac{2x}{L(z)}\right)(1 + C_3 r^2). \quad (12)$$

For the full penetration weld pool oscillating in the symmetric mode, parameter n is determined to -2 from the volume constant condition, and C 's are determined from no-slip condition. The displacement field is obtained as:

$$F(x, y, z) = K \left(1 + (1-p) \frac{z}{H}\right)^{-2} \left(1 - \left(\left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2\right)\right), \quad (13a)$$

$$p = \frac{W(-H)}{W(0)}, \quad (13b)$$

where p represents the sidewall slope of the tapered cylinder. When p equals to 1, which is a vertical sidewall, the amplitude function becomes 1 and this implies that the amplitude is uniform along the pool depth. For the sloshing mode, C_3 in

Eqn. 12 is determined to -1 from the no-slip condition, but parameter n is not determined explicitly because the volume constant condition is satisfied automatically. The displacement function for the sloshing mode is obtained as:

$$F(x, y, z) = K \left(1 + (1-p) \frac{z}{H}\right)^n \left(1 - \left(\left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2\right)\right). \quad (14)$$

In the partial penetration condition, displacement functions for symmetric and sloshing modes are derived in similar ways:

$$F(x, y, z) = K \left(1 + \frac{z}{D}\right)^n \left(1 - \left(\left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2\right)\right) \left(1 - 3 \left(\left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2\right)\right) \quad (15)$$

$$F(x, y, z) = K \left(1 + \frac{z}{D}\right)^n \left(\frac{2x}{L(z)}\right) \left(1 - \left(\left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2\right)\right). \quad (16)$$

In the transition penetration condition, as there is no displacement at the solid-liquid interface and the perimeter of the ellipse described by $L_1(z)$ and $W_1(z)$, the displacement function becomes:

$$F(x, y, z) = K \left(1 + (1-p) \frac{z}{H}\right)^n \left(1 - \left(\left(\frac{2x}{L(z)}\right)^2 + \left(\frac{2y}{W(z)}\right)^2\right)\right) \left(1 - \left(\left(\frac{2x}{L_1(z)}\right)^2 + \left(\frac{2y}{W_1(z)}\right)^2\right)\right) \quad (17)$$

From the volume constant condition, the inner pool width on the top surface, $W_1(0)$, is calculated as:

$$W_1(0) = \frac{W(0)}{\sqrt{3-2p^{n+2}}} \quad (18)$$

The ellipse described by $W_1(z)$ and $L_1(z)$ can be determined when the eccentricity, sidewall slope and parameter n are given. When $L_1(z)$ and $W_1(z)$ approach to $L(z)$ and $W(z)$, the displacement function becomes same as that of full penetration. When $L_1(-H)$ and $W_1(-H)$ approach to 0 ($p=0$), the displacement function becomes same as that of partial penetration. This implies that the displacement function of transition penetration has the common feature of full and partial penetration.

Fig. 3 shows topographic plots of the resultant displacement functions on the elliptical free surface for a pool length-to-width ratio of 2. The symmetric modes of full and partial penetration welds are shown in Fig. 3a and 3b, respectively, and the sloshing mode is shown in Fig. 3c.

Fig. 3 Topographical plots of displacement function. (a) full penetration, symmetric mode, (b) partial penetration, symmetric mode, (c) sloshing mode.

2.3 Calculation of Natural Frequency

With the displacement function for the specific weld geometry and oscillation mode, the natural frequency is calculated from Eqn. 6. Non-dimensional natural frequency is introduced to describe the natural frequency in terms of non-dimensional quantities such as eccentricity, sidewall slope and parameter n , independent of the material properties and pool dimensions.

For the full penetration weld pool, non-dimensional natural frequencies, ω^* ($=\omega / \left(\frac{1}{W(0)} \sqrt{\sigma / \rho H} \right)$), of the symmetric and sloshing modes are calculated to give:

$$\omega^* = 2\sqrt{2(2-e^2)(p+p^{-3})}, \quad (19a)$$

$$\omega^* = 4\sqrt{(2n+3)(4-3e^2) \frac{(1+p^{2n})(1-p)}{(1-p^{2n+3})}}, \quad (19b)$$

$$e^2 = 1 - \left(\frac{W(z)}{L(z)} \right)^2, \quad (19c)$$

where e is the eccentricity of the elliptic weld pool surface. For singular cases of the sloshing mode, the natural frequency becomes:

$$\text{if } n = -\frac{3}{2},$$

$$\omega^* = 4\sqrt{(4-3e^2) \frac{(1+p^{-3})(\log p)}{(p-1)}}, \quad (20a)$$

$$\text{if } p=1,$$

$$\omega^* = 4\sqrt{4-3e^2}. \quad (20b)$$

For the partial penetration weld pool, non-dimensional natural frequencies ($=\omega / \left(\frac{1}{W(0)} \sqrt{\sigma / \rho D} \right)$) for the symmetric and sloshing mode are calculated to give:

$$\omega^* = 2\sqrt{15(2-e^2) \frac{(n+1)(2n+3)}{(n+2)}}, \quad (21a)$$

$$\omega^* = 4\sqrt{(4-3e^2) \frac{(n+1)(2n+3)}{(n+2)}}. \quad (21b)$$

For the transition penetration weld pool, non-dimensional natural frequency ($=\omega / \left(\frac{1}{W(0)} \sqrt{\sigma / \rho H} \right)$) of the symmetric mode is calculated to give:

$$\omega^* = \left(\frac{4\sqrt{5}}{I_1} \right) (2p^{2n+4} + p^{2n} - 4p^{n+2} + 3) \sqrt{2+e^2}, \quad (22a)$$

$$I_1 = \frac{1}{W(0)L(0)H} \int_{-H}^0 \left(1 + (1-p) \frac{z}{H} \right)^{2n} L(z)W(z)(i(z)^4 - 5i(z)^2 + 10)dz, \quad (22b)$$

$$i(z) = \frac{L(z)}{L_1(z)} = \frac{W(z)}{W_1(z)}. \quad (22c)$$

The integration is calculated numerically using

the Simpson's rule.

3. Results and Discussion

From the definition of non-dimensional natural frequency, the natural frequency decreases with increase in pool dimension, and the pool width has more effect on the frequency than the plate thickness or pool depth because these are inside the square root. As for the effects of material properties, the natural frequency decreases when the surface tension decreases or mass density increases. Effects of the sidewall slope, eccentricity and parameter n on the natural frequency are discussed for the specific penetration condition.

3.1 Full penetration weld pool

Non-dimensional natural frequencies of symmetric and sloshing modes are shown in Fig. 4 and 5, respectively, with respect to the sidewall slope, eccentricity and parameter n which is not determined explicitly in the sloshing mode. Fig. 4 illustrates the effects of the sidewall slope and pool length-to-width ratio (L/W) on the non-dimensional frequency for pool oscillation in the symmetric mode. Instead of the eccentricity, L/W is used in the plot for simplicity. When L/W increases which is equivalent to increase in eccentricity, the frequency decreases due to mass increase of the weld pool. When the sidewall slope becomes gradual, the frequency increases due to mass decrease and more constraint imposed by the sidewall of the weld pool. With the sidewall slope less than 0.5, the predicted frequency increases rapidly and becomes unreasonably high. In this range, the displacement on the bottom pool surface increases significantly because the area on the bottom pool surface is much smaller than that on the top surface. It produces the high potential energy, and results in

Fig. 4 Natural frequency of full penetration weld in symmetric mode.

high natural frequencies. Therefore, it is speculated that the weld pool may oscillate in the symmetric mode of the transition penetration rather than full penetration when the sidewall slope is less than 0.5 because the depression of the top surface cannot be fully balanced by sag of the bottom surface.

For the sloshing mode, natural frequencies with respect to the sidewall slope, L/W and parameter n are shown in Fig. 5a and 5b with L/W of 1.5 and 2, respectively. As the sloshing mode is used for the fast traveling condition, the circular surface shape ($L/W=1$) is excluded. Fig. 5a shows the large frequency variation with the different values of parameter n , when the sidewall slope is gradual. As the sidewall slope becomes steeper, the frequency decreases and is less affected by the variation of parameter n . The frequencies for the symmetric mode are included in the plot for comparison. In general, frequencies of the slosh-

Fig. 5 Natural frequency of full penetration weld in sloshing mode. (a) $L/W=1.5$, (b) $L/W=2$.

ing mode are higher than those of symmetric mode because the sloshing mode is higher vibration mode. However, the frequency of the symmetric mode becomes higher than that of the sloshing mode when the sidewall slope is less than approximately 0.5 due to small bottom pool surface. In the case of $L/W=2$ as shown in Fig. 5b, general trends are similar to the case of $L/W=1.5$, except lower frequencies due to mass increase from larger eccentricity.

Although parameter n cannot be determined explicitly in the sloshing mode, it can be approximated using the characteristics of the upper bound solution of energy method⁷⁾ such that when the natural frequency calculated using a given parameter n becomes lower than others with other n values, the given parameter n approximates pool oscillation more accurately. Therefore, parameter n can be determined by selecting the value which produces the lowest natural frequency. From Fig. 5, the value of parameter n which gives lowest frequency is approximately 0.5.

3.2 Partial Penetration Weld Pool

Non-dimensional natural frequencies of symmetric and sloshing modes with respect to L/W and parameter n are shown in Fig. 6. In the symmetric mode as shown in Fig. 6a, the frequency decreases with increase in L/W due to mass increase. As parameter n is not determined explicitly, frequencies with parameter n varying between 0.5 and 3 are calculated. Positive values of parameter n are selected because the amplitude should decrease along the pool depth and be zero on the bottom of the weld pool. As the value of parameter n increases, the amplitude decays faster, which results in decrease of the kinetic energy. The natural frequency thus increases with larger value of parameter n . For the sloshing mode, similar trends to the symmetric mode are shown in Fig. 6b. The natural frequency of the sloshing mode is always lower than that of the symmetric mode with fixed L/W and parameter n because the sloshing mode is lower vibration mode than the symmetric mode.

As parameter n becomes smaller, the natural frequency decreases continuously. Therefore, the characteristics of the upper bound solution cannot be applied to determine parameter n in the partial penetration condition. One way to determine parameter n is to use the experimental data. By comparing the experimental frequencies with the calculated frequencies using various n values, the proper value of n can be selected which fits the experimental frequency most closely.

Fig. 6 Natural frequency of partial penetration weld. (a) symmetric mode, (b) sloshing mode.

3.3 Transition Penetration Weld Pool

Non-dimensional natural frequencies of symmetric mode are shown in Fig. 7, with respect to the sidewall slope, L/W and parameter n . The frequency of the full penetration weld oscillating in the symmetric mode is plotted in the figure for comparison. For the stationary weld ($L/W=1$) as shown in Fig. 7a, the natural frequency decreases as the sidewall slope becomes steeper. When the sidewall slope is gradual, less than 0.2, the frequency becomes lower for positive values of parameter n ($n=0.5$). When the sidewall slope becomes steeper, negative values of parameter n give lower natural frequencies than the positive ones. Therefore, the value of parameter n which gives the lowest frequency varies with the sidewall slope. The full penetration frequency becomes lower than the transition penetration

(a)

(b)

Fig. 7 Natural frequency of transition penetration weld in symmetric mode. (a) $L/W = 1$, (b) $L/W = 1.5$.

frequency when the sidewall slope becomes larger than approximately 0.47. It can be speculated that the weld pool may oscillate in transition penetration mode when the sidewall slope is less than 0.47, and that the full penetration oscillation mode may be dominant, or both transition and full penetration oscillation modes may exist when the sidewall slope is greater than 0.47⁸⁾. Effects of the sidewall slope and parameter n with the fixed L/W of 1.5, are shown in Fig. 7b. While general trends are similar to the stationary case, the frequency becomes lower than the stationary case because of mass increase.

Variation of parameter n which gives the minimum natural frequency with respect to the sidewall slope is shown in Fig. 8. As the sidewall slope increases, parameter n changes from the positive to negative value. In the range of the slope greater than 0.5, parameter n approaches

Fig. 8 Variation of parameter n vs. sidewall slope for lowest natural frequency.

rapidly to -2 which is the value of the full penetration condition. This trend appears to be reasonable because transition penetration is the intermediate state between the partial and full penetration conditions.

While the transition from the partial penetration condition can be clearly distinguished by the onset of melt-through to the bottom surface, distinction between the transition and full penetration conditions is imprecise. Two criteria are suggested to identify the boundary between transition and full penetration conditions. The first criterion is that when the bottom pool area becomes larger than the inner area of the elliptic top surface defined by $W_1(0)$ and $L_1(0)$ (Fig. 2e), the top surface will begin to oscillate in the symmetric mode of the full penetration weld. Using this criterion, the sidewall slope is in the range between 0.65 and 0.75 depending on values of parameter n as shown in Fig. 7, where calculation stops if $W_1(0)$ becomes smaller than $W(-H)$. Another criterion is that when the value of parameter n approaches to -2 which is the value in the full penetration condition, the weld pool starts to oscillate in the full penetration mode. From Fig. 8, the sidewall slope is in the range between 0.5 and 0.6. From both criteria, the sidewall slope which can distinguish the transition penetration from full penetration appears to exist approximately between 0.5 and 0.75. It agrees with the fact that natural frequencies of the full penetration symmetric mode become unreasonably high when the sidewall slope is less than 0.5 (Fig. 4), and become lower than those of the transition penetration welds when the sidewall slope is larger than 0.47 (Fig. 7).

3.4 Model Expansion and Limitation

The developed energy model has several advan-

tages over other models such that effects of more complex pool geometries, penetration conditions and oscillation modes can be analyzed. By selecting the higher-order trial function as a displacement function, higher oscillation modes can be analyzed, though it may not be useful in practice because clear pool oscillation signals are not easily detected experimentally at high frequency range⁸⁾. Displacement functions other than polynomial functions used in this work are to be tested for accuracy in predicting the natural frequency. Another advantage is that the equivalent mass and spring constant of the weld pool can be derived from the kinetic and potential energies, which result in a lumped model.

While energy model has some advantages, it also has some shortcomings. First of all, the effect of gravity is ignored in estimating the potential energy because it is small compared with that of surface tension. However, when the pool dimensions becomes larger, the effect of gravity cannot be ignored and should be included in estimating the potential energy. The molten pool is assumed to oscillate only in the z-direction, so that effects of weld pool convection cannot be considered. The most critical shortcoming of energy model is that parameter n is not determined explicitly except the symmetric mode of the full penetration weld. Although the value of parameter n can be determined using the property of the upper bound solution, it cannot be applied to the partial penetration case. Parameter n in the partial penetration condition may be determined from the experimental data, or other types of the displacement function may be chosen.

4. Conclusions

A theoretical model is derived using energy method which can be predict the effects of weld pool conditions on natural frequencies. Results of this work lead to the following conclusions:

- (1) A general model is developed which can be applied to various pool geometries, penetration conditions and oscillation modes by selecting a polynomial as the displacement function to approximate pool oscillation.
- (2) In general, the natural frequency decreases with large eccentricity and steep sidewall slope due to mass increase and less constraint. When the sidewall slope is less than 0.5 for the full

penetration weld oscillating in the symmetric mode, the predicted frequency becomes unreasonably high.

- (3) In the transition penetration condition, natural frequencies decrease continuously as melting progresses, and the corresponding parameter n changes from positive to negative values. The sidewall slope representing the boundary between the transition and full penetration conditions is in the range between 0.5 and 0.75.
- (4) Although parameter n is not determined explicitly in some cases, the characteristics of the upper bound solution and experimental data can be utilized to determine the value of parameter n for specific condition.

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